1

$f(x) = x^2$

$x + \Delta x$

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

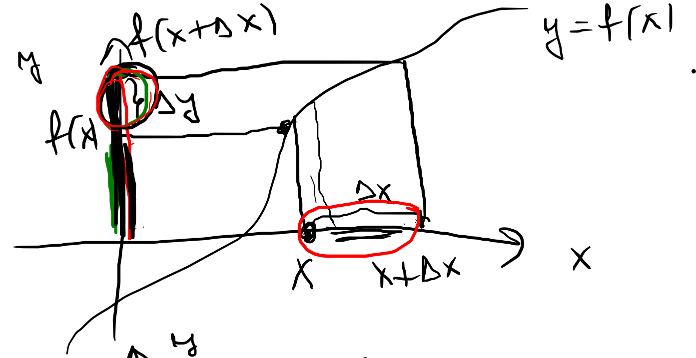
$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 - \cancel{x^2}}{\Delta x}$$

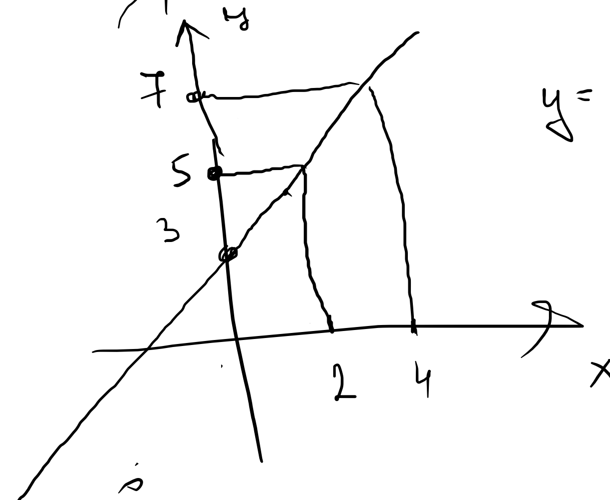
$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} (2x + \cancel{\Delta x})$$

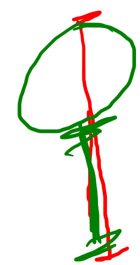
$$= 2x$$

$\Delta y = f(x+\Delta x) - f(x)$





$y = x + 3$



$f(x) = x^3 + 2x$

$f(x+\Delta x) = (x+\Delta x)^3 + 2(x+\Delta x)$

2

$$f(x) = \frac{1}{x+5}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x+5} - \frac{1}{x+5}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{x+5-(x+\Delta x+5)}{(x+\Delta x+5)(x+5)}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{\cancel{x}+\cancel{5}-\cancel{x}-\Delta x-\cancel{5}}{(x+\Delta x+5)(x+5)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\cancel{\Delta x}}{\cancel{\Delta x}(x+\Delta x+5)(x+5)} = \lim_{\Delta x \to 0} \frac{-1}{(x+\underset{\to 0}{\cancel{\Delta x}}+5)(x+5)} =$$

$$= \frac{-1}{(x+5)(x+5)} = -\frac{1}{(x+5)^2}$$

$$\frac{1}{x+5} + \Delta x = f(x) + \Delta x$$

③ $f(x) = \sqrt{5x + x^2} \longrightarrow f'(x) = \dfrac{1}{2\sqrt{5x+x^2}} \cdot (5+2x)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{5(x+\Delta x)+(x+\Delta x)^2} - \sqrt{5x+x^2}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{\overset{A}{\sqrt{5x+5\Delta x+x^2+2x\Delta x+\Delta x^2}} - \overset{B}{\sqrt{5x+x^2}}}{\Delta x} \cdot \frac{\overset{A}{\sqrt{5x+5\Delta x+x^2+2x\Delta x+\Delta x^2}} + \overset{B}{\sqrt{5x+x^2}}}{-\|-}$$

$$= \lim_{\Delta x \to 0} \frac{5\cancel{x}+5\Delta x+\cancel{x^2}+2x\Delta x+\Delta x^2-\cancel{5x}-\cancel{x^2}}{\Delta x\left(\sqrt{5x+5\Delta x+x^2+2x\Delta x+\Delta x^2}+\sqrt{5x+x^2}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(5+2x+\cancel{\Delta x}^{\,0})}{\cancel{\Delta x}\left(\sqrt{5x+\cancel{5\Delta x}^{\,0}+x^2+2x\cancel{\Delta x}^{\,0}+\cancel{\Delta x^2}^{\,0}}+\sqrt{5x+x^2}\right)} =$$

$$= \frac{5+2x}{\sqrt{5x+x^2}+\sqrt{5x+x^2}} = \frac{5+2x}{2\sqrt{5x+x^2}}$$

4

$$f(x)=\begin{cases} b(3+2x)^{x+2}, & x<0 \\ a, & x=0 \\ \left(1+\frac{5x^2}{x+7}\right)^{\frac{1}{x^2}}, & x>0 \end{cases}$$

$X=0$

$a=e^{\frac{5}{7}}=9b$

$a=e^{\frac{5}{7}}$

$b=\frac{1}{9}e^{\frac{5}{7}}$

0

$$\overset{a}{f(0)}=\lim_{x\to 0^+} f(x) \overset{e^{\frac{5}{7}}}{=} \overset{9b}{\lim_{x\to 0^-} f(x)}$$

$f(0)=a$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\left(1+\frac{5x^2}{x+7}\right)^{\frac{1}{x^2}} \overset{1^\infty}{=} \lim_{x\to 0^+}\left(1+\frac{5x^2}{x+7}\right)^{\frac{x+7}{5x^2}\cdot\frac{5\cancel{x^2}}{x+7}\cdot\frac{1}{\cancel{x^2}}}$$

$$=e^{\lim_{x\to 0^+}\frac{5}{x+7}}=e^{\frac{5}{7}}$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-} b(3+2x)^{x+2}=b(3+2\cdot 0)^{0+2}=b\cdot 3^2=9b$$

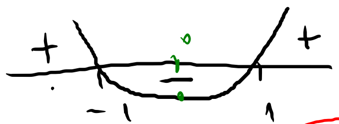
5) $f(x) = x\cdot\sqrt{x^2-1}$

1) $x^2-1 \geq 0$

$x^2-1=0$

$x^2=1$

$x=\pm 1$



$\boxed{D=(-\infty,-1]\cup[1,\infty)}$

2) $y=0$ $\quad x\sqrt{x^2-1}=0$

$\cancel{x=0}$ $\notin D$ $\quad x^2-1=0$

$x=\pm 1$

3) $y>0$ $\quad x\sqrt{x^2-1}>0$ $\quad (\sqrt{x^2-1} \geq 0)$

$x>0$

$y>0,\ x\in(1,\infty)$

$y<0 \quad x\in(-\infty,-1)$

4) $f(-x) = -x\sqrt{(-x)^2-1} = -x\sqrt{x^2-1} = -f(x)$

$\Rightarrow$ NEPARNA

5) V.A. - NEMA

H.A. $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} x\sqrt{x^2-1} = \pm\infty$

NEMA H.A.

K.A. $y=kx+n$

$k=\lim\limits_{x\to\pm\infty}\frac{f(x)}{x} = \lim\limits_{x\to\pm\infty}\frac{\cancel{x}\sqrt{x^2-1}}{\cancel{x}} = +\infty$

NEMA K.A.

6) $y' = \sqrt{x^2-1} + x\frac{1}{2\sqrt{x^2-1}}\cdot 2x$

$= \sqrt{x^2-1} + \frac{x^2}{\sqrt{x^2-1}}$

$= \frac{x^2-1+x^2}{\sqrt{x^2-1}}$

$= \frac{2x^2-1}{\sqrt{x^2-1}}, \quad x\in(-\infty,-1)\cup(1,\infty)$

$y'=0 \quad 2x^2-1=0$

$2x^2=1$

$x^2=\frac{1}{2}$

$x=\pm\frac{1}{\sqrt{2}}$

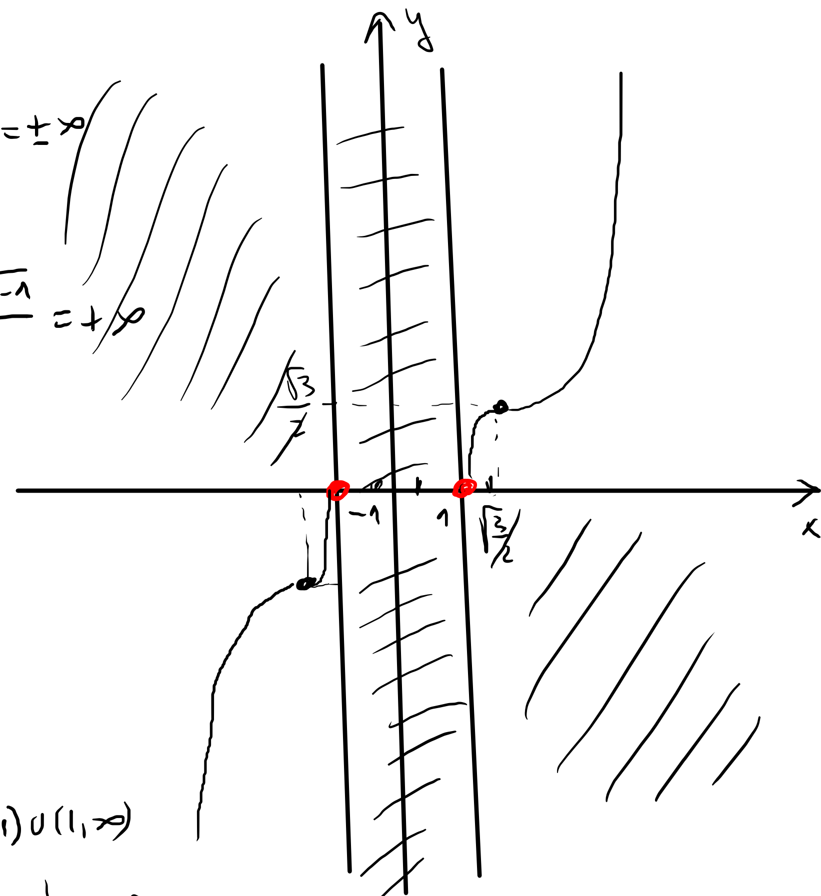
$x=\pm\frac{\sqrt{2}}{2}$

$0<\frac{\sqrt{2}}{2}<1$

$-1<-\frac{\sqrt{2}}{2}<0$

$x=\pm\frac{\sqrt{2}}{2}\notin D$

NEMA E.V.



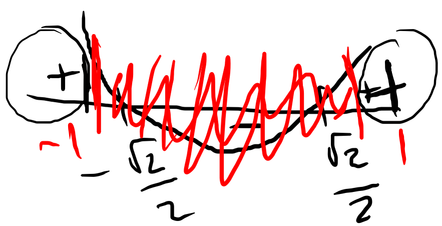
$$y' = \frac{2x^2-1}{\sqrt{x^2-1}}$$

$$y' > 0 \qquad \frac{2x^2-1}{\sqrt{x^2-1}} > 0 \qquad \sqrt{x^2-1} \geq 0$$

$$2x^2-1 > 0$$

$$2x^2-1 = 0$$

$$x = \pm \frac{\sqrt{2}}{2}$$



$y' > 0$, $y \nearrow \ \forall x \in D$

7, $$y'' = \frac{4x\sqrt{x^2-1} - (2x^2-1)\frac{1}{2\sqrt{x^2-1}}\cdot 2x}{x^2-1}$$

$$= \frac{4x\sqrt{x^2-1} - \frac{2x^3-x}{\sqrt{x^2-1}}}{x^2-1}$$

$$= \frac{\left[\frac{4x(x^2-1)-(2x^3-x)}{\sqrt{x^2-1}}\right]}{\left[\frac{x^2-1}{1}\right]}$$

$$= \frac{4x(x^2-1) - x(2x^2-1)}{(x^2-1)\sqrt{x^2-1}}$$

$$= \frac{x(4x^2-4-2x^2+1)}{(x^2-1)\sqrt{x^2-1}}$$

$$= \frac{x(2x^2-3)}{(x^2-1)\sqrt{x^2-1}}$$

$$y'' = \frac{x(2x^2-3)}{(x^2-1)\sqrt{x^2-1}}, \quad x \in (-\infty; -1) \cup (1, \infty)$$

$y'' = 0 \qquad x \cdot (2x^2-3) = 0$

$\cancel{x=0}$ $\notin D \qquad 2x^2 - 3 = 0$

$2x^2 = 3$

$x^2 = \frac{3}{2}$

$x = \pm\sqrt{\frac{3}{2}}$

$y'' > 0 \qquad \frac{x(2x^2-3)}{(x^2-1)\sqrt{x^2-1}} > 0$

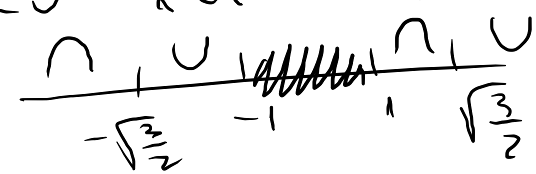
$(x^2-1) > 0 \qquad \sqrt{x^2-1} > 0$

$x(2x^2-3) > 0$

	$-\sqrt{\frac{3}{2}}$		-1	0	1		$\sqrt{\frac{3}{2}}$	
x	−		−	0				+
$2x^2-3$	+	0				−	0	+
y''	−		+			−		+

$y'' > 0 \quad x \in \left(-\sqrt{\frac{3}{2}}, -1\right) \cup \left(\sqrt{\frac{3}{2}}, \infty\right)$ $y \cup$

$y'' < 0 \quad x \in \left(-\infty, -\sqrt{\frac{3}{2}}\right) \cup \left(1, \sqrt{\frac{3}{2}}\right)$ $y \cap$



$x = \sqrt{\frac{3}{2}}$

$y_{PI} = \sqrt{\frac{3}{2}}\sqrt{\frac{3}{2}-1} = \sqrt{\frac{3}{2}} \cdot \sqrt{\frac{1}{2}} = \frac{\sqrt{3}}{2}$

$x = -\sqrt{\frac{3}{2}} \qquad y_{PI} = -\sqrt{\frac{3}{2}}\sqrt{\frac{3}{2}-1} = -\frac{\sqrt{3}}{2}$

$$\lim_{x\to x_0} f(x) = \lim_{x\to x_0} g(x) = 0 \,, \qquad \lim_{x\to x_0} \frac{f'(x)}{g'(x)} = A$$

$$\Rightarrow \lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{f'(x)}{g'(x)} = A$$

$\frac{0}{0}, \frac{\infty}{\infty}$: $$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{f'(x)}{g'(x)}$$

Př: $$\lim_{x\to 0} \frac{x^3+5x}{2x+4x^2} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{(x^3+5x)'}{(2x+4x^2)'} = \lim_{x\to 0} \frac{3x^2+5}{2+8x} = \frac{5}{2}$$

$$\lim_{x\to \infty} \frac{\ln x}{x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to \infty} \frac{(\ln x)'}{(x)'} = \lim_{x\to \infty} \frac{\frac{1}{x}}{1} = \lim_{x\to \infty} \frac{1}{x} = 0$$

$0\cdot\infty$:

$$\lim_{x\to x_0} \cancel{f(x)}^{0}\cdot \cancel{g(x)}^{\infty} = \left\langle \begin{array}{l} \lim_{x\to x_0} \dfrac{f(x)}{\frac{1}{g(x)}} = \frac{0}{0} \quad L.P. \\ \lim_{x\to x_0} \dfrac{g(x)}{\frac{1}{f(x)}} = \frac{\infty}{\infty} \quad L.P. \end{array}\right.$$

$$\lim_{x\to 0} \cancel{\frac{\sin x}{x}} = 1$$

PR :

$$\lim_{x\to 0} x\ln x \overset{0(-\infty)}{=} \lim_{x\to 0} \frac{\ln x}{\frac{1}{x}} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to 0} \frac{(\ln x)'}{\left(\frac{1}{x}\right)'} = \lim_{x\to 0} \frac{\frac{1}{\cancel{x}}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0} (-x) = 0$$

$$\lim_{x\to 0} (\operatorname{tg} x\cdot \ln x) \overset{0(-\infty)}{=} \lim_{x\to 0} \frac{\ln x}{\frac{1}{\operatorname{tg} x}} = \lim_{x\to 0} \frac{\ln x}{\operatorname{ctg} x} \overset{\frac{\infty}{\infty}}{=}$$

$$= \lim_{x\to 0} \frac{(\ln x)'}{(\operatorname{ctg} x)'} = \lim_{x\to 0} \frac{\frac{1}{x}}{-\frac{1}{\sin^2 x}} = \lim_{x\to 0} \frac{-\sin^2 x}{x} \overset{\frac{0}{0}}{=}$$

$$\hookrightarrow \lim_{x\to 0} -\frac{\sin x\cdot \sin x}{x} = 0$$

$$= \lim_{x\to 0} \frac{\left(-(\sin x)^2\right)'}{(x)'} = \lim_{x\to 0} \frac{-2\sin x\cos x}{1} = \frac{-2\cdot 0\cdot 1}{1} = 0$$

$\infty - \infty$: $\lim_{x \to x_0} (\cancel{f(x)}^{\to\infty} - \cancel{g(x)}^{\to\infty}) = \lim_{x \to x_0} \cancel{f(x)}^{\to\infty} \left(1 - \frac{g(x)}{f(x)}\right)^{\to 0}$

PR. $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x}\right) = \lim_{x \to 0^+} \frac{\sin x - x}{x \sin x} \overset{\frac{0}{0}}{=}$

$= \lim_{x \to 0^+} \frac{(\sin x - x)'}{(x \sin x)'} = \lim_{x \to 0^+} \frac{\cos x - 1}{\sin x + x \cos x} \overset{\frac{0}{0}}{=}$

$= \lim_{x \to 0^+} \frac{(\cos x - 1)'}{(\sin x + x \cos x)'} = \lim_{x \to 0^+} \frac{-\sin x}{\cos x + \cos x + x(-\sin x)} = \frac{0}{1+1+0} = 0$

$\lim_{x \to \infty} (\sqrt{x+3} - \sqrt{x+5})^{\infty - \infty} \cdot \frac{\sqrt{x+3} + \sqrt{x+5}}{\sqrt{x+3} + \sqrt{x+5}} = \lim_{x \to \infty} \frac{\cancel{x} + 3 - (\cancel{x} + 5)}{\sqrt{x+3} + \sqrt{x+5}} = \frac{-2}{\infty}$

$= 0$

$$(\sin x + x\cos x)' = (\sin x)' + (x\cdot\cos x)'$$

$$= \cos x + (x)'\cos x + x\cdot(\cos x)'$$

$$= \cos x + \cos x + x(-\sin x)$$

$$= 2\cos x - x\sin x$$

1^{∞} — ISTO SE RADE

$0^0, \infty^0$ — MORAJU L.P.

$$\lim_{x \to x_0} f(x)^{g(x)} = A \quad / \ln$$

$$\ln A = \ln \lim_{x \to x_0} f(x)^{g(x)} = \lim_{x \to x_0} \ln f(x)^{g(x)}$$

$$= \lim_{x \to x_0} g(x) \cdot \ln f(x) = \infty \cdot 0$$

PR:

$$\lim_{x \to 0^+} x^{\frac{2026}{1+\ln x}} \overset{0^0}{=} A \quad / \ln$$

$$0^{\frac{2006}{1+\ln 0^+}} = 0^{\frac{2006}{1-\infty}} = 0^0$$

$\ln A = 2026$

$A = e^{2026}$

$$\ln A = \ln \lim_{x \to 0^+} x^{\frac{2026}{1+\ln x}} = \lim_{x \to 0^+} \ln x^{\frac{2026}{1+\ln x}} = \lim_{x \to 0^+} \left(\frac{2026}{1+\ln x} \cdot \ln x \right) \overset{0 \infty}{=}$$

$$= \lim_{x \to 0^+} \frac{2026 \ln x}{1+\ln x} \overset{\frac{-\infty}{-\infty}}{=} 2026 \lim_{x \to 0^+} \frac{(\ln x)'}{(1+\ln x)'} = 2026 \lim_{x \to 0^+} \frac{\frac{1}{\cancel{x}}}{\frac{1}{\cancel{x}}} = 2026$$

$$\ln A = B \iff A = e^{B}$$

$$\ln A = 2026 \implies A = e^{2026}$$

$$\log_c A = B \iff A = c^{B}$$

$$\left(1+\square\right)^{\frac{1}{\square}}$$

$$\lim_{x\to\frac{\pi}{2}}\left(1+\cos x\right)^{\frac{1}{\cos x}} \overset{1^{\infty}}{=} e$$