① $y = \frac{x^2}{x-2}$

1) $\boxed{D = \mathbb{R} \setminus \{2\} = (-\infty, 2) \cup (2, \infty)}$

$x - 2 \neq 0$

$x \neq 2$

2) $y = 0$ $\quad x^2 = 0$

$x = 0$

3) $y > 0$ $\quad \frac{x^2}{x-2} > 0$ $\quad x^2 \geq 0$

$x - 2 > 0$

$x > 2$

$y > 0, \; x \in (2, \infty)$

$y < 0, \; x \in (-\infty, 2)$

4) NI - NI JER DOMEN NIJE SIMETRIČAN

5) V.A.

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^2}{x-2} = \frac{4}{0^-} = -\infty$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{x^2}{x-2} = \frac{4}{0^+} = +\infty$

$\boxed{x = 2 \text{ V.A.}}$

$\lim_{x \to 2} \frac{x^2}{x-2}$ = NE POSTOJI

H.A.

$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x^2}{x-2} = \pm\infty$

NEMA

K.A. $y = kx + n$

$k = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{\frac{x^2}{x-2}}{\frac{x}{1}} = \lim_{x \to \pm\infty} \frac{x^2}{x(x-2)} = 1$

$n = \lim_{x \to \pm\infty} (f(x) - kx) = \lim_{x \to \pm\infty} \left(\frac{x^2}{x-2} - \frac{x}{1} \right) = \lim_{x \to \pm\infty} \frac{x^2 - x(x-2)}{x-2}$

$= \lim_{x \to \pm\infty} \frac{\cancel{x^2} - \cancel{x^2} + 2x}{x-2} = \lim_{x \to \pm\infty} \frac{2x}{x-2} = 2$

$\boxed{y = x + 2}$ K.A.

$x = 0 \to y = 2$

$x = 2 \to y = 4$

6) $y' = \frac{2x(x-2) - x^2}{(x-2)^2} = \frac{x(2x - 4 - x)}{(x-2)^2} = \frac{x(x-4)}{(x-2)^2} = \frac{x^2 - 4x}{(x-2)^2}$

$x \neq 2$

$y' = 0$ $\quad x(x-4) = 0$

$\boxed{x = 0}$ $\boxed{x = 4}$

STACIONARNE TAČKE

$y' > 0$ $\quad \frac{x(x-4)}{(x-2)^2} > 0$ $\quad (x-2)^2 > 0$ UVEK

$x(x-4) > 0$

	0		4	
x	$-$	0 $+$		$+$
$x-4$		$-$	0 $-$	$+$
	$+$		$-$	$+$

$y' > 0 \quad x \in (-\infty, 0) \cup (4, \infty) \quad y \nearrow$

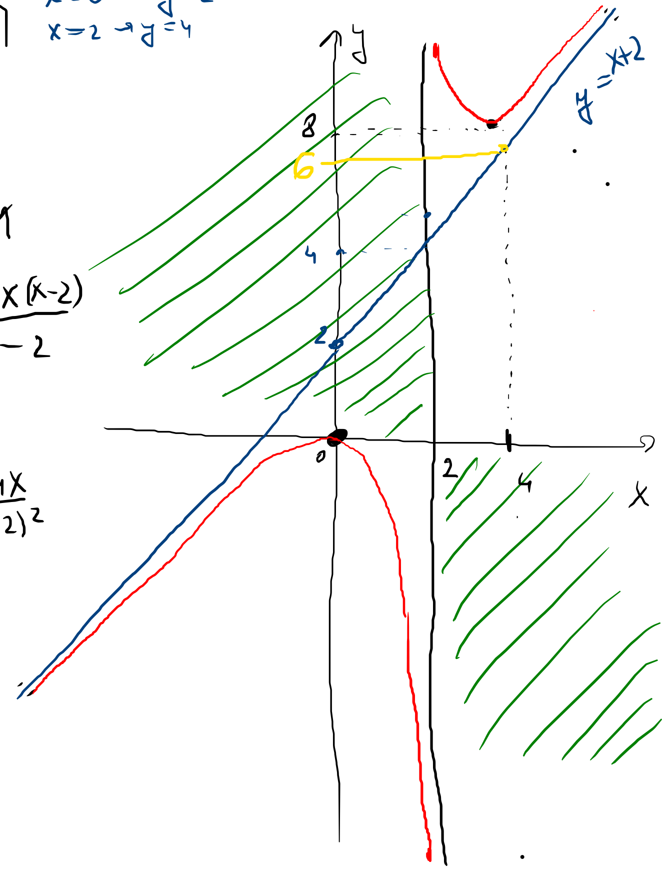
$y' < 0 \quad x \in (0, 2) \cup (2, 4) \quad y \searrow$

$\nearrow \; 0 \; \searrow \; 2 \; \searrow \; 4 \; \nearrow$

lokalni max $\quad$ ∉D $\quad$ lokalni min

$x = 0 \quad y_{MAX} = \frac{0^2}{0-2} = 0$

$x = 4 \quad y_{MIN} = \frac{4^2}{4-2} = \frac{16}{2} = 8$



7)

$$y' = \frac{x^2-4x}{(x-2)^2}$$

$$y'' = \frac{(2x-4)(x-2)^2 - (x^2-4x)\cdot 2(x-2)}{(x-2)^4} = \frac{\cancel{(x-2)}\left((2x-4)(x-2) - 2(x^2-4x)\right)}{(x-2)^{\cancel{4}\,3}} = \frac{\cancel{2x^2} - \cancel{4x} - \cancel{4x} + 8 - \cancel{2x^2} + \cancel{8x}}{(x-2)^3}$$

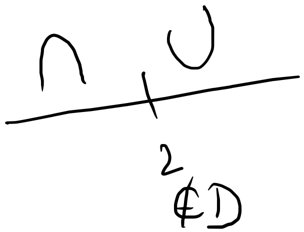
$$= \frac{8}{(x-2)^3}, \quad x \neq 2$$

$y'' \neq 0 \quad \forall x \in D \Rightarrow$ NEMA P.T.

$$y'' > 0 \qquad \frac{8}{(x-2)^3} > 0$$

$$x-2 > 0$$

$$x > 2$$



$y'' > 0$, $y \cup$ $x \in (2, \infty)$

$y'' < 0$ $y \cap$ $x \in (-\infty, 2)$

② $y = \frac{4x}{4-x^2}$

1) $4 - x^2 \neq 0$

$x^2 \neq 4$

$x \neq \pm 2$

$D = \mathbb{R} \setminus \{\pm 2\}$

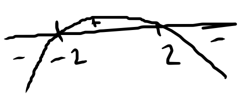
$= (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

2) $y = 0$ $\quad 4x = 0$

$x = 0$

3) $y > 0$ $\quad \frac{4x}{4-x^2} > 0$

		-2	0	2	
0 $\quad 4x$	$-$	$-$	0 $+$	$+$	
± 2 $\quad 4-x^2$	$-$	0 $+$	$+$	0 $-$	
y	$+$	$-$	$+$	$-$	



$y > 0 \quad x \in (-\infty, -2) \cup (0, 2)$

$y < 0 \quad x \in (-2, 0) \cup (2, \infty)$

4) $f(-x) = \frac{4(-x)}{4-(-x)^2} = -\frac{4x}{4-x^2} = -f(x)$

NEPRNA

5) V.A.

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} \frac{4x}{4-x^2} = \frac{8}{0^-} = -\infty$

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} \frac{4x}{4-x^2} = \frac{8}{0^+} = +\infty$

$\boxed{x = 2}$ V.A.

$\boxed{x = -2}$ JE V.A. JER JE FUNKCIJA NEPARNA

H.A.

$\lim\limits_{x \to \pm\infty} f(x) = \lim\limits_{x \to \pm\infty} \frac{4x}{4-x^2} = 0$

$\boxed{y = 0}$ H.A.

K.A. NEMA JER IMA H.A.

6) $y' = \frac{4(4-x^2) - 4x(-2x)}{(4-x^2)^2} = \frac{16 - 4x^2 + 8x^2}{(4-x^2)^2} = \frac{16 + 4x^2}{(4-x^2)^2}$

$x \neq \pm 2$

$y' = 0 \quad 16 + 4x^2 = 0 \quad \perp$

$y' \neq 0 \quad \forall x \in D \Rightarrow$ NEMA E.V.

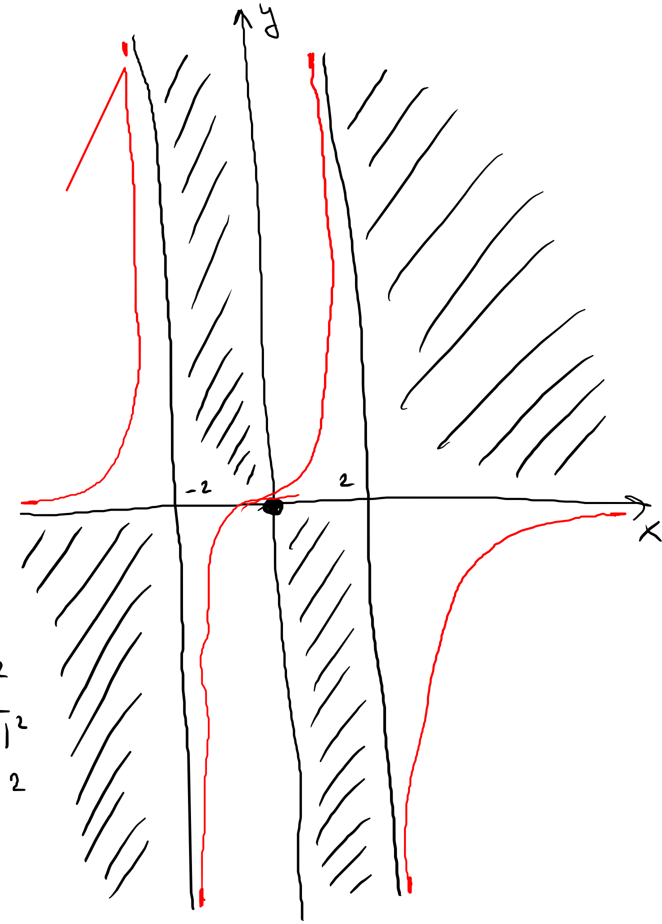
$y' > 0 \quad \frac{16 + 4x^2}{(4-x^2)^2} > 0$

$16 + 4x^2 > 0$

$(4-x^2)^2 > 0$

$y' > 0 \quad \forall x \in D$

$y \nearrow$



7, $y' = \dfrac{16+4x^2}{(4-x^2)^2}$

$$y'' = \frac{8x(4-x^2)^2 - (16+4x^2)\cdot 2(4-x^2)\cdot(-2x)}{(4-x^2)^4} = \frac{x\cancel{(4-x^2)}\left(8(4-x^2)+4(16+4x^2)\right)}{(4-x^2)^{\cancel{4}\,3}}$$

$$= \frac{x(32-8x^2+64+16x^2)}{(4-x^2)^3} = \frac{x(8x^2+96)}{(4-x^2)^3} = \frac{8x(x^2+12)}{(4-x^2)^3} \qquad x \neq \pm 2$$

$y'' = 0$ $\qquad 8x\underbrace{(x^2+12)}_{>0} = 0$

$$\boxed{x = 0}$$

$y'' > 0$ $\qquad \dfrac{8x\overbrace{(x^2+12)}^{>0}}{(4-x^2)^3} > 0$

	-2		0		2		
$8x$	$-$		$-\ 0$	$+$		$+$
$4-x^2$	$-$	0	$+$	$+$	0	$-$
	$+$		$-$	$+$		$-$

$y'' > 0$ $\quad y\ \cup$ $\quad x \in (-\infty, -2) \cup (0, 2)$

$y'' < 0$ $\quad y\ \cap$ $\quad x \in (-2, 0) \cup (2, \infty)$

$x = 0$

$y_{PT} = \dfrac{4\cdot 0}{4-0^2} = 0$

$\cup$		$\cap$		$\cup$		$\cap$
	-2		0		2	
	$\notin D$		PT		$\notin D$	

(3) $y=\dfrac{x^2-1}{(x+2)^2}$

1) $x+2\neq 0$

$x\neq -2$

$\boxed{D=\mathbb{R}\setminus\{-2\} =(-\infty,-2)\cup(-2,\infty)}$

2, $y=0$ $\quad x^2-1=0$

$x^2=1$

$x=\pm 1$

$x=0 \quad y=\dfrac{0^2-1}{(0+2)^2}=\dfrac{-1}{4}$

3) $y>0 \quad \dfrac{x^2-1}{(x+2)^2}>0$

$(x+2)^2>0$

$x^2-1>0$

$y>0 \quad x\in(-\infty,-2)\cup(-2,-1)\cup(1,\infty)$

$y<0 \quad x\in(-1,1)$

4, NI-NI

D-NIJE SIMETRICAN

5, V.A.

$\lim\limits_{x\to -2^+} f(x)=\lim\limits_{x\to -2^+}\dfrac{x^2-1}{(x+2)^2}=\dfrac{3}{0^+}=+\infty$

$\lim\limits_{x\to -2^-} f(x)=\lim\limits_{x\to -2^-}\dfrac{x^2-1}{(x+2)^2}=\dfrac{3}{0^+}=+\infty$

$\boxed{x=-2}$ V.A.

H.A. $\lim\limits_{x\to\pm\infty}\dfrac{x^2-1}{(x+2)^2}=1$

$\boxed{y=1}$ H.A.

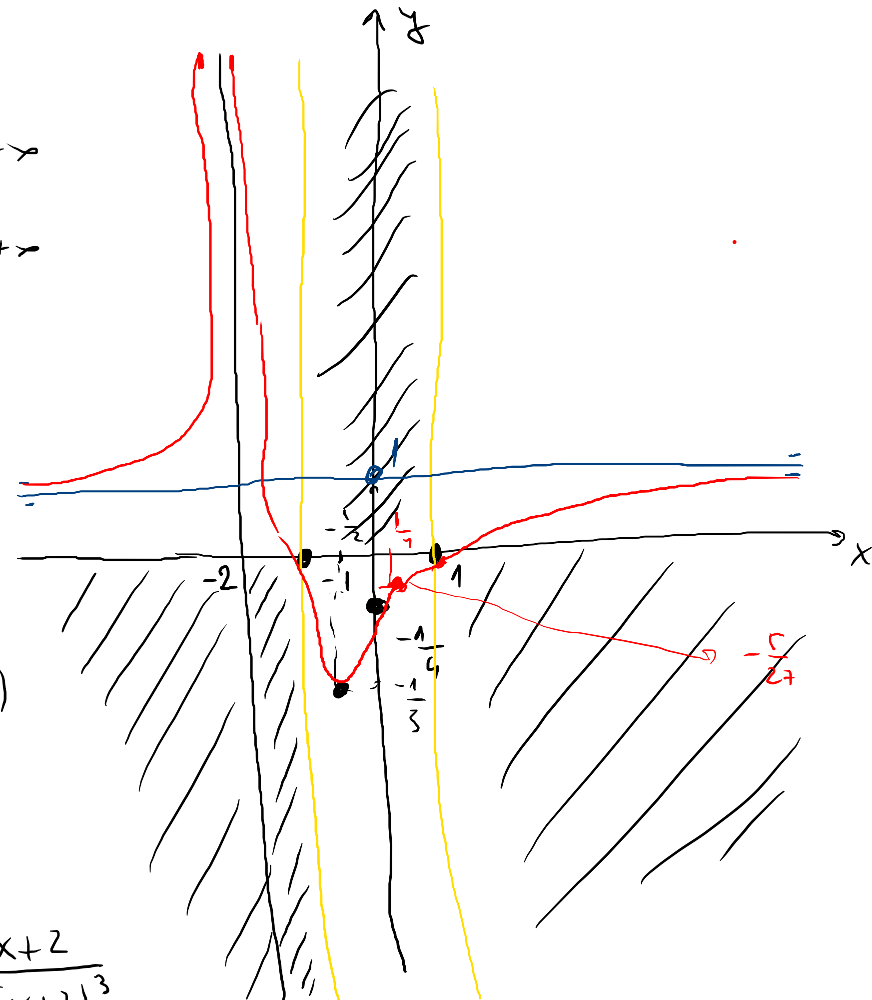
K.A. NEMA

6, $y'=\dfrac{2x(x+2)^2-(x^2-1)2(x+2)}{(x+2)^4}$

$=\dfrac{2(x+2)\left(x(x+2)-(x^2-1)\right)}{(x+2)^{4\,3}}$

$=\dfrac{2(x^2+2x-x^2+1)}{(x+2)^3}$

$=\dfrac{2(2x+1)}{(x+2)^3}=\dfrac{4x+2}{(x+2)^3} \quad x\neq -2$



$y' = \frac{4x+2}{(x+2)^3}$

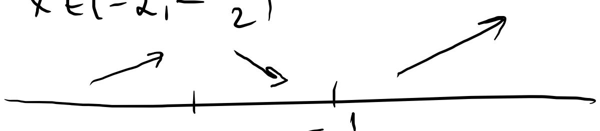
$y' = 0 \quad 4x+2 = 0 \quad \boxed{x = -\frac{1}{2}}$

$y' > 0 \quad \frac{4x+2}{(x+2)^3} > 0$

		-2		$-\frac{1}{2}$	
$-\frac{1}{2}$	$4x+2$	$-$	$-$	0	$+$
-2	$x+2$	$-$	0 $+$		$+$
	y'	$+$	$-$		$+$
	y	$\nearrow$	$\searrow$		$\nearrow$

$y \nearrow \quad x \in (-\infty, -2) \cup (-\frac{1}{2}, \infty)$

$y \searrow \quad x \in (-2, -\frac{1}{2})$



-2 $\notin D$ $\quad -\frac{1}{2}$ min

$x = -\frac{1}{2}$

$y_{MIN} = \frac{(-\frac{1}{2})^2 - 1}{(-\frac{1}{2}+2)^2} = \frac{\frac{1}{4} - 1}{(\frac{3}{2})^2} = \frac{-\cancel{\frac{3}{4}}}{\cancel{\frac{9}{4}}} = -\frac{1}{3}$

$x = \frac{1}{4}$

$y_{PI} = \frac{(\frac{1}{4})^2 - 1}{(\frac{1}{4}+2)^2} = \frac{-\frac{15}{16}}{\frac{81}{16}} = -\frac{5}{27}$

7, $y'' = \frac{4(x+2)^3 - (4x+2)\cdot 3(x+2)^2}{(x+2)^6}$

$= \frac{\cancel{(x+2)^2}(4(x+2) - 3(4x+2))}{(x+2)^{\cancel{6}\,4}} = \frac{4x+8-12x-6}{(x+2)^4}$

$= \frac{2-8x}{(x+2)^4} = \frac{2(1-4x)}{(x+2)^4} \quad x \neq -2$

$y'' = 0 \quad 1-4x = 0$

$4x = 1$

$\boxed{x = \frac{1}{4}}$

$y'' > 0 \quad \frac{2(1-4x)}{(x+2)^4} > 0$ $\quad 2 > 0, \ (x+2)^4 > 0$

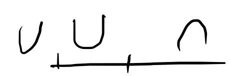
$1-4x > 0$

$4x < 1$

$x < \frac{1}{4}$

$y'' > 0 \quad x \in (-\infty, \frac{1}{4}) \setminus \{-2\} \quad y \cup$

$y'' < 0 \quad x \in (\frac{1}{4}, \infty) \quad y \cap$



$-2 \quad \frac{1}{4}$