$(A, *, \circ)$ - PRSTEN $\quad A \neq \emptyset$

1. $(A, *)$ - ABELOVA GRUPA (ZATVORENOST, KOMUTATIVNOST, ASOCIJATIVNOST, L.N.E I L.I.E)

2. $(A, \circ)$ - POLUGRUPA (ZATVORENOST, ASOCIJATIVNOST)

3. DISTRIBUTIVNOST $\circ$ PREMA $*$

$$\forall x, y, z \in A \quad x \circ (y * z) = (x \circ y) * (x \circ z)$$
$$(y * z) \circ x = (y \circ x) * (z \circ x)$$

$(A, *, \circ)$ - POLJE

1. $(A, *, \circ)$ - PRSTEN

2. $(A \setminus \{e\}, \circ)$ - ABELOVA GRUPA

$\uparrow$ N.E. ZA $*$

$(\mathbb{R},+,\cdot)$ – JE PRSTEN

1, $(\mathbb{R},+)$ – JESTE ABELOVA GRUPA JER JE $(\mathbb{R},+)$ GRUPOID

$+$ ASOCIJATIVNO, KOMUTATIVNO

0 JE N.E., SVAKO $x\in\mathbb{R}$ IMA

ODGOVARAJUĆE $-x\in\mathbb{R}$ $x+(-x)=0$,

TJ. SVAKI ELEMENAT IMA INVERZNI

2, $(\mathbb{R},\cdot)$ – JESTE POLUGRUPA JER JE $(\mathbb{R},\cdot)$ GRUPOID

I $\cdot$ JE ASOCIJATIVNO

3, DISTRIBUTIVNOST $\cdot$ PREMA $+$ VAŽI JER

$$\forall x,y,z\in\mathbb{R},\quad x(y+z)=xy+xz$$

$$(y+z)x=yx+zx$$

$(\mathbb{R},+,\cdot)$ JE POLJE

1, $(\mathbb{R},+,\cdot)$ PRSTEN ✓

2, $(\mathbb{R}\setminus\{0\},\cdot)$ – JESTE ABELOVA GRUPA JER JE

$(\mathbb{R}\setminus\{0\},\cdot)$ GRUPOID

$\cdot$ JE ASOCIJATIVNO, KOMUTATIVNO

1 JE NEUTRALNI ELEMENT

ZA SVAKO $x\in\mathbb{R}\setminus\{0\}$, POSTOJI

$\frac{1}{x}\in\mathbb{R}\setminus\{0\}$, $x\cdot\frac{1}{x}=1$, T.

SVAKI ELEMENAT IMA

INVERZNI

$(\mathbb{N}\cup\{0\},+)$

$([0,\infty),+)$

1. $(\mathbb{Q},+)$ ✓
2. $(\mathbb{Q},\cdot)$ ✓
3. DIST.

$x\cdot(y+z)=xy+xz$

1. $(\mathbb{Z},+)$
2. $(\mathbb{Z},\cdot)$
3. DIST.

$(\{5,0,-5\},+)$

	PRSTEN	POLJE	KOMUTATIVAN PRSTEN	PRSTEN SA JEDINICOM
$(\mathbb{N}\cup\{0\},+,\cdot)$	X —	—	—	—
$(\mathbb{Q},+,\cdot)$	+	+	+	+
$(\mathbb{Z},+,\cdot)$	+	—	+	+
$([0,\infty),+,\cdot)$	X —	—	—	—
$(\{5,0,-5\},+,\cdot)$	—	—	—	—
$(\mathbb{Q},\cdot,+)$	—	—	—	—

$-(\mathbb{Q}\setminus\{0\},\cdot)$

$x\cdot\boxed{\frac{1}{x}}=1$

$+(\mathbb{Z}\setminus\{0\},\cdot)$

$2\cdot\boxed{\frac{1}{2}}=1$

$\notin \mathbb{Z}\setminus\{0\}$

↳ $x+(yz)=(x+y)(x+z)$

$2+(3\cdot 4)\neq(2+3)\cdot(2+4)$

$14 = 5\cdot 6$

$(\mathbb{Q},\cdot,+)\neq(\mathbb{Q},+,\cdot)$

$(\mathbb{Q}, +, \cdot)$ JESTE POLJE $\Rightarrow$ JESTE DOMEN INTEGRITETA

$(\mathbb{Z}, +, \cdot)$ JESTE KOMUTATIVAN PRSTEN SA JEDINICOM I NEMA DELITELJA NULE

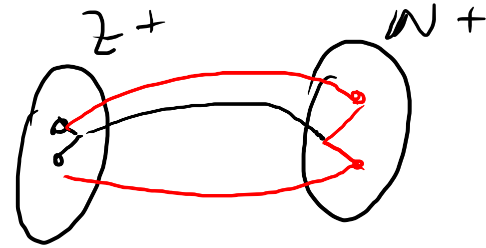
$$\forall x, y \in \mathbb{Z}, \quad x \cdot y = 0 \Rightarrow x = 0 \lor y = 0$$

$\Rightarrow$ JESTE DOMEN INTEGRITETA

$$\underline{\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}} \cdot \underline{\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}} = \underline{\underline{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}}$$

$(\mathbb{N}, +)$, $(\mathbb{Z}, +)$

GRUPOID GRUPOID



$$f : \mathbb{Z} \longrightarrow \mathbb{N}, \qquad f(x) = |x|$$

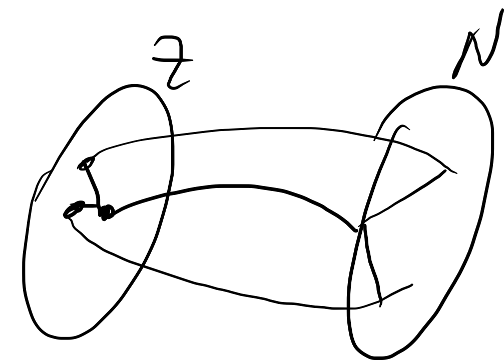
DA LI JE f HOMOMORFIZAM GRUPOIDA $(\mathbb{N}, +)$ U GRUPOID $(\mathbb{Z}, +)$?

$$\forall x, y \in \mathbb{Z}, \quad f(x+y) = f(x) + f(y) \; ?$$

$$\left.\begin{aligned} f(x+y) &= |x+y| \\ f(x) &= |x| \\ f(y) &= |y| \end{aligned}\right\} \quad \begin{aligned} &|x+y| = |x| + |y| \\ &x = 3, \; y = -5 \\ &|3-5| = |3| + |-5| \\ &\;\;\; 2 \;\;= 3 + 5 \end{aligned}$$

$(\mathbb{N}, \cdot)$ GRUPOID

$(\mathbb{Z}, \cdot)$ GRUPOID



$f: \mathbb{Z} \to \mathbb{N}$, $\quad f(x) = |x|$

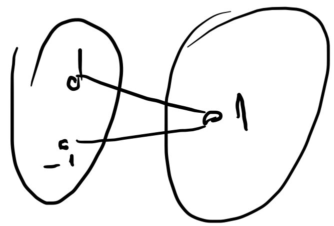
$\forall x, y \in \mathbb{Z}$, $\quad f(x \cdot y) = f(x) \cdot f(y)$

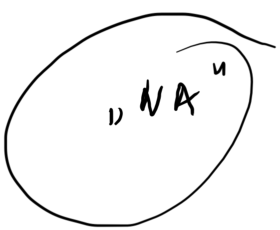
$$f(x \cdot y) = |x \cdot y| = |x| \cdot |y| = f(x) \cdot f(y) \quad ✓$$

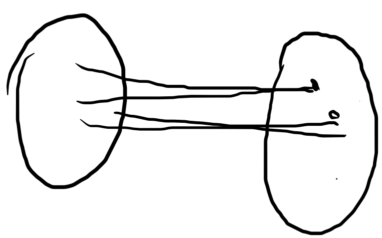
~~„1-1"~~

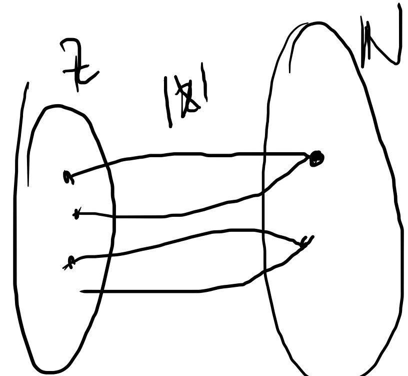
$f(1) = 1$

$f(-1) = 1$

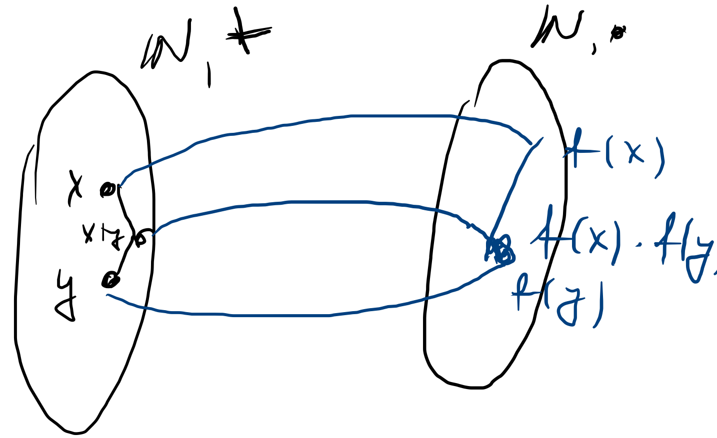








$(\mathbb{N}, \cdot)$ $\quad$ $(\mathbb{N}, +)$



$f: \mathbb{N} \to \mathbb{N}$

$f(x) = 2^x$

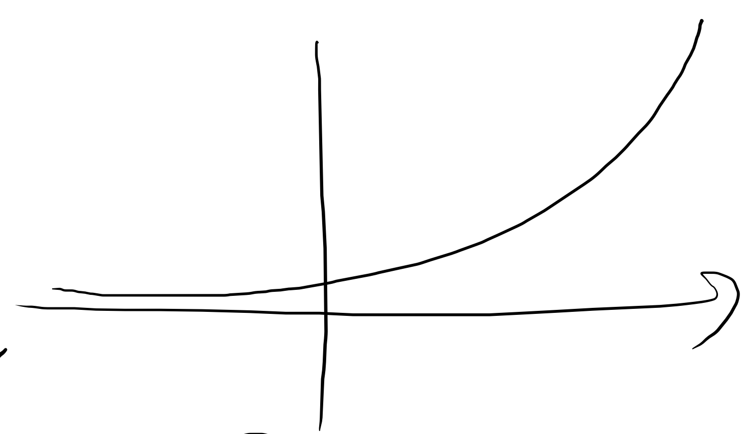
$(\mathbb{N}, +) \to (\mathbb{N}, \cdot)$

$\forall x, y \in \mathbb{N} \quad f(x+y) = f(x) \cdot f(y) \quad ?$

$$\left.\begin{array}{l} f(x+y) = 2^{x+y} \\ f(x) = 2^x \\ f(y) = 2^y \end{array}\right\} \quad 2^{x+y} = 2^x \cdot 2^y$$

$a^x \cdot a^y$

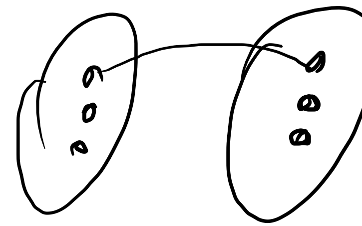
$f: \mathbb{N} \to \mathbb{N}, \quad f(x) = 2^x$



"1-1" ✓ $\quad f(x) = f(y) \Rightarrow 2^x = 2^y$

$\Rightarrow x = y$ ✓

"NA" ✓ $\quad \forall y \in \mathbb{N}, \exists x \in \mathbb{N}, \quad f(x) = y$?



$f(x) = y$

$$\begin{cases} 2^x = y \quad \Big/ \log_2 \\ \log_2 2^x = \log_2 y \end{cases}$$

$x = \log_2 y$

c

$$(z+3i)^2 \cdot (2-i) + \overline{4+5i} = 3+6i$$

NE KVADRIRATI!

$$(z+3i)^2 \cdot (2-i) + \underbrace{4-5i} = 3+6i$$

$$(z+3i)^2 (2-i) = 3+6i-4+5i$$

$$(z+3i)^2 \cdot (2-i) = -1+11i$$

$$(z+3i)^2 = \frac{-1+11i}{2-i} \cdot \frac{2+i}{2+i}$$

$$(z+3i)^2 = \frac{-2-i+22i-11}{4+1}$$

$$(z+3i)^2 = \frac{-13+21i}{5}$$

$$z+3i = \sqrt{-\frac{13}{5}+\frac{21}{5}i}$$

$$z = -3i + \sqrt{-\frac{13}{5}+\frac{21}{5}i}$$

$f(x) = 2x + 3$ $\qquad f : \mathbb{R} \to \mathbb{R}$

$g(x) = x^2 - x - 2$ $\qquad g : \mathbb{R} \to \mathbb{R}$

$f \circ f(x) = f(f(x)) = f(2x+3) = 2(2x+3) + 3$

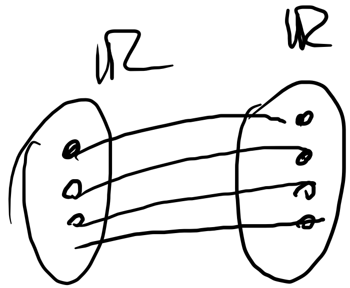
$f \circ g(x) = f(g(x)) = f(x^2 - x - 2) = 2(x^2 - x - 2) + 3$

$g \circ f(x) = g(f(x)) = g(2x+3) = (2x+3)^2 - (2x+3) - 2$

$g \circ g(x) = g(g(x)) = g(x^2 - x - 2) = (x^2 - x - 2)^2 - (x^2 - x - 2) - 2$

$f(x) = 2x+3$ $\quad f:\mathbb{R}\to\mathbb{R}$

„1-1": $f(x)=f(y) \Rightarrow 2x+\cancel{3} = 2y+\cancel{3} \Rightarrow x=y$ ✓



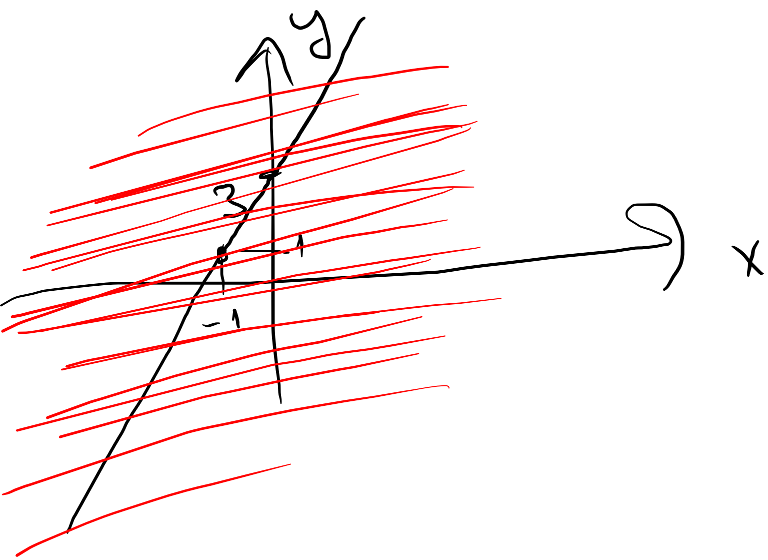
„NA" $\quad \forall y\in\mathbb{R}, \exists x\in\mathbb{R}, f(x)=y$?

$f(x)=y$

$2x+3=y$

$2x=y-3$

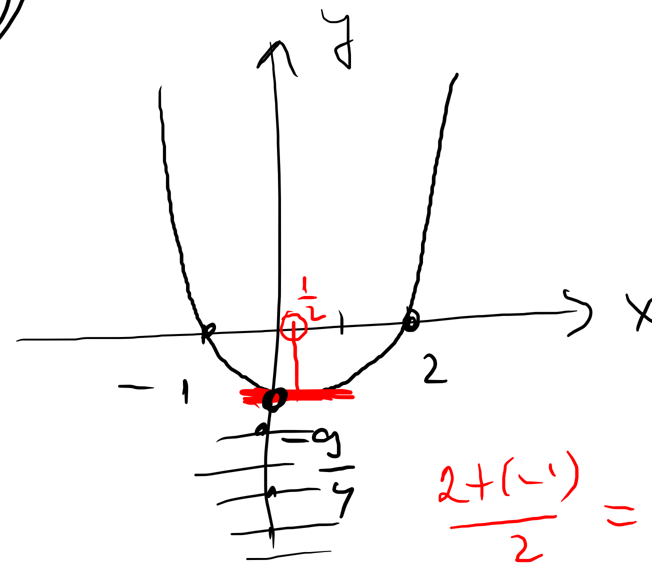
$x=\dfrac{y-3}{2}$ ✓



$g(x) = x^2 - x - 2$ $\qquad g: \mathbb{R} \to \mathbb{R}$

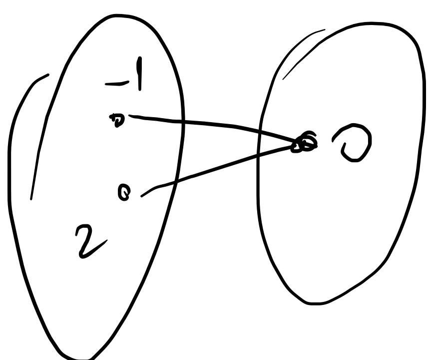
$x^2 - x - 2 = 0$

$x_{1,2} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \begin{cases} 2 \\ -1 \end{cases}$



$\frac{2+(-1)}{2} = \frac{1}{2}$

$\left.\begin{array}{l} g(2) = 0 \\ g(-1) = 0 \end{array}\right\} g(2) = g(-1)$

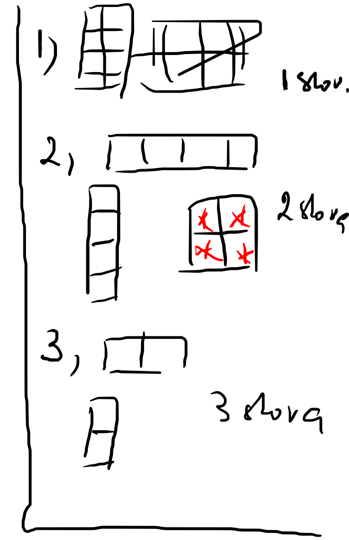


NIJE „1-1"

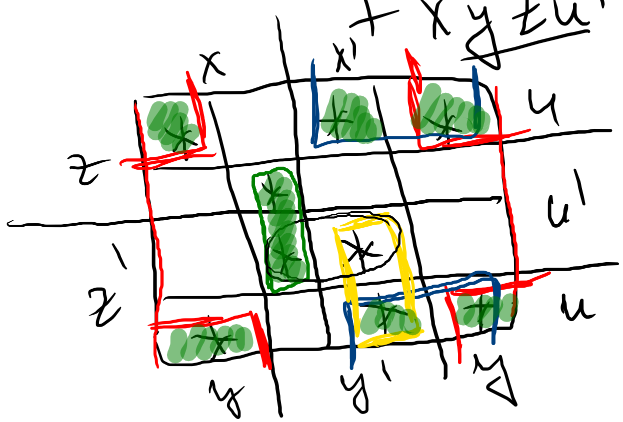
$g\left(\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{2} - 2 = \frac{1-2-8}{4} = -\frac{9}{4}$

NIJE „NA"

x	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
y	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
z	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
u	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$f(x,y,z,u)$	1	1	0	1	0	1	0	1	1	0	1	0	0	1	0	1



$$SDNF(f(x,y,z,u)) = x'y'z'u' + x'y'z'u + x'yzu + x'yz'u + x'yzu + xy'z'u' + xy'zu' + xyz'u + xyzu$$



PI: yu, $x'u$, $xy'u'$, $x'y'z'$, $y'z'u'$

$$MDNF_1(f(x,y,z,u)) = yu + x'u + xy'u' + x'y'z'$$

$$MDNF_2(f(x,y,z,u)) = yu + x'u + xy'u' + y'z'u'$$

$(G, *)$ – JE GRUPOID , $G \neq \emptyset$ $\quad * : G^2 \to G$

$G \neq \emptyset$

$\forall x, y \in G, \ x * y \in G \to$ ZATVORENOST

1) ASOCIJATIVNOST $\quad \forall x, y, z \in G, \ (x * y) * z = x * (y * z)$

2) KOMUTATIVNOST $\quad \forall x, y \in G, \ x * y = y * x$

3) IDEMPOTENTNOST $\quad \forall x \in G, \ x * x = x$

4) L.N.E. $\ \exists e \in G, \forall x \in G, \ e * x = x$, D.N.E. $\exists e \in G, \forall x \in G \ x * e = x$

5) I.N MORA $\underline{\underline{\exists \text{ N.E.}}}$ $\quad$ L.I.E. $\ \forall x \in G, \exists x' \in G \quad x' * x = e$

D.I.E. $\ \forall x \in G, \exists x' \in G \quad x * x' = e$

6) NILPOTENTNI $\quad \exists \theta \in G, \forall x \in G, \ \theta * x = \theta$

7) KANCELACIJA $\quad x * \theta = \theta$

$\forall x, y, z \in G \quad \cancel{x} * y = \cancel{x} * z \Rightarrow y = z$

$y * x = z * x \Rightarrow y = z$

$A = \{a, b, c, d\}$

$*$	a	b	c	d
a	a	c	d	b
b	c	b	a	d
c	d	a	c	b
d	b	d	b	d

$$\left.\begin{array}{l} d * a = b \\ d * c = b \end{array}\right\} \quad \cancel{d} * a = \cancel{d} * c$$

$$a \neq c$$

$*$	a	b	c	d
a	d	b	c	a
b	b	$\underline{b}$	$\underline{b}$	b
c	c	b	a	c
d	a	b	c	d

$$\left.\begin{array}{l} b * b = b \\ b * c = b \end{array}\right\} \quad \cancel{b} * b = \cancel{b} * c$$

$$b \neq c$$

$$\sqrt{-\frac{13}{5}+\frac{21}{5}i} = x+yi \;\Big/^2 \qquad x, y \in \mathbb{R}$$

$$-\frac{13}{5}+\frac{21}{5}i = x^2+2xyi-y^2$$

$$-\frac{13}{5}+\frac{21}{5}i = (x^2-y^2)+2xyi$$

$$x^2-y^2=-\frac{13}{5}$$

$$2xy=\frac{21}{5} \Longrightarrow y=\frac{21}{10x}$$

$$x^2-\frac{21^2}{100x^2}=-\frac{13}{5} \quad \Big/\cdot 100x^2$$

$$100x^4+1300x^2-21^2=0$$

$$\sqrt{3-4i} = x+yi \;/^2, \quad x, y \in \mathbb{R}$$

$$3-4i = (x^2-y^2) + 2xyi$$

$$x^2-y^2=3$$

$$2xy=-4 \Rightarrow y=-\frac{2}{x}$$

$$x^2-\frac{4}{x^2}=3 \;/\cdot x^2$$

$$x^4-x^2-4=0$$

$$x^2=t \qquad t^2-3t-4=0$$

$$t_{1,2}=\frac{3\pm\sqrt{9+16}}{2}=\frac{3\pm5}{2} \;\begin{cases} 4 \\ -1 \end{cases}$$

$$x^2=4$$

$$x_1=2$$

$$x_2=-2$$

$$y_1=-1$$

$$y_2=1$$

$$x^2=-1$$

$$\cancel{x=i}$$

$$\cancel{x=-i}$$

$$\sqrt{3-4i} \;\begin{cases} 2-i \\ -2+i \end{cases}$$