$\vec{AB} = \vec{AO} + \vec{OB}$

$\alpha \vec{a}$

$\underbrace{\alpha \in \mathbb{R}, \quad \alpha \neq 0, \quad \vec{a} \neq 0}$ $\quad (\alpha = 0 \lor a = 0 \Rightarrow \alpha a = 0)$

PROIZVOD SKALARA I VEKTORA JE VEKTOR

1) $\alpha\vec{a}$ IMA ISTI PRAVAC KAO $\vec{a}$

2) SMER $\alpha\vec{a}$ ZAVISI OD α
- $\alpha > 0$ ISTI KAO $\vec{a}$
- $\alpha < 0$ SUPROTAN OD $\vec{a}$

3) $|\alpha\vec{a}| = |\alpha|\,|\vec{a}|$

$\vec{a}, \alpha\vec{a}$

$\vec{a}, \vec{b} \Rightarrow \vec{b} = \alpha\vec{a}$

SKALARNI PROIZVOD

REZULTAT JE SKALAR

$\vec{a} \cdot \vec{b}$, $\vec{a}, \vec{b} \neq 0$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cdot \cos\angle(\vec{a}, \vec{b})$$

1, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

2, $\vec{a} \cdot \vec{a} = |\vec{a}|^2$

3, $\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0$

4, $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

5, $\alpha (\vec{a} \cdot \vec{b}) = (\alpha \vec{a}) \vec{b} = \vec{a} (\alpha \vec{b})$

$\alpha \in \mathbb{R}$

$\vec{a} = 0 \vee \vec{b} = 0 \Rightarrow \vec{a} \cdot \vec{b} = 0$

$\vec{a} \times \vec{b} = 0$

VEKTORSKI PROIZVOD

REZULTAT JE VEKTOR

$\vec{a} \times \vec{b}$, $\vec{a}, \vec{b} \neq 0$

1, $\vec{a} \times \vec{b} \perp \vec{a} \wedge \vec{a} \times \vec{b} \perp \vec{b}$

2, $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin \angle(\vec{a}, \vec{b})$

3, SMER DESNE RUKE

1, $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

2, $\vec{a} \times \vec{a} = 0$

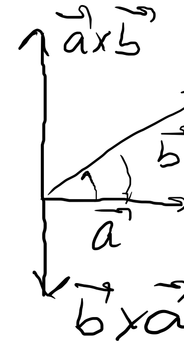
3, $\vec{a} \parallel \vec{b} \Leftrightarrow \vec{a} \times \vec{b} = 0$

4, $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

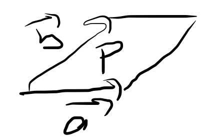
5, $\alpha (\vec{a} \times \vec{b}) = (\alpha \vec{a}) \times \vec{b}$

$= \vec{a} \times (\alpha \vec{b})$

$\alpha \in \mathbb{R}$



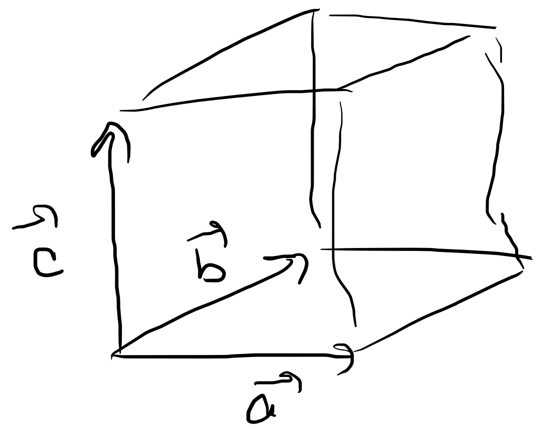
6, $P_{\square} = |\vec{a} \times \vec{b}|$



MEŠOVITI PROIZVOD

$$\vec{a} \cdot (\overbrace{\vec{b} \times \vec{c}}^{\vec{d}}) \longrightarrow \text{REZULTAT JE SKALAR}$$

1. $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{c}(\vec{a} \times \vec{b}) = \vec{b}(\vec{c} \times \vec{a})$

2. 

$$V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$

3. 

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$$

KOMPLANARNI

① $\vec{a} = 2\vec{m} + 3\vec{n}$ $\qquad |\vec{m}| = 2$ $\qquad \angle(\vec{m}, \vec{n}) = \frac{\pi}{4}$

$\vec{b} = -\vec{m} + 4\vec{n}$ $\qquad |\vec{n}| = 1$

1) $\vec{a} \cdot \vec{b}$

2) $\vec{a} \times \vec{b}$, $|\vec{a} \times \vec{b}|$

3) $|\vec{a}|$

4) $|\vec{b}|$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \angle(\vec{a}, \vec{b})$

5) $\angle(\vec{a}, \vec{b})$?

1) $\vec{a} \cdot \vec{b} = (2\vec{m} + 3\vec{n}) \cdot (-\vec{m} + 4\vec{n})$

$= -2\vec{m} \cdot \vec{m} + 8\vec{m} \cdot \vec{n} - 3\vec{n} \cdot \vec{m} + 12\vec{n} \cdot \vec{n}$

$= -2|\vec{m}|^2 + 5\vec{m} \cdot \vec{n} + 12|\vec{n}|^2$

$= -2 \cdot 4 + 5|\vec{m}| \cdot |\vec{n}| \cdot \cos \angle(\vec{m}, \vec{n}) + 12 \cdot 1$

$= -8 + 5 \cdot 2 \cdot 1 \cdot \cos \frac{\pi}{4} + 12$

$= 4 + \cancel{10}^{5} \cdot \frac{\sqrt{2}}{\cancel{2}} = 4 + 5\sqrt{2}$

2. $\vec{a} \times \vec{b} = (2\vec{m} + 3\vec{n}) \times (-\vec{m} + 4\vec{n})$

$= \cancel{-2\vec{m} \times \vec{m}}^{0} + 8\vec{m} \times \vec{n} - 3\vec{n} \times \vec{m} + \cancel{12\vec{n} \times \vec{n}}^{0}$

$= 8\vec{m} \times \vec{n} + 3\vec{m} \times \vec{n}$

$= 11\vec{m} \times \vec{n}$

$\rightarrow \quad |\vec{a} \times \vec{b}| = |11\vec{m} \times \vec{n}| = 11|\vec{m} \times \vec{n}| = 11|\vec{m}||\vec{n}| \cdot \sin\angle(\vec{m}, \vec{n})$

$= 11 \cdot 2 \cdot 1 \sin\frac{\pi}{4} = 22 \cdot \frac{\sqrt{2}}{2} = 11\sqrt{2}$

3. $|\vec{a}|^2 = \vec{a} \cdot \vec{a} = (2\vec{m} + 3\vec{n}) \cdot (2\vec{m} + 3\vec{n})$

$= 4\vec{m} \cdot \vec{m} + 6\vec{m} \cdot \vec{n} + 6\vec{n} \cdot \vec{m} + 9\vec{n} \cdot \vec{n}$

$= 4|\vec{m}|^2 + 12\vec{m} \cdot \vec{n} + 9|\vec{n}|^2$

$= 4 \cdot 4 + 12|\vec{m}||\vec{n}| \cdot \cos\angle(\vec{m}, \vec{n}) + 9 \cdot 1$

$= 16 + 12 \cdot 2 \cdot 1 \cdot \cos\frac{\pi}{4} + 9 = 25 + 12\sqrt{2}$

$|\vec{a}|^2 = 25 + 12\sqrt{2}$

$|\vec{a}| = \sqrt{25 + 12\sqrt{2}}$

4) $|\vec{b}|^2 = \vec{b} \cdot \vec{b} = (-\vec{m} + 4\vec{n}) \cdot (-\vec{m} + 4\vec{n})$

$= \vec{m} \cdot \vec{m} - 4\vec{m} \cdot \vec{n} - 4\vec{n} \cdot \vec{m} + 16\,\vec{n} \cdot \vec{n}$

$= |\vec{m}|^2 - 8\,\underline{\vec{m} \cdot \vec{n}} + 16\,|\vec{n}|^2$

$= 4 - 8\,|\vec{m}| \cdot |\vec{n}| \cdot \cos\sphericalangle(\vec{m}, \vec{n}) + 16$

$= 4 - 8 \cdot \cancel{2} \cdot 1 \cdot \frac{\sqrt{2}}{\cancel{2}} + 16$

$= 20 - 8\sqrt{2}$

$|\vec{b}|^2 = 20 - 8\sqrt{2} \implies |\vec{b}| = \sqrt{20 - 8\sqrt{2}}$

5) $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\sphericalangle(\vec{a}, \vec{b})$

$\underline{\underline{\cos\sphericalangle(\vec{a}, \vec{b})}} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{4 + 5\sqrt{2}}{\sqrt{25 + 12\sqrt{2}} \cdot \sqrt{20 - 8\sqrt{2}}}$

(2)

$\vec{p} = 2\vec{a} + 3\vec{b}$
$\vec{q} = k\,\vec{a} - 4\vec{b}$

$|\vec{a}| = |\vec{b}| = 1$
$\sphericalangle(\vec{a}, \vec{b}) = \frac{\pi}{4}$

$\left.\begin{array}{l} \vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\sphericalangle(\vec{a},\vec{b}) \\ = 1\cdot 1\cdot\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} \end{array}\right\}$

1, $k = ?$ $\vec{p}$ i $\vec{q}$ BUDU NORMALNI
2, $k = ?$ $\vec{p}$ i $\vec{q}$ BUDU KOLINEARNI, AKO $\vec{a}$ i $\vec{b}$ NISU KOLINEARNI

1, $\vec{p} \perp \vec{q} \iff \vec{p}\cdot\vec{q} = 0$

$\iff (2\vec{a} + 3\vec{b})(k\vec{a} - 4\vec{b}) = 0$

$\iff 2k\vec{a}\cdot\vec{a} - 8\vec{a}\,\vec{b} + 3k\,\vec{a}\,\vec{b} - 12\vec{b}\,\vec{b} = 0$

$\iff 2k|\vec{a}|^2 - 8\cdot\frac{\sqrt{2}}{2} + 3k\frac{\sqrt{2}}{2} - 12|\vec{b}|^2 = 0$

$\iff 2k - 4\sqrt{2} + \frac{3\sqrt{2}}{2}k - 12 = 0$

$\iff \left(2 + \frac{3\sqrt{2}}{2}\right)k = 12 + 4\sqrt{2}$

$k = \dfrac{12 + 4\sqrt{2}}{2 + \frac{3\sqrt{2}}{2}}$

2. $p \parallel q \iff \vec{p} \times \vec{q} = 0$

$\iff (2\vec{a} + 3\vec{b}) \times (k\vec{a} - 4\vec{b}) = 0$

$\iff 2k \underbrace{\cancel{\vec{a} \times \vec{a}}}_{0} - 8\,\vec{a} \times \vec{b} + 3k\,\vec{b} \times \vec{a} - 12 \underbrace{\cancel{\vec{b} \times \vec{b}}}_{0} = 0$

$\iff -8\,\vec{a} \times \vec{b} - 3k\,\vec{a} \times \vec{b} = 0$

$\iff (-8 - 3k)\,\vec{a} \times \vec{b} = 0$

$\iff -8 - 3k = 0 \quad \vee \quad \cancel{\vec{a} \times \vec{b} = 0}$

$\iff k = -\dfrac{8}{3}$

$$\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k} = (a_1, a_2, a_3)$$
$$\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k} = (b_1, b_2, b_3)$$

$$\vec{a} \pm \vec{b} = (a_1, a_2, a_3) \pm (b_1, b_2, b_3) = (a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3)$$

$$\alpha\vec{a} = \alpha(a_1, a_2, a_3) = (\alpha a_1, \alpha a_2, \alpha a_3)$$

$$\vec{a} \cdot \vec{b} = (a_1, a_2, a_3) \cdot (b_1, b_2, b_3) = \cancel{(a_1 b_1, a_2 b_2, a_3 b_3)}$$
$$= a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$|\vec{a}| = \sqrt{\vec{a} \cdot \vec{a}} = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}$$
$$= (a_2 b_3 - a_3 b_2)\vec{i} - (a_1 b_3 - a_3 b_1)\vec{j} + (a_1 b_2 - b_1 a_2)\vec{k}$$

$$P = |\vec{a} \times \vec{b}|$$

$$V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$

$\vec{a} = (1, 2, 3)$

$\vec{b} = (-2, 4, k)$

$\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0$

$(1, 2, 3) \cdot (-2, 4, k) = 0$

$-2 + 8 + 3k = 0$

$3k = -6$

$k = -2$

$$\vec{a} = 3\vec{i} - 4\vec{k} = (3, 0, -4)$$

$$\vec{b} = 2\vec{j} + 5\vec{k} = (0, 2, 5)$$

$$2\vec{a} + 3\vec{b} = 2(3,0,-4) + 3(0,2,5) = (6,0,-8) + (0,6,15) = (6,6,7)$$

$$-\vec{a} - 2\vec{b} = -(3,0,-4) - 2(0,2,5) = (-3,0,4) + (0,-4,-10) = (-3,-4,-6)$$

$$\vec{a} \cdot \vec{b} = (3,0,-4) \cdot (0,2,5) = 0 + 0 + (-20) = -20$$

$$|\vec{a}| = \sqrt{3^2 + 0^2 + (-4)^2} = \sqrt{9+16} = 5$$

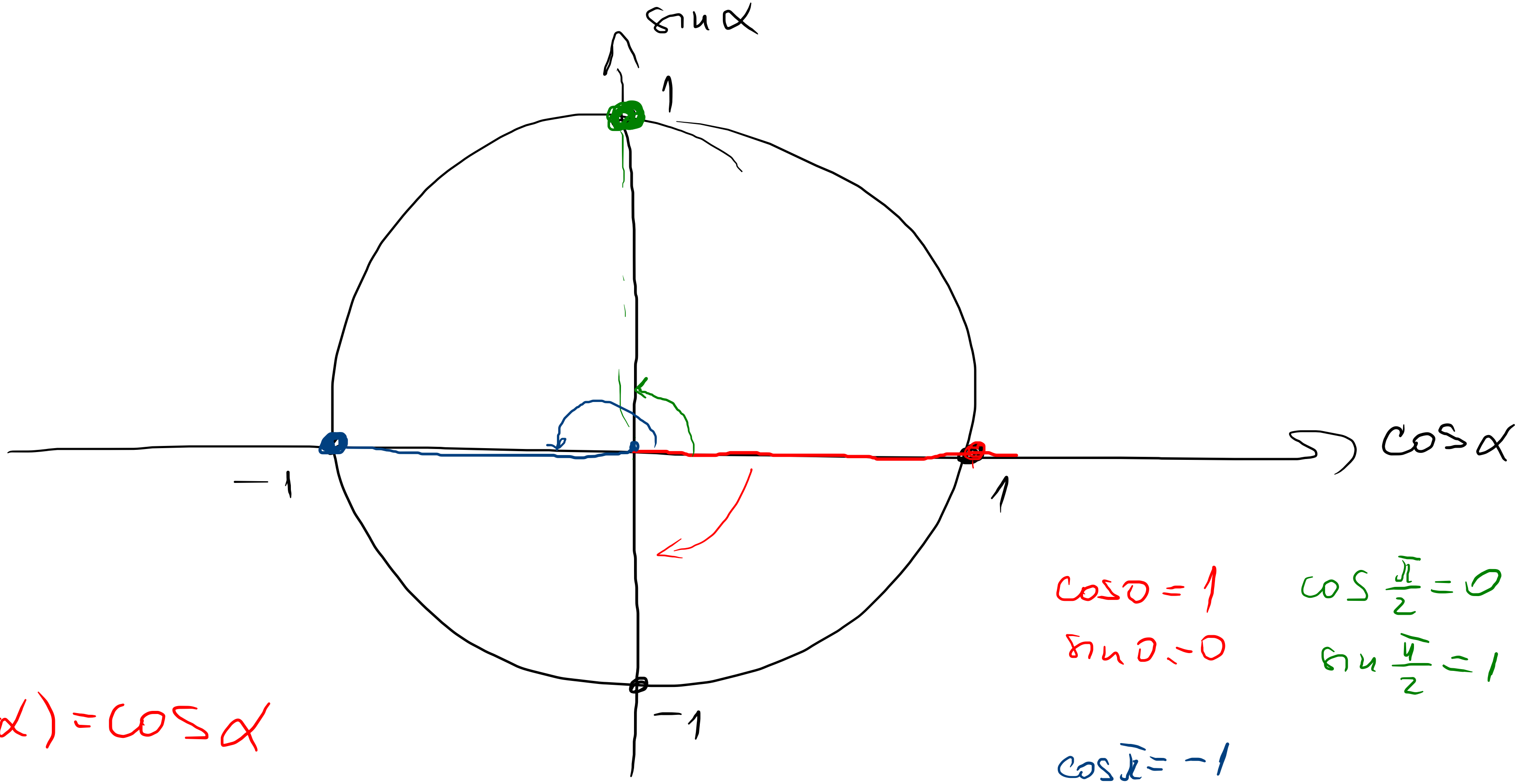
$$|\vec{b}| = \sqrt{0^2 + 2^2 + 5^2} = \sqrt{4+25} = \sqrt{29}$$

$$\cos \measuredangle(\vec{a}, \vec{b}) = \frac{\vec{a} \circ \vec{b}}{|\vec{a}||\vec{b}|} = \frac{-20}{5\sqrt{29}}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \measuredangle(\vec{a}, \vec{b})$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & -4 \\ 0 & 2 & 5 \end{vmatrix} = \begin{vmatrix} 0 & -4 \\ 2 & 5 \end{vmatrix} \vec{i} - \begin{vmatrix} 3 & -4 \\ 0 & 5 \end{vmatrix} \vec{j} + \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} \vec{k}$$

$$= 8\vec{i} - 15\vec{j} + 6\vec{k} = (8, -15, 6)$$



$\cos 0 = 1$

$\sin 0 = 0$

$\cos \frac{\pi}{2} = 0$

$\sin \frac{\pi}{2} = 1$

$\cos \pi = -1$

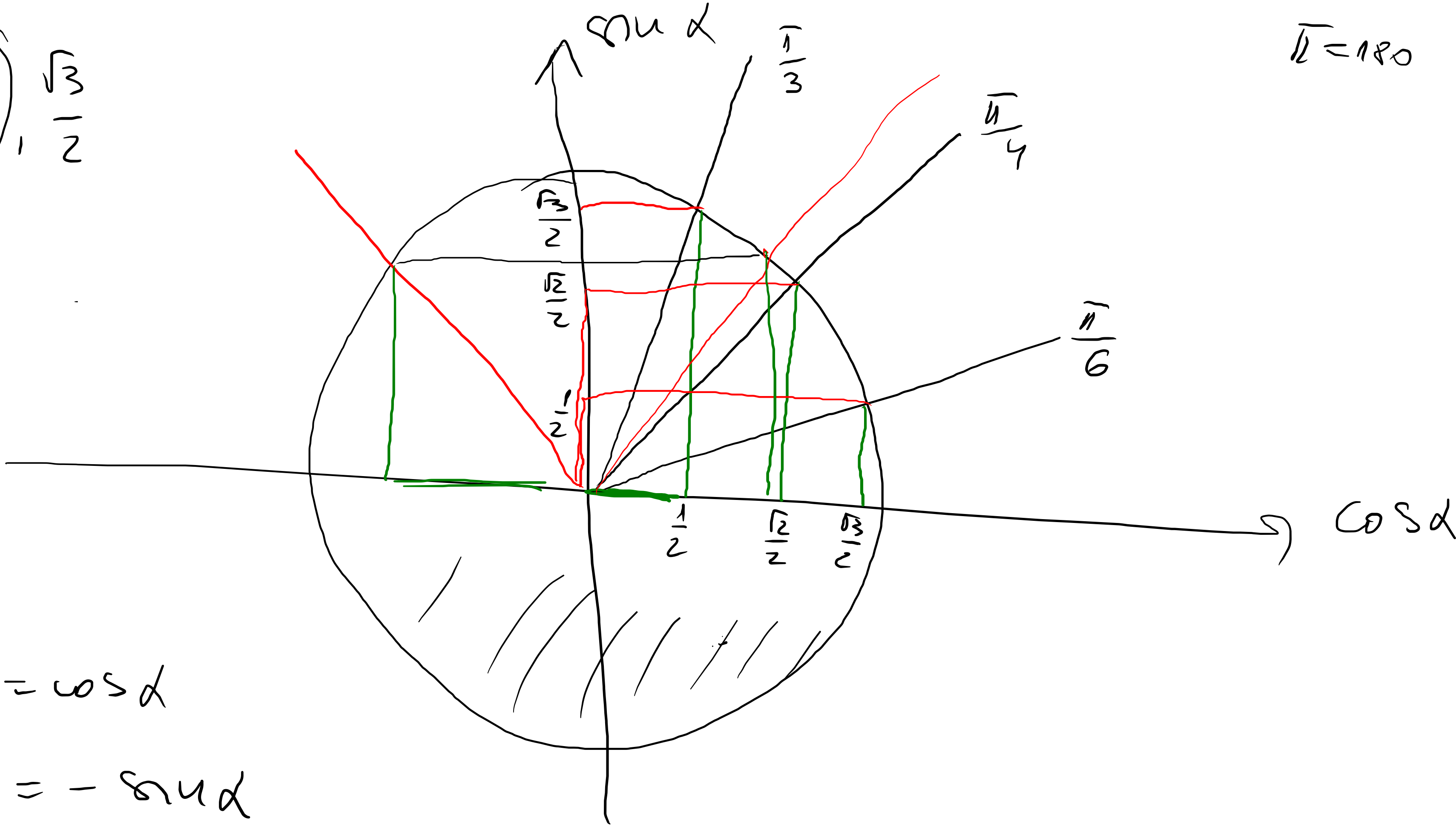
$\sin \pi = 0$

$\cos(-\alpha) = \cos\alpha$

$\sin(-\alpha) = -\sin\alpha$

$\frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}$

$\pi = 180$



$\cos(-\alpha) = \cos\alpha$

$\sin(-\alpha) = -\sin\alpha$

$(V, +, \cdot, F)$

$+ : V^2 \to V \qquad a + b = c \quad , \quad a, b, c \in V$

$\cdot : F \times V \to V \qquad \alpha \cdot a = \vec{b} \quad , \alpha \in \mathbb{R}, \quad a, b \in V$

LINEARNA KOMBINACIJA $(a_1, \ldots, a_n)$

$$\alpha_1 a_1 + \alpha_2 a_2 + \ldots \ldots + \alpha_n a_n$$

$$\alpha_1, \alpha_2, \ldots, \alpha_n \in \mathbb{R}$$

$\vec{a}, \vec{b}, \vec{c} \qquad \alpha \vec{a} + \beta \vec{b} + \gamma \vec{c} \quad , \quad \alpha, \beta, \gamma \in \mathbb{R}$

$(\mathbb{R}^3, +, \cdot, \mathbb{R})$

$(\mathbb{R}^n, +, \cdot, \mathbb{R})$

$a = (1, 2, 3)$

$b = (-1, 4, 5)$

$$\alpha a + \beta b = \alpha (1,2,3) + \beta(-1,4,5)$$

$$= (\alpha, 2\alpha, 3\alpha) + (-\beta, 4\beta, 5\beta)$$

$$= (\alpha - \beta, 2\alpha + 4\beta, 3\alpha + 5\beta)$$

$$\boxed{\alpha_1 a_1 + \alpha_2 a_2 + \cdots + \alpha_n a_n = 0}$$

$\Rightarrow \alpha_1 = \alpha_2 = \cdots = \alpha_n = 0$ LIN. NEZAVISNI

AKO $\exists \alpha_i \neq 0 \Rightarrow$ LIN. ZAVISNI

$$\left.\begin{aligned} a &= (1,2,3) \\ b &= (-2,0,1) \\ c &= (-1,2,4) \end{aligned}\right\}$$

$c = a + b$

$(-1,2,4) = (1,2,3) + (-2,0,1)$

$a + b - c = 0$

$a = (1, 2, 3)$

$b = (0, 0, 1)$

$c = (0, 5, 0)$

$$\alpha a + \beta b + \gamma c = 0$$

$$\alpha(1,2,3) + \beta(0,0,1) + \gamma(0,5,0) = 0$$

$$(\alpha, 2\alpha, 3\alpha) + (0, 0, \beta) + (0, 5\gamma, 0) = 0$$

$$(\alpha, 2\alpha + 5\gamma, 3\alpha + \beta) = 0$$

$$\alpha = 0$$
$$2\alpha + 5\gamma = 0$$
$$3\alpha + \beta = 0$$

$$\alpha = 0$$
$$\gamma = 0$$
$$\beta = 0$$

$$\left.\begin{aligned}\vec{i} &= (1,0,0)\\ \vec{j} &= (0,1,0)\\ \vec{k} &= (0,0,1)\end{aligned}\right\}$$

$$\cancel{\vec{m} = (1,1,2)}$$

$$\vec{a} = \underline{\underline{a_1\vec{i} + a_2\vec{j}}} + a_3\vec{k}$$

BAZA = LINEARNO NEZAVISNIH + GENERATORNIH

BAZA = MAKSIMALAN SKUP LINEARNO NEZAVISNIH VEKTORA

BAZA = MINIMALAN SKUP GENERATORA

$\underline{\underline{\dim(V)}}$ – BROJ VEKTORA U BAZI

$(\mathbb{R}^3, +, \cdot, \mathbb{R})$ $\quad \dim(\mathbb{R}^3) = 3$ $\quad \dim(\mathbb{R}^4) = 4$

$$\boxed{\begin{aligned} \vec{i} &= (1,0,0) \\ \vec{j} &= (0,1,0) \\ \vec{k} &= (0,0,1) \end{aligned}}$$

$$\boxed{\begin{gathered} (1,1,1) \\ (1,1,0) \\ (1,0,0) \end{gathered}}$$

$$\alpha(1,1,1) + \beta(1,1,0) + \gamma(1,0,0) = 0$$

$$(\alpha+\beta+\gamma, \alpha+\beta, \alpha) = 0$$

$$\begin{aligned} \alpha+\beta+\gamma &= 0 \\ \alpha+\beta &= 0 \\ \alpha &= 0 \end{aligned}$$

$$\alpha = \beta = \gamma = 0$$