① $\begin{array}{r|l} p(x) & x-1 \\ \hline & \\ -5 & \end{array}$ $\begin{array}{r|l} p(x) & x-2 \\ \hline & \\ -25 & \end{array}$ $\begin{array}{r|l} p(x) & (x-1)\cdot(x-2) \\ \hline & \\ 7 & \end{array}$

$p(x) : (x-1)(x-2) = s(x)$

$r(x)$

$\deg(r(x)) < \deg((x-1)(x-2))$

$x^2 - 3x + 2$

$\deg(r(x)) < 2$

$r(x) = ax + b$

$p(x) = (x-1)(x-2)\cdot s(x) + r(x)$

$p(x) = (x-1)(x-2)\cdot s(x) + ax + b$

$p(1) = a + b \Rightarrow a + b = -5$

$p(2) = 2a + b \Rightarrow 2a + b = -25$

$a = -20$

$b = 15$

$r(x) = -20x + 15$

BEZUOV STAV

$\begin{array}{r|l} p(x) & x-a \\ \hline & \\ p(a) & \end{array}$

$\begin{array}{r|l} p(x) & x-1 \\ \hline & \\ & \end{array}$

- PO BEZUOVOM OSTATAK JE $p(1)$
- PO USLOVU ZADATKA OSTATAK JE -5

$p(1) = -5$

$\begin{array}{r|l} p(x) & x-2 \\ \hline & \\ -25 & \end{array}$

$p(2) = -25$

$7 : 2 = 3$ ostatak 1

$7 = 2 \cdot 3 + 1$

(2)

$\frac{p(x)}{x+1}$ → $\frac{p(x)}{x+1}$ → $p(-1)$

$\frac{p(x)}{x-1}$ → $\frac{p(x)}{x-1}$ → $p(1)$

$\frac{p(x)}{x-2}$ → $\frac{p(x)}{x-2}$ → $p(2)$

$p(-1)=2 \qquad p(1)=3 \qquad p(2)=-1$

$\frac{p(x)}{(x+1)(x-1)(x-2)}$

?

$p(x) : (x+1)(x-1)(x-2) = S(x)$

$r(x)$

$\deg(r(x)) < \deg((x+1)(x-1)(x-2)) = 3$

$r(x) = ax^2+bx+c$

$p(x) = (x+1)(x-1)(x-2) \cdot S(x) + r(x)$

$p(x) = (x+1)(x-1)(x-2) \cdot S(x) + ax^2+bx+c$

$p(-1) = a-b+c \Rightarrow a-b+c=2$

$p(1) = a+b+c \Rightarrow a+b+c=3$

$p(2) = 4a+2b+c \Rightarrow 4a+2b+c=-1$

$a-b+c=2 \quad /\cdot(-1)$

$a+b+c=3$

$4a+2b+c=-1 \quad /\cdot(-4)$

$a-b+c=2$

$2b=1$

$6b-3c=-9 \quad /\cdot(-3)$

$a-b+c=2$

$2b+2c=1$

$-3c=-12$

$c=4$

$b=\frac{1}{2}$

$a-\frac{1}{2}+4=2$

$a=-\frac{3}{2}$

$r(x) = -\frac{3}{2}x^2+\frac{1}{2}x+4$

(3)

$p(x) = x^5 - 3x^4 + 9x^3 - 23x^2 + 36$

$k \mid 36 \Rightarrow k \in \{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36\}$

$m \mid 1 \Rightarrow m \in \{1\}$

$\left.\begin{array}{l} \\ \\ \end{array}\right\} \frac{k}{m} = k$

	1	−3	9	−23	0	36
1	1	−2	7	−16	−16	20
−1	1	−4	13	−36	36	0
−1	1	−5	18	−54	90	
2	1	−2	9	−18	0	
2	1	0	9	0		

$x^2 + 9$

$x^2 + 9 = 0$

$x^2 = -9$

$x = \pm 3i$

NULE (KOREN)

$-1, \pm 3i$ – JEDNOSTRUKE

2 – DVOSTRUKA

$\left.\begin{array}{l} \\ \end{array}\right\}$ 5

$p(x) = (x+1)(x-3i)(x+3i)(x-2)^2$ NAD $\mathbb{C}$

$p(x) = (x+1)(x-2)^2(x^2+9)$ NAD $\mathbb{R}$

(4) $r(x) = \dfrac{5x^2-4x+8}{x^2(x^2+4)} = \dfrac{A1}{x} + \dfrac{B}{x^2} + \dfrac{C1x+D3}{x^2+4}$ $/ \cdot x^2(x^2+4)$

$(x-0)^2$

$5x^2-4x+8 = Ax(x^2+4) + B(x^2+4) + (Cx+D)x^2$

$5x^2-4x+8 = Ax^3+4Ax+Bx^2+4B+Cx^3+Dx^2$

$5x^2-4x+8 = (A+C)x^3 + (B+D)x^2 + 4Ax + 4B$

$A+C=0 \Rightarrow C=1$

$B+D=5 \Rightarrow D=3$

$4A=-4 \Rightarrow A=-1$

$4B=8 \Rightarrow B=2$

$$r(x) = -\frac{1}{x} + \frac{2}{x^2} + \frac{x+3}{x^2+4}$$

(5)

a) $\frac{1}{x^2(x+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+3}$

b) $\frac{1}{x^3(x^2+x+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+x+1} + \frac{Fx+G}{(x^2+x+1)^2}$

$x_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} \notin \mathbb{R}$

c) $\frac{x^5+2x^4+x+1}{x^2+3x} = x^3 - x^2 + 3x - 9 + \frac{28x+1}{x^2+3x}$

$(x^5+2x^4+x+1) : (x^2+3x) = x^3 - x^2 + 3x - 9$

$-(x^5+3x^4)$

$-x^4+x+1$

$-(-x^4-3x^3)$

$3x^3+x+1$

$-(3x^3+9x^2)$

$-9x^2+x+1$

$-(-9x^2-27x)$

$28x+1$

$\frac{28x+1}{x^2+3x} = \frac{28x+1}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3}$

(6) $ax + 8y = 3a - 2$

$2x + ay = a + 1$

ODREDEN	$a \neq \pm 4$
NEODREDEN	$a = 4$
NEMOGUĆ	$a = -4$

$$D_S = \begin{vmatrix} a & 8 \\ 2 & a \end{vmatrix} = a^2 - 16$$

I $D_S \neq 0$ ZA $a \neq 4 \wedge a \neq -4 \Rightarrow$ SUSTAV ODREDEN

II $D_S = 0$ ZA $a = 4 \vee a = -4 \Rightarrow$ SUSTAV NEMOGUĆ ILI NEODREDEN

$a = 4$

$4x + 8y = 10 \;/:(-2)$

$2x + 4y = 5$ ↓

$4x + 8y = 10$

$0 = 0$ ✓

$4x = 10 - 8y$

1 X NEODREDEN

$y = t, \; t \in \mathbb{R}$

$x = \frac{5}{2} - 2t$

$R_S = \{(\frac{5}{2} - 2t, t) \mid t \in \mathbb{R}\}$

$a = -4$

$-4x + 8y = -14 \;/:2$

$2x - 4y = -3$

$-4x + 8y = -14$

$0 = -10$

$R_S = \emptyset$

(I) $x + 2y - z = 9$

$3x + ay + 4z = 2 - 3a$

$5x + 5y + (a+1)z = 26 - 9a$

a) $a \in \mathbb{R}$ DISKUTOVATI PRIRODU RJEŠENJA I RJEŠITI U SLUČAJU NEODREĐENOSTI

b) ZA $a = 3$ RJEŠITI MATRIČNOM METODOM

c) ZA $a = 3$ RJEŠITI GAUSSOVIM POSTUPKOM

d) ZA $a = 3$ RJEŠITI KRAMEROVIM PRAVILOM

a)

$$D_S = \begin{vmatrix} 1 & 2 & -1 \\ 3 & a & 4 \\ 5 & 5 & a+1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & a \\ 5 & 5 \end{matrix}$$

$= a(a+1) + 40 - 15 - (-5a + 20 + 6(a+1))$

$= a^2 + a + 25 - (-5a + 20 + 6a + 6)$

$= a^2 + a + 25 - (a + 26)$

$= a^2 + a + 25 - a - 26$

$= a^2 - 1$

$= (a+1)(a-1)$

I $D_S \neq 0$ ZA $a \neq \pm 1$ $\Rightarrow$ SISTEM ODREĐEN

II $D_S = 0$ ZA $a = 1 \vee a = -1$

$\Rightarrow$ SISTEM NEMOGUĆ ILI NEODREĐEN

$a = 1$

$x + 2y - z = 9 \quad /\cdot(-3)$

$3x + y + 4z = -1$

$5x + 5y + 2z = 17 \quad /\cdot(-5)$

$x + 2y - z = 9$

$-5y + 7z = -28 \quad /\cdot(-1)$

$-5y + 7z = -28$

$x + 2y - z = 9$

$-5y + 7z = -28$

$0 = 0 \; /$

$x + 2y = 9 + z$

$-5y = -28 - 7z$

1 x NEODREĐEN

$z = t, \; t \in \mathbb{R}$

$y = \frac{-28 - 7t}{-5}$

$x = 9 + t - 2\,\frac{-28 - 7t}{-5}$

$= \frac{45 + 5t - 56 - 7t}{5}$

$= \frac{11 - 7t}{5}$

$R_S = \left\{ \left(\frac{11 - 7t}{5}, \frac{28 + 7t}{5}, t \right) \mid t \in \mathbb{R} \right\}$

a = −1

$$\begin{aligned} x + 2y - z &= 9 \\ 3x - y + 4z &= 5 \quad /\cdot(-3) \\ 5x + 5y &= 35 \quad /\cdot(-5) \end{aligned}$$

$$\begin{aligned} x + 2y - z &= 9 \\ -7y + 7z &= -22 \\ -5y + 5z &= -10 \quad /:(-5) \end{aligned}$$

$$\begin{aligned} x + 2y - z &= 9 \\ y - z &= 2 \quad /\cdot 7 \\ -7y + 7z &= -22 \end{aligned}$$

$$\begin{aligned} x + 2y - z &= 9 \\ y - z &= 2 \\ 0 &= -8 \end{aligned}$$

$L_S = \emptyset$

MATRIČNA METODA

b) $x + 2y - z = 9$
$3x + 3y + 4z = -7$
$5x + 5y + 4z = -1$

(1, 2, −4)

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 3 & 4 \\ 5 & 5 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 9 \\ -7 \\ -1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$AX = B \;/\cdot A^{-1}$ so leve strane also množim

$A^{-1}AX = A^{-1}B$

E

$\boxed{X = A^{-1}B}$

$A^{-1} = \frac{1}{\det(A)} \operatorname{ady}(A)$

$$\det(A) = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 3 & 4 \\ 5 & 5 & 4 \end{vmatrix} = 8$$

$$\operatorname{ady}(A) = \begin{bmatrix} + \begin{vmatrix} 3 & 4 \\ 5 & 4 \end{vmatrix} & - \begin{vmatrix} 3 & 4 \\ 5 & 4 \end{vmatrix} & + \begin{vmatrix} 3 & 3 \\ 5 & 5 \end{vmatrix} \\ - \begin{vmatrix} 2 & -1 \\ 5 & 4 \end{vmatrix} & + \begin{vmatrix} 1 & -1 \\ 5 & 4 \end{vmatrix} & - \begin{vmatrix} 1 & 2 \\ 5 & 5 \end{vmatrix} \\ + \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} & - \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} & + \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} -8 & 8 & 0 \\ -13 & 9 & 5 \\ 11 & -7 & -3 \end{bmatrix}^T = \begin{bmatrix} -8 & -13 & 11 \\ 8 & 9 & -7 \\ 0 & 5 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{ady}(A)$$

$$= \frac{1}{8} \begin{bmatrix} -8 & -13 & 11 \\ 8 & 9 & -7 \\ 0 & 5 & -3 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{8}\begin{bmatrix} -8 & -13 & 11 \\ 8 & 9 & -7 \\ 0 & 5 & -3 \end{bmatrix}\begin{bmatrix} 9 \\ -7 \\ -1 \end{bmatrix}$$

$3\times 3 \longleftrightarrow 3\times 1$

$$= \frac{1}{8}\begin{bmatrix} -72+91-11 \\ 72-63+7 \\ -35+3 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 8 \\ 16 \\ -32 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x=1 \quad y=2 \quad z=-4$

$R_S = \{(1,2,-4)\}$

GAUSOV POSTUPAK

c) $x+2y-z=9$
$3x+3y+4z=-7$ $\cdot(-3)$
$5x+5y+4z=-1$ $\cdot(-5)$

$x+2y-z=9$
$-3y+7z=-34 \mid :5$
$-5y+9z=-46 \mid :(-3)$

$x+2y-z=9$
$-15y+35z=-170$
$15y-27z=138$ $+$

$x+2y-z=9$
$-3y+7z=-34$
$8z=-32$

$z=-4$
$-3y=-34+28$
$y=2$
$x=9-4-4$
$x=1$
$R_S=\{(1,2,-4)\}$

KRAMER

d) $x+2y-z=9$
$3x+3y+4z=-7$
$5x+5y+4z=-1$

$D_S=\begin{vmatrix}1&2&-1\\3&3&4\\5&5&4\end{vmatrix}=8$

$D_x=\begin{vmatrix}9&2&-1\\-7&3&4\\-1&5&4\end{vmatrix}=8$

$D_y=\begin{vmatrix}1&9&-1\\3&-7&4\\5&-1&4\end{vmatrix}=16$

$D_z=\begin{vmatrix}1&2&9\\3&3&-7\\5&5&-1\end{vmatrix}=-32$

$x=\frac{D_x}{D_S}=\frac{8}{8}=1 \quad y=\frac{D_y}{D_S}=\frac{16}{8}=2$

$z=\frac{D_z}{D_S}=\frac{-32}{8}=-4 \quad R_S=\{(1,2,-4)\}$

8,

$$\begin{bmatrix} 2 & 3 & 5 \\ 1 & -2 & -3 \\ 0 & -1 & 4 \end{bmatrix} \quad \begin{bmatrix} -2 & 4 \\ -1 & 1 \\ 2 & -3 \end{bmatrix}$$

3×3 ⟷ 3×2

$$= \begin{bmatrix} 2\cdot(-2)+3\cdot(-1)+5\cdot 2 & 2\cdot 4+3\cdot 1+5\cdot(-3) \\ 1\cdot(-2)+(-2)\cdot(-1)+(-3)\cdot 2 & 1\cdot 4+(-2)\cdot 1+(-3)(-3) \\ 0\cdot(-2)+(-1)(-1)+4\cdot 2 & 0\cdot 4+(-1)\cdot 1+4(-3) \end{bmatrix}$$

$X = EX = XE$

$2X = (2E)X$

9)

$$AX + 2X = B$$

$$(\underbrace{A + 2E}_{M})X = B$$

$$MX = B \quad / M^{-1}$$ SA LEVE STRANE NALO POSTOJI

$$\underbrace{M^{-1}M}_{E}X = M^{-1}B$$

$$\boxed{X = M^{-1}B}$$

10

$$2X - B + XA = C$$

$$2X + XA = B + C$$

$$X(\underbrace{2E + A}_{M}) = \underbrace{B + C}_{N}$$

$XM = N \quad / \cdot M^{-1}$ co desne strane ako postoji

$$XMM^{-1} = NM^{-1}$$

$$\underbrace{M}_{E}M^{-1} = NM^{-1}$$

$$\boxed{X = NM^{-1}}$$