$\underline{\underline{A_{m \times n}}}$

$$\boxed{A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)}$$

$$ax = b$$

$$x = \frac{b}{a} \qquad a \neq 0$$

$$x = ba^{-1}$$
$$= a^{-1}b$$

$$\underline{\underline{AB \neq BA}}$$

$$ax - 2x = b$$
$$(a-2)x = b$$

$$ax - 2 \cdot 1 \cdot x = b$$
$$(a - 2 \cdot 1)x = b$$

5. Rešiti matrične jednačine:

5.1 $AX = B$ ako je $A = \begin{bmatrix} 1 & 2 \\ 6 & 3 \end{bmatrix}$ i $B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

$AX = B \quad | \; A^{-1}$ sa leve strane, ako postoji

$$\underline{A^{-1}A}X = A^{-1}B$$

NIJE DOBRO: $\cancel{A^{-1}AX = BA^{-1}}$

$$AXA^{-1} = BA^{-1}$$

$$AA^{-1} = A^{-1}A = E$$

$$XE = EX = X$$

$$EX = A^{-1}B$$

$$\boxed{X = A^{-1}B}$$

NAPOMENA: $XA = B \quad | \;$ sa A^{-1} sa desne strane, ako postoji

$$X\underline{AA^{-1}} = BA^{-1}$$

$$XE = BA^{-1}$$

$$X = BA^{-1}$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) = -\frac{1}{9}\begin{bmatrix} 3 & -2 \\ -6 & 1 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 \\ 6 & 3 \end{vmatrix} = 3 - 12 = -9 \neq 0$$

$$\operatorname{adj}(A) = \begin{bmatrix} 3 & -2 \\ -6 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= -\frac{1}{9}\underbrace{\begin{bmatrix} 3 & -2 \\ -6 & 1 \end{bmatrix}}_{2\times 2}\underbrace{\begin{bmatrix} 2 \\ 0 \end{bmatrix}}_{2\times 1}$$

$$= -\frac{1}{9}\begin{bmatrix} 6 \\ -12 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2}{3} \\ \frac{4}{3} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

$$= \frac{2}{3}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

5.2 $\boxed{AX - 2X = B}$ ako je $A = \begin{bmatrix} 3 & -5 \\ 1 & -1 \end{bmatrix}$ i $B = \begin{bmatrix} 4 & 5 \\ -8 & -3 \end{bmatrix}$.

$$AX - 2X = B$$

$$\underbrace{(A - 2E)}_{M} X = B$$

$$2X = 2EX = 2XE$$

$$M = A - 2E$$

$$MX = B \quad / \, M^{-1} \text{ sa leve strane ako postoji}$$

$$\underbrace{M^{-1}M}_{E} X = M^{-1}B$$

$$\boxed{X = M^{-1}B}$$

$$(A-2E)^{-1}$$

$$(A-2E)^{-1}(A-2E)X = (A-2E)^{-1}B$$

$$X = (A-2E)^{-1}B$$

$$M = A - 2E = \begin{bmatrix} 3 & -5 \\ 1 & -1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad +(-2)$$

$$= \begin{bmatrix} 3 & -5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$\boxed{M = \begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix}}$$

$$\det(M) = \begin{vmatrix} 1 & -5 \\ 1 & -3 \end{vmatrix} = -3 + 5 = 2 \neq 0$$

$$\operatorname{adj}(\overset{A-2E}{M}) = \begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix}$$

$$\underset{(A-2E)^{-1}}{M^{-1}} = \frac{1}{\det(M)} \operatorname{adj}(M) = \frac{1}{2}\begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix}$$

$$X = M^{-1}B = \frac{1}{2}\begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ -8 & -3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -52 & -30 \\ -12 & -8 \end{bmatrix}$$

$2 \times 2 \quad 2 \times 2$

$$= \begin{bmatrix} -26 & -15 \\ -6 & -4 \end{bmatrix}$$

$2XA = X2A = XA2$

5.3 $X - 2XA = B$ ako je $A = \begin{bmatrix} -2 & 1 \\ 3 & -2 \end{bmatrix}$ i $B = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$.

$$X - 2XA = B$$

$$X\underbrace{(E - 2A)}_{M} = B$$

$XM = B \,/\, M^{-1}$ sa desne strane, ako postoji

$$X\underbrace{MM^{-1}}_{E} = BM^{-1}$$

$$\boxed{X = BM^{-1}}$$

$$M = E - 2A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} -2 & 1 \\ 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -6 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} 5 & -2 \\ -6 & 5 \end{bmatrix}}$$

$$\det(M) = \begin{vmatrix} 5 & -2 \\ -6 & 5 \end{vmatrix} = 25 - 12 = 13 \neq 0$$

$$\operatorname{adj}(M) = \begin{bmatrix} 5 & 2 \\ 6 & 5 \end{bmatrix}$$

$$M^{-1} = \frac{1}{\det(M)} \operatorname{adj}(M) = \frac{1}{13}\begin{bmatrix} 5 & 2 \\ 6 & 5 \end{bmatrix}$$

$$X = BM^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \cdot \frac{1}{13} \cdot \begin{bmatrix} 5 & 2 \\ 6 & 5 \end{bmatrix} = \frac{1}{13} \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} 5 & 2 \\ 6 & 5 \end{bmatrix}}_{2\times 2} = \frac{1}{13}\begin{bmatrix} 6 & 5 \\ -17 & -12 \end{bmatrix}$$

6. Rešiti matričnu jednačinu $AX - B = X$ za $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ i

$$B = \begin{bmatrix} 0 & 2 & -6 \\ 4 & -2 & -2 \\ -3 & 7 & -3 \end{bmatrix}.$$

$$AX - B = X$$

$$AX - X = B$$

$$\underbrace{(A - E)}_{M} X = B$$

$MX = B \quad / M^{-1}$ sa leve strane, ako postoji

$$\underbrace{M^{-1}M}_{E} X = M^{-1}B$$

$$\boxed{X = M^{-1}B}$$

$$M = A - E = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 2 & 0 \\ -1 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix} \qquad M^{-1} = \frac{1}{\det(M)} \operatorname{adj}(M)$$

$$\det(M) = \left| \begin{matrix} 0 & 2 & 0 \\ -1 & 0 & 1 \\ 3 & 1 & 0 \end{matrix} \right| \begin{matrix} 0 & 2 \\ -1 & 0 \\ 3 & 1 \end{matrix} = 6 \neq 0$$

$$\operatorname{adj}(M) = \begin{bmatrix} +\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} -1 & 1 \\ 3 & 0 \end{vmatrix} & +\begin{vmatrix} -1 & 0 \\ 3 & 1 \end{vmatrix} \\ -\begin{vmatrix} 2 & 0 \\ 1 & 0 \end{vmatrix} & +\begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} & -\begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix} \\ +\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 0 \\ -1 & 1 \end{vmatrix} & +\begin{vmatrix} 0 & 2 \\ -1 & 0 \end{vmatrix} \end{bmatrix}^T = \begin{bmatrix} -1 & 3 & -1 \\ 0 & 0 & 6 \\ 2 & 0 & 2 \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 0 & 0 \\ -1 & 6 & 2 \end{bmatrix}$$

$$M^{-1} = \frac{1}{\det(M)} \operatorname{adj}(M) = \frac{1}{6} \begin{bmatrix} -1 & 0 & 2 \\ 3 & 0 & 0 \\ -1 & 6 & 2 \end{bmatrix}$$

$$X = M^{-1} B = \frac{1}{6} \begin{bmatrix} -1 & 0 & 2 \\ 3 & 0 & 0 \\ -1 & 6 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 & -6 \\ 4 & -2 & -2 \\ -3 & 7 & 3 \end{bmatrix}$$

$$\underline{3} \times 3 \longleftrightarrow 3 \times \underline{3}$$

$$= \frac{1}{6} \begin{bmatrix} -6 & 12 & 12 \\ 0 & 6 & -18 \\ 18 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 1 & -3 \\ 3 & 0 & 0 \end{bmatrix}$$

7. Rešiti matričnu jednačinu $ABX = 4X + C$ za $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 3 & 1 \end{bmatrix}$,

$B = A^T$ i $C = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}$.

$$ABX = 4X + C$$

$$ABX - 4X = C$$

$$\underbrace{(AB - 4E)}_{M}X = C$$

$$MX = C \quad / \; M^{-1}$$ sa leve strane ako postoji

$$\underbrace{M^{-1}M}_{E}X = M^{-1}C$$

$$\boxed{X = M^{-1}C}$$

$$M = AB - 4E = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 1 & 2 & 1 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$3 \times 2 \qquad 2 \times 3$

$$= \begin{bmatrix} 2 & 2 & 4 \\ 2 & 4 & 2 \\ 4 & 2 & 10 \end{bmatrix} + \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$M = \begin{bmatrix} -2 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 6 \end{bmatrix}$$

$$\det(M) = \begin{vmatrix} -2 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 6 \end{vmatrix} \begin{matrix} -2 & 2 \\ 2 & 0 \\ 4 & 2 \end{matrix} = 16 + 16 - (-8 + 24)$$

$$= 16 \neq 0$$

$$M = \begin{bmatrix} -2 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 6 \end{bmatrix}$$

$$\operatorname{adj}(M) = \begin{bmatrix} -4 & -4 & +4 \\ -4 & -28 & +12 \\ +4 & +12 & -4 \end{bmatrix}^T = \begin{bmatrix} -4 & -4 & 4 \\ -4 & -28 & 12 \\ 4 & 12 & -4 \end{bmatrix}$$

$$M^{-1} = \frac{1}{\det(M)} \operatorname{adj}(M) = \frac{1}{16} \begin{bmatrix} -4 & -4 & 4 \\ -4 & -28 & 12 \\ 4 & 12 & -4 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -1 & -1 & 1 \\ -1 & -7 & 3 \\ 1 & 3 & -1 \end{bmatrix}$$

$$X = M^{-1} C = \frac{1}{4} \underbrace{\begin{bmatrix} -1 & -1 & 1 \\ -1 & -7 & 3 \\ 1 & 3 & -1 \end{bmatrix}}_{3\times 3} \underbrace{\begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}}_{3\times 1} = \frac{1}{4} \begin{bmatrix} 4 \\ 8 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

8. Rešiti matričnu jednačinu $AXB = C$ za $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & -4 \\ 3 & 1 & 2 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ -1 & -2 & 0 \end{bmatrix}$ i $C = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$.

$AXB = C \quad / A^{-1}$ sa leve strane ako postoji

$$\underbrace{A^{-1}A}_{E}XB = A^{-1}C$$

$XB = A^{-1}C \quad / B^{-1}$ sa desne strane ako postoji

$$X\underbrace{BB^{-1}}_{E} = A^{-1}CB^{-1}$$

$$\boxed{X = A^{-1}CB^{-1}}$$

9. Matričnim računom rešiti sistem linearnih jednačina

$$\begin{aligned} -x - 2y + z &= 1 \\ x - y - z &= -1 \\ y - z &= 0 \end{aligned}.$$

$$A = \begin{bmatrix} -1 & -2 & 1 \\ 1 & -1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$AX = B \,/\, A^{-1}$ sa leve strane ako postoji

$$\underbrace{A^{-1}A}_{E} X = A^{-1}B$$

$$\boxed{X = A^{-1}B}$$

$$\det(A) = \begin{vmatrix} -1 & -2 & 1 \\ 1 & -1 & -1 \\ 0 & 1 & -1 \end{vmatrix} \begin{matrix} -1 & -2 \\ 1 & -1 \\ 0 & 1 \end{matrix}$$

$$= \cancel{-1} + \cancel{1} - (1 + 2) = -3 \neq 0$$

$$\operatorname{adj}(A) = \begin{bmatrix} +2 & 1 & +1 \\ -1 & +1 & +1 \\ +3 & 0 & +3 \end{bmatrix}^T = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) = -\frac{1}{3} \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

$$X = -\frac{1}{3} \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = -\frac{1}{3} \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$x = -1, \; y = 0, \; z = 0$

$R_S = \{(-1, 0, 0)\}$