

① ZAKLJUČENI PRSTENI KOJI NISU POJE

~~a)~~  $(\mathbb{N}, +, \cdot)$  - NEMA N.E. ZA  $(\mathbb{N}, +)$

~~b)~~  $(\mathbb{Z} \setminus \{0\}, +, \cdot)$  - NEMA N.E. ZA  $(\mathbb{Z} \setminus \{0\}, +)$

~~c)~~  $(\mathbb{R}, +, \cdot)$  - JE POJE

~~d)~~  $(\mathbb{R} \setminus \{0\}, +, \cdot)$  - NEMA N.E. ZA  $(\mathbb{R} \setminus \{0\}, +)$

e)  $(\mathbb{Z}, +, \cdot)$

~~f)~~  $(\mathbb{Z} - 1, +, \cdot)$   $1+1=2$

~~g)~~  $(\mathbb{Z} - 1, \cdot, +)$   $1+1=2$

$(\mathbb{N}, +, \cdot)$  PRSTEN?

~~1)~~  $(\mathbb{N}, +)$  - A.G. - NISU NEMA N.E.

2,  $(\mathbb{N}, \cdot)$  - POLUGRUPA

3. DISTRIBUTIVNOST - PREMA +

$(\mathbb{Z} \setminus \{0\}, +, \cdot)$  - PRSTEN?

~~$(\mathbb{Z} \setminus \{0\}, +)$  - A.G.?~~ - NISU NEMA N.E.

$(\mathbb{Z} \setminus \{0\}, +)$

$(\mathbb{Z}, +)$

$(\mathbb{Z}, \cdot)$

$(\mathbb{Z}, +, \cdot)$  JE PRSTEN

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$(\mathbb{Z} \setminus \{0\}, \cdot)$  - NEMA I.N. SVAKI E.L. NISU POJE

$(A, +, \circ)$  - PRSTEN

1,  $(A, +)$  - ABELOVA GRUPA

2,  $(A, \circ)$  - POLUGRUPA (ASOCIJATIVAN GRUPOID)

3,  $\circ$  PREMA + TREBA DA JE DISTRIBUTIVNO

$\forall x, y, z \in A$

$$x \circ (y + z) = xy + xz$$

$$(y + z) \circ x = yx + zx$$

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$(A, +, \circ)$  - POYE

1,  $(A, +, \circ)$  - PRSTEN

2,  $(A \setminus \{0\}, \circ)$  - ABELOVA GRUPA

$(\mathbb{A}\{0\}, +)$  - NIJE GRUPA

$(\mathbb{A}, \cdot)$

$0 \in \mathbb{A}$  - NIJE GRUPA

JEER 0 NIKAD NEMA INVERZIJU

$(\mathbb{R}, \cdot)$ ,  $(\mathbb{Q}, \cdot)$

$([0, \infty), \cdot)$

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# GRUPOVNE

*	e	a	b
e	e	a	b
a	a		
b	b	a	a

e - N.E.  
 NE MOŽE e

NEMA I.E.

*	e	a	b	c
e	e	a	b	c
a	a			e
b	b	e		
c	c		e	

NEMA I.E.

1, N.E. U SVAKOJ VISTI  
 2, N.E. U SVAKOJ  
 KOLONI

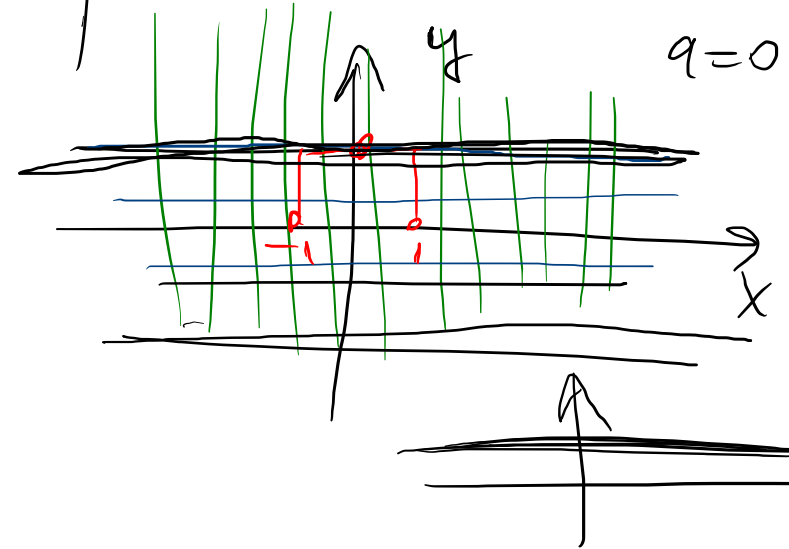
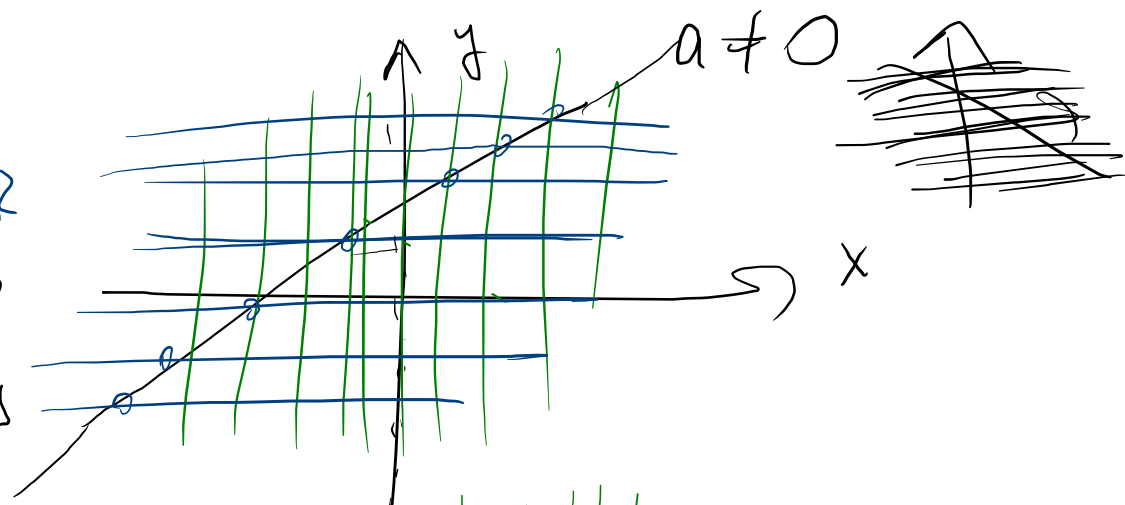
3, SAMO ŽEDNO

4, SIMETRIČNO  
 RASPOREĐEN U ODNOSU NA  
 GLAVNU DIJAGONALU

②  $a, b \in \mathbb{R}$   $f: \mathbb{R} \rightarrow \mathbb{R}$

$f(x) = ax + b$

- 1, FUNKCIJA,  $a, b \in \mathbb{R}$
- 2,  $f$  - 1-1,  $a \notin \mathbb{R} \setminus \{0\}, b \in \mathbb{R}$
- 3,  $f$  - NA,  $a \in \mathbb{R} \setminus \{0\}, b \in \mathbb{R}$
- 4,  $f$  - PROJEKCIJNA,  $a \in \mathbb{R} \setminus \{0\}, b \in \mathbb{R}$



③ Задача:  $f(x) = x^2$  и  $g(x) = 2x + 3$

$f: \mathbb{R} \rightarrow \mathbb{R}$

$f(x) = x^2$

$g: \mathbb{R} \rightarrow \mathbb{R}$

$g(x) = 2x + 3$

~~a) f je "1-1"~~

c) g je "1-1"

~~b) f je "no"~~

~~d) g je "no"~~



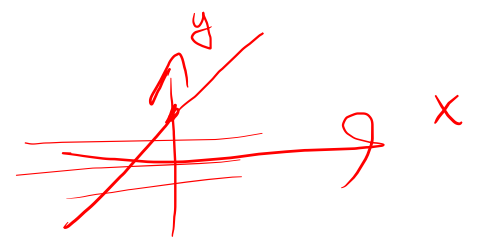
$f(1) = 1$   
 $f(-1) = 1$



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$f(x) = x^2$

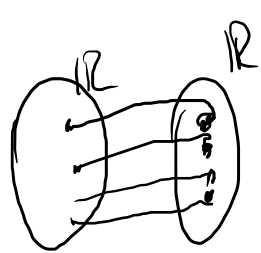
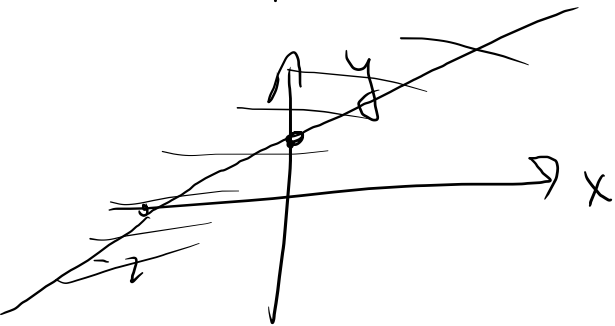
$f: \mathbb{R}^+ \cup \{0\} \rightarrow \mathbb{R}^+ \cup \{0\}$



4)  $f^{-1}$   $f: \mathbb{R} \rightarrow \mathbb{R}$

$f(x) = 1 - 3x$

2a) Czy  $f: \mathbb{R} \rightarrow \mathbb{R}$  ma odwrotność? Sprawdź metodą sumy i różnicy.



"1-1" :  $f(x) = f(y) \Rightarrow x = y$  ? ✓

$f(x) = f(y) \Rightarrow 1 - 3x = 1 - 3y \Rightarrow -3x = -3y \Rightarrow x = y$

"NA" :  $\forall y \in \mathbb{R}, \exists x \in \mathbb{R}, f(x) = y$  ?

$f(x) = y$   
 $1 - 3x = y$   
 $-3x = y - 1$

$x = \frac{y-1}{-3}$   
 jest "no"

$\Rightarrow f$  je bijekcja  
 $f^{-1}(f(x)) = x$   
 $f^{-1}(1 - 3x) = x$   
 $f^{-1}(t) = \frac{t-1}{-3}$   
 $1 - 3x = t$   
 $-3x = t - 1$   
 $x = \frac{t-1}{-3}$   
 $f^{-1}(x) = \frac{x-1}{-3}$

⑤  $f = \begin{pmatrix} a & b & c \\ c & a & b \end{pmatrix}$   $f^{-1} = \begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix}$

$$g = \begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix}$$

$$h = \begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix}$$

$\rho$	$f$	$g$	$h$
$f$	$g$	$h$	$f$
$g$	$h$	$f$	$g$
$h$	$f$	$g$	$h$

$$\begin{aligned} f^{-1} &= g \\ g^{-1} &= f \\ h^{-1} &= h \end{aligned}$$

$(\underbrace{f, g, h}, \rho) \rightarrow \text{Gr?}$

$$f \circ f = \begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix} = g$$

$$f \circ g = \begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix} = h$$

$$f \circ h = \begin{pmatrix} a & b & c \\ c & a & b \end{pmatrix} = f$$

$$g \circ f = \begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix} = h$$

$$g \circ g = \begin{pmatrix} a & b & c \\ c & a & b \end{pmatrix} = f$$

$$g \circ h = \begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix} = g$$

$$h \circ f = \begin{pmatrix} a & b & c \\ c & a & b \end{pmatrix} = f$$



$e - N.E.$

$$X \cdot \boxed{e} = e$$

↑

$I.E. \cdot \cancel{A} \cdot e$

$$X^T \cdot X = e$$

$$\underline{e} \cdot e = e$$

$$a \cdot e = a$$

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$(\mathbb{N}, +, 0)$

~~PRSTEN~~

~~D.I.~~

~~POSLE~~

~~$(\mathbb{N}, +)$  - PRSTEN VA GRUPA~~

$$-a + a = 0$$

$$\frac{1}{a} \cdot a = 1$$

GRUPE

~~⊗~~

~~$(\mathbb{Z}, +)$~~

$(\mathbb{R} \setminus \{0\}, \cdot)$

$(\mathbb{R}, +)$

$(\mathbb{Z} \setminus \{1\}, \cdot)$

~~$(\mathbb{Z} \setminus \{0\}, +)$~~

$\{f \mid f: A \xrightarrow{f} A\}$

~~$(\mathbb{R}, \cdot)$~~

$$f: A \rightarrow A$$

$$g: A \rightarrow A$$

$$f \circ g: A \rightarrow A$$

$i_A(x) = x$

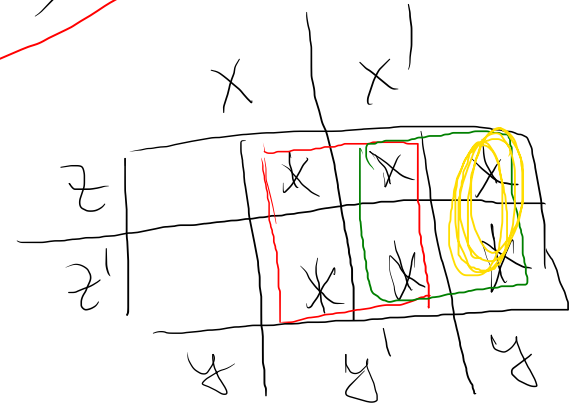
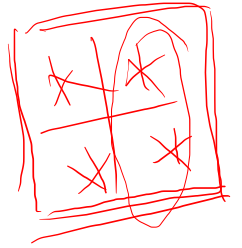
→ D NEMA I.E.

$(\mathbb{R}, +)$

~~$(\mathbb{Z} \setminus \{0\}, +)$~~   
NEMA N.I.E

$(\mathbb{Z}, \circ)$

$(\mathbb{Z} \setminus \{0\}, \circ)$



$$f = (x \cdot y)' + (x + y)' \cdot z + x'z$$

$$= x' + y' + x' \cdot y \cdot z + x'z \leftarrow \text{DNF}$$

$$= x'(y + y')(z + z') + y'(x + x')(z + z') + x'y'z + x'(y + y')z$$

$$= x'yz + x'y'z + x'yz + x'y'z + x'yz + x'y'z + x'y'z + x'y'z$$

PI:  $y', x'$   
MANT (f) =  $y' + x'$