no sleole 'cos shrau

1) $\cancel{\int e^{2x}\sin 4x\,dx =}$

$\int p_n(x)e^{\alpha x}dx$

$\int p_n(x)\sin\alpha x\,dx$

$\int p_n(x)\cos\alpha x\,dx$

$u = p_n(x)$

$dv = \begin{cases} e^{\alpha x}dx \\ \sin\alpha x\,dx \\ \cos\alpha x\,dx \end{cases}$

$$\int (x-2)^2 \sin 3x\,dx = (x-2)^2 \cdot \left(-\frac{1}{3}\cos 3x\right) - \int \left(-\frac{1}{3}\cos 3x\right)\cdot 2(x-2)\,dx$$

$u = (x-2)^2$ $\quad dv = \sin 3x\,dx \longrightarrow v = -\frac{1}{3}\cos 3x$

$du = 2(x-2)dx$

$\int \sin 3x\,dx = -\frac{1}{3}\cos 3x + C$

$3x = t$

$x = \frac{1}{3}t$

$dx = \frac{1}{3}dt$

$\frac{1}{3}\int \sin t\,dt$

$= -\frac{1}{3}\cos 3x + C$

$\int \sin\alpha x\,dx = -\frac{1}{\alpha}\sin\alpha x + C$

$$= -\frac{1}{3}(x-2)^2\cos 3x + \frac{2}{3}\boxed{\int (x-2)\cos 3x\,dx}$$

$u = x-2$ $\quad dv = \cos 3x\,dx$

$du = dx$ $\quad v = \frac{1}{3}\sin 3x$

$$= -\frac{1}{3}(x-2)^2\cos 3x + \frac{2}{3}\left((x-2)\frac{1}{3}\sin 3x - \int \frac{1}{3}\sin 3x\,dx\right)$$

$$= -\frac{1}{3}(x-2)^2\cos 3x + \frac{2}{9}(x-2)\sin 3x - \frac{2}{9}\int \sin 3x\,dx$$

(2)

$$\boxed{\int e^{2x}\sin 4x\,dx} = \overset{u}{e^{2x}}\cdot\overset{v}{\left(-\frac{1}{4}\cos 4x\right)} - \int \overset{v}{\left(-\frac{1}{\cancel{4}}\cos 4x\right)}\cdot \overset{du}{\cancel{2}e^{2x}\,dx}$$

$u = e^{2x}$ $\qquad dv = \sin 4x\,dx$

$du = 2e^{2x}\,dx$ $\qquad v = -\frac{1}{4}\cos 4x$

$u = e^{2x}$ $\qquad dv = \cos 4x\,dx$

$du = 2e^{2x}\,dx$ $\qquad v = \frac{1}{4}\sin 4x$

$$= -\frac{1}{4}e^{2x}\cos 4x + \frac{1}{2}\boxed{\int e^{2x}\cos 4x\,dx} = -\frac{1}{4}e^{2x}\cos 4x + \frac{1}{2}\left(\overset{u}{e^{2x}}\cdot\overset{v}{\frac{1}{4}\sin 4x} - \int \overset{v}{\frac{1}{4}\sin 4x}\cdot \underset{du}{2e^{2x}\,dx}\right)$$

$$= \boxed{-\frac{1}{4}e^{2x}\cos 4x + \frac{1}{8}e^{2x}\sin 4x} - \frac{1}{4}\boxed{\int e^{2x}\sin 4x\,dx}$$

$I = A - \frac{1}{4}I$

$I + \frac{1}{4}I = A$

$$\int e^{2x}\sin 4x\,dx + \frac{1}{4}\int e^{2x}\sin 4x\,dx = -\frac{1}{4}e^{2x}\cos 4x + \frac{1}{8}e^{2x}\sin 4x$$

$$\frac{5}{4}\int e^{2x}\sin 4x\,dx = -\frac{1}{4}e^{2x}\cos 4x + \frac{1}{8}e^{2x}\sin 4x \Rightarrow \int e^{2x}\sin 4x\,dx = \frac{4}{5}\left(-\frac{1}{4}e^{2x}\cos 4x + \frac{1}{8}e^{2x}\sin 4x\right) + C$$

$\underline{a} = 24 - \frac{1}{4}a$

$a + \frac{1}{4}a = 24$

$\frac{5}{4}a = 24$

$a = \frac{4}{5} \cdot 24$

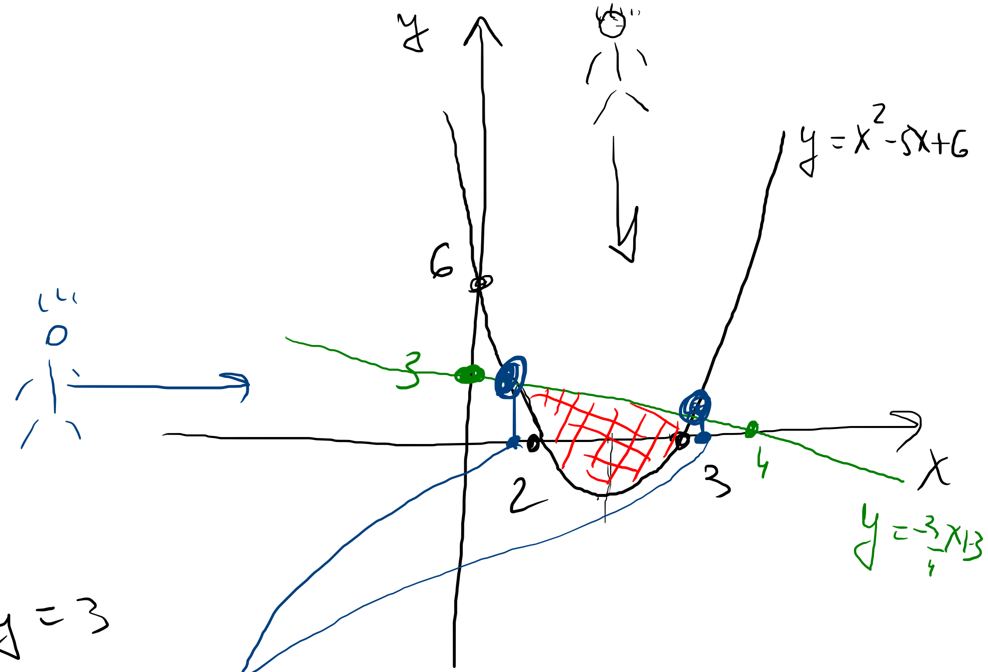
$$\underline{\overbrace{\int e^{2x}\sin 4x\,dx}^{a}} = \underbrace{\overbrace{-\frac{1}{4}e^{2x}\cos 4x + \frac{1}{8}e^{2x}\sin 4x}}_{} - \frac{1}{4}\underline{\overset{\tilde{a}}{\int e^{2x}\sin 4x}}$$

$$\int x + \frac{1}{4}\int \cancel{x} =$$

(3)

$$y = x^2 - 5x + 6$$

$$y = -\frac{3}{4}x + 3$$



$y = x^2 - 5x + 6$

$y = 0 \quad x^2 - 5x + 6 = 0$

$x_1 = 3 \qquad x_2 = 2$

$x = 0 \qquad y = 6$

$y = -\frac{3}{4}x + 3$

$x = 0 \rightarrow y = 3$

$x = 4 \rightarrow y = 0$

$$P = \int_{x_1}^{x_2} \left(-\frac{3}{4}x + 3 - (x^2 - 5x + 6)\right) dx$$

$y = x^2 - 5x + 6$

$y = -\frac{3}{4}x + 3$

$x^2 - 5x + 6 = -\frac{3}{4}x + 3 \quad / \cdot 4$

$4x^2 - 20x + 24 = -3x + 12$

$4x^2 - 17x + 12 = 0$

$x_{1,2} = \frac{17 \pm \sqrt{17^2 - 16 \cdot 12}}{8}$

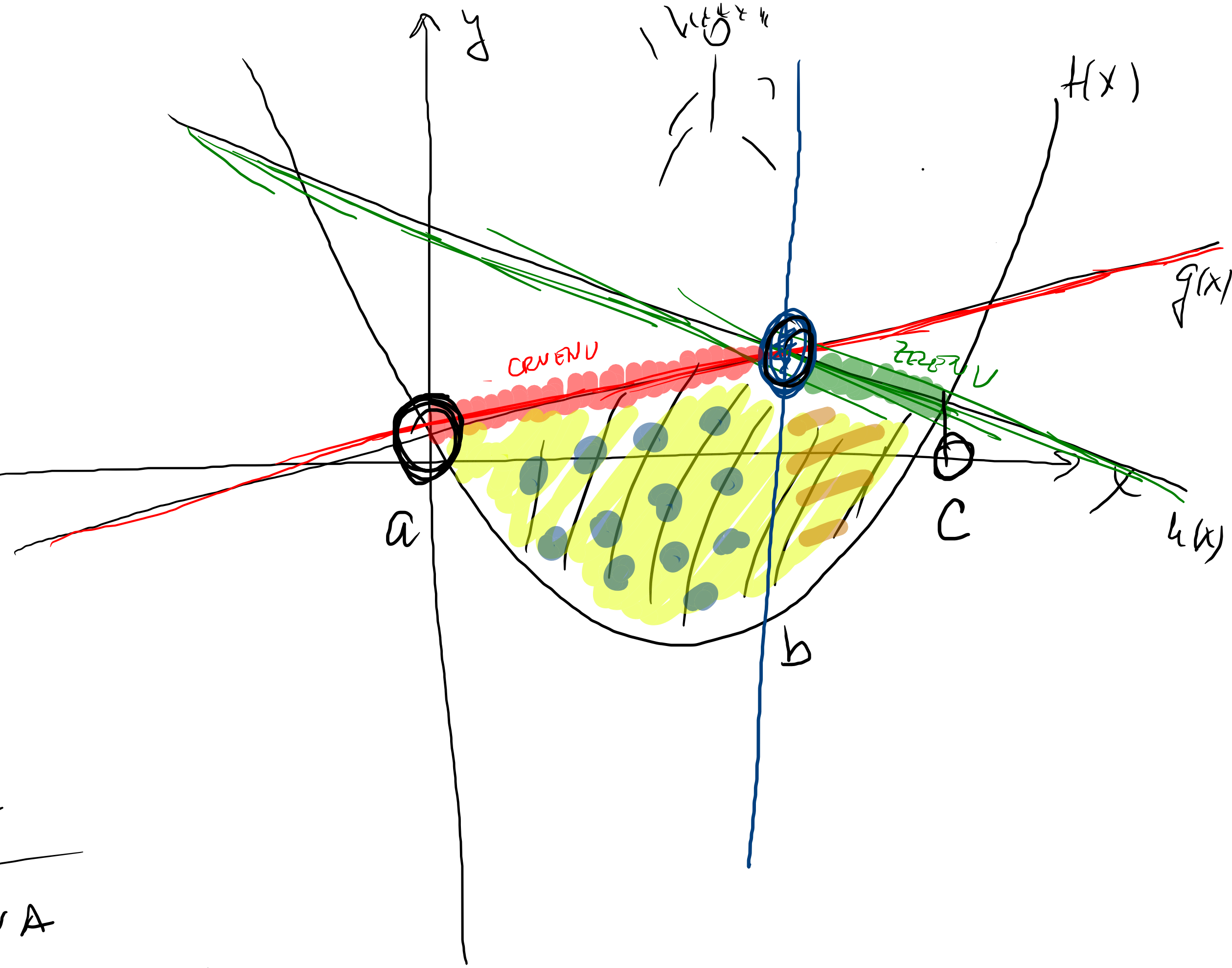
x_1

x_2

$x_1 < x_2$

$$P_{::} = \int_a^b \left(g(x) - f(x) \right) dx$$

$$P_{\equiv} = \int_b^c \left(h(x) - f(x) \right) dx$$



$$P = P_{::} + P_{\equiv}$$

a: PRESEK CRNA i CRVENA

b: PRESEK CRVENE i ZELENE

c: PRESEK CRNE i ZELENE

(1) $$\int \frac{\sin^2 x}{\cos x}\,dx = \int \frac{1-\cos^2 x}{\cos x}\,dx = \int \frac{1}{\cos x} - \int \cos x\,dx$$

$\sin x$

$$\int \frac{1}{\cos x}\,dx = \int \frac{1}{\frac{1-t^2}{\cancel{1+t^2}}}\cdot\frac{2\,dt}{\cancel{1+t^2}} = 2\int \frac{1}{(1-t)(1+t)}\,dt = \int\frac{dt}{1-t} + \int\frac{dt}{1+t}$$

$$\frac{1}{(1-t)(1+t)} = \frac{A}{1-t} + \frac{B}{1+t} \quad / \cdot (1-t)(1+t)$$

$$1 = A(1+t) + B(1-t)$$

$$1 = A + At + B - Bt$$

$$A + B = 1$$

$$A - B = 0$$

$$2A = 1$$

$$A = \frac{1}{2} = B$$

$$= -\ln|1-t| + \ln|1+t| + C$$

$$= -\ln\left|1-\mathrm{tg}\frac{x}{2}\right| + \ln\left|1+\mathrm{tg}\frac{x}{2}\right| + C$$

$$\int \frac{1}{\cos x}\,dx = \int \frac{1}{\cos^2 x}\cdot\cos x\,dx = \int \frac{1}{1-\sin^2 x}\cos x\,dx$$

$\sin x = u$

$\cos x\,dx = du$

$$\int \frac{1}{1-u^2}\,du = \int \frac{1}{(1-u)(1+u)}\,du$$

⑤ $z(x,y) = -x^3 - y^3 + 3xy$

$$\frac{\partial z}{\partial x} = \boxed{-3x^2 + 3y}$$

$$\frac{\partial z}{\partial y} = -3y^2 + 3x$$

$$\frac{\partial^2 z}{\partial x^2} = -6x$$

$$\frac{\partial^2 z}{\partial x \partial y} = 3$$

$$\frac{\partial^2 z}{\partial y^2} = -6y$$

$$-3x^2 + 3y = 0 \quad /:3$$
$$-3y^2 + 3x = 0 \quad /:3$$

$$y = x^2$$
$$x = y^2$$

$$x = x^4$$

$$x - x^4 = 0$$

$$x(1 - x^3) = 0$$

$$x = 0 \qquad x = 1$$

$$y = 0 \qquad y = 1$$

$T_1(0,0)$, $T_2(1,1)$

$\boxed{T_1(0,0)}$:

$$r = \frac{\partial^2 z}{\partial x^2}(0,0) = 0$$

$$s = \frac{\partial^2 z}{\partial y^2}(0,0) = 0$$

$$t = \frac{\partial^2 z}{\partial x \partial y}(0,0) = 3$$

$rs - t^2 = 0 - 9 = -9 < 0$

NEMA E.V. u $T_1(0,0)$

$\boxed{T_2(1,1)}$:

$$r = \frac{\partial^2 z}{\partial x^2}(1,1) = -6$$

$$s = \frac{\partial^2 z}{\partial y^2}(1,1) = -6$$

$$t = \frac{\partial^2 z}{\partial x \partial y}(1,1) = 3$$

$rs - t^2 = (-6)(-6) - 9 = 36 - 9 = 25 > 0$

$\Rightarrow$ IMA E.V. u $T_2(1,1)$

$\boxed{r = -6 < 0}$

$\Rightarrow$ MAX

$z_{MAX}(1,1) = -1 - 1 + 3 = 1$

⑥ $z = x^4 + y^4 - x^2 - 2xy - y^2$

$T_1(0,0)$,

$r = (12x^2 - 2)(0,0) = -2$

$s = (12y^2 - 2)(0,0) = -2$

$t = -2$

$rs - t^2 = 4 - 4 = 0$

NEMA ODGOVORA

$\frac{\partial z}{\partial x} = 4x^3 - 2x - 2y$

$\frac{\partial z}{\partial y} = 4y^3 - 2x - 2y$

$\frac{\partial^2 z}{\partial x^2} = 12x^2 - 2$

$\frac{\partial^2 z}{\partial x \partial y} = -2$

$\frac{\partial^2 z}{\partial y^2} = 12y^2 - 2$

$d^2z = \frac{\partial^2 z}{\partial x^2}dx^2 + 2\frac{\partial^2 z}{\partial x \partial y}dxdy + \frac{\partial^2 z}{\partial y^2}dy^2$

$= (12x^2 - 2)dx^2 - 4dxdy + (12y^2 - 2)dy^2$

$d^2z(0,0) = -2dx^2 - 4dxdy - 2dy^2$

$= -2(dx^2 + 2dxdy + dy^2) = -2(dx+dy)^2$

$< 0 \Rightarrow z_{max}(0,0) = 0$

7

$$y' - y = x^2 y^3$$

$$y = uv \Rightarrow y' = u'v + uv'$$

$$u'v + \underbrace{uv' - uv} = x^2 u^3 v^3$$

$$u'v + u(\underbrace{v' - v}_{=0}) = x^2 u^3 v^3$$

$$v' - v = 0$$

$$v' = v$$

$$\frac{dv}{dx} = v$$

$$dv = v\,dx$$

$$\int \frac{1}{v}\,dv = \int dx$$

$$\ln v = x$$

$$\boxed{v = e^x}$$

$$u'\cancel{e^x} = x^2 u^3 (e^x)^3$$

$$u' = x^2 u^3 e^{2x}$$

$$\frac{du}{dx} = x^2 u^3 e^{2x}$$

$$du = x^2 u^3 e^{2x}\,dx$$

$$\int \frac{1}{u^3}\,du = \int x^2 e^{2x}\,dx$$

$$\frac{u^{-2}}{-2} = \left(\frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4}\right)e^{2x} + C$$

$$\frac{1}{u^2} = \left(-x^2 + x - \frac{1}{2}\right)e^{2x} + C$$

$$u = \frac{1}{\sqrt{(-x^2 + x - \frac{1}{2}) e^{2x} + C}}$$

$$\int x^2 e^{2x}\,dx =$$

$u = x^2$ $\quad dv = e^{2x}dx$

$du = 2x\,dx$ $\quad v = \frac{1}{2}e^{2x}$

$$= \frac{1}{2}x^2 e^{2x} - \int x e^{2x}\,dx =$$

$u = x$ $\quad dv = e^{2x}dx$

$du = dx$ $\quad v = \frac{1}{2}e^{2x}$

$$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{2}\int e^{2x}\,dx$$

$$= \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x} + C$$

8

$$x^2y' - y^2 = x^2 e^{\frac{y}{x}} \quad / : x^2$$

$$y' - \frac{y^2}{x^2} = e^{\frac{y}{x}}$$

$$y' = \left(\frac{y}{x}\right)^2 + e^{\frac{y}{x}}$$

$$\frac{y}{x} = t \rightarrow \quad y = t \cdot x$$

$$y' = t'x + t$$

$$t'x + t = t^2 + e^t$$

$$t'x = t^2 + e^t - t$$

$$\frac{dt}{dx} x = t^2 - t + e^t$$

$$dt \cdot x = (t^2 - t + e^t)\, dx$$

$$\int \frac{dt}{t^2 - t + e^t} = \int \frac{dx}{x}$$

NE ANDIAMO DA RETIMO