① $y = f(x) = \dfrac{x^2+1}{(x-2)^2}$

1) $D = \mathbb{R} \setminus \{2\}$

$x - 2 \neq 0$

$x \neq 2$

2) $y = 0$ $\quad x^2+1 \neq 0 \ \forall x \in \mathbb{R}$

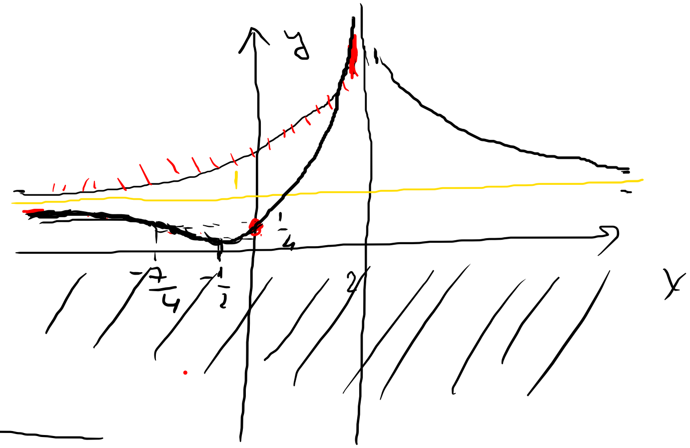
NEMA NULE

$x = 0 \quad f(0) = \frac{1}{4}$

3) $y > 0 \quad \forall x \in D$

$\dfrac{x^2+1}{(x-2)^2} > 0$

4) NI - NI



$\dfrac{1}{x}$ $\quad x \to 0$

$\dfrac{1}{x} \to \infty$ $\quad x \to 0^+$

$\dfrac{1}{x} \to -\infty$ $\quad x \to 0^-$

5) V. A.

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^2+1}{(x-2)^2} = \frac{5}{0^+} = +\infty$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{x^2+1}{(x-2)^2} = \frac{5}{0^+} = +\infty$$

$\boxed{x = 2}$ je V. A.

H. A.

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x^2+1}{(x-2)^2} = \lim_{x \to \pm\infty} \frac{x^2+1}{x^2-4x+4} = 1$$

$\boxed{y = 1}$ JE H. A.

K. A. NEMA JER IMA HORIZONTALNA A.

6. $y = \dfrac{x^2+1}{(x-2)^2}$

$$y' = \frac{2x(x-2)^2 - (x^2+1)\cdot 2(x-2)}{(x-2)^4} = \frac{2\cancel{(x-2)}\left(x(x-2) - (x^2+1)\right)}{(x-2)^{\cancel{4}\,3}} =$$

$$= \frac{2(\cancel{x^2} - 2x - \cancel{x^2} - 1)}{(x-2)^3} = \frac{-2(2x+1)}{(x-2)^3} = \frac{-4x-2}{(x-2)^3}, \quad x \neq 2$$

$y' = 0 \qquad 2x+1 = 0$

$x = -\frac{1}{2}$ KANDIDAT ~~ZA~~ E.V

$y' > 0 \qquad \dfrac{-2(2x+1)}{(x-2)^3} > 0$

	$-\frac{1}{2}$		2	
-2	$-$		$-$	$-$
$2x+1$	$-$ 0		$+$	$+$
$(x-2)^3$	$-$		$-$ 0	$+$
y'	$-$		$+$	$-$
y	$\searrow$	min	$\nearrow$ $\notin D$	$\searrow$

$y \nearrow \quad x \in \left(-\frac{1}{2}, 2\right)$

$y \searrow \quad x \in \left(-\infty, -\frac{1}{2}\right) \cup (2, +\infty)$

$x = -\frac{1}{2} \qquad y_{min}\left(-\frac{1}{2}\right) = \dfrac{\left(-\frac{1}{2}\right)^2+1}{\left(-\frac{1}{2}-2\right)^2} = \dfrac{\frac{5}{\cancel{4}}}{\frac{25}{\cancel{4}}} = \frac{1}{5}$

7) $y' = \dfrac{-2(2x+1)}{(x-2)^3} = \dfrac{-4x-2}{(x-2)^3}$

$$y'' = \frac{-4(x-2)^{3} - (-4x-2)\,3\cdot(x-2)^2}{(x-2)^6} = \frac{\cancel{(x-2)^2}\left(-4(x-2) - 3\cdot(-4x-2)\right)}{(x-2)^{\cancel{6}\,4}}$$

$$= \frac{-4x+8+12x+6}{(x-2)^4} = \frac{8x+14}{(x-2)^4}, \quad x \neq 2$$

$y''=0$

$8x+14=0$

$x = -\frac{14}{8}$

$\boxed{x=-\frac{7}{4}}$

KANDIDAT ZA PREVOJNU T.

$y''>0 \quad \dfrac{8x+14}{(x-2)^4} > 0$

$(x-2)^4 > 0 \ \forall x \in D$

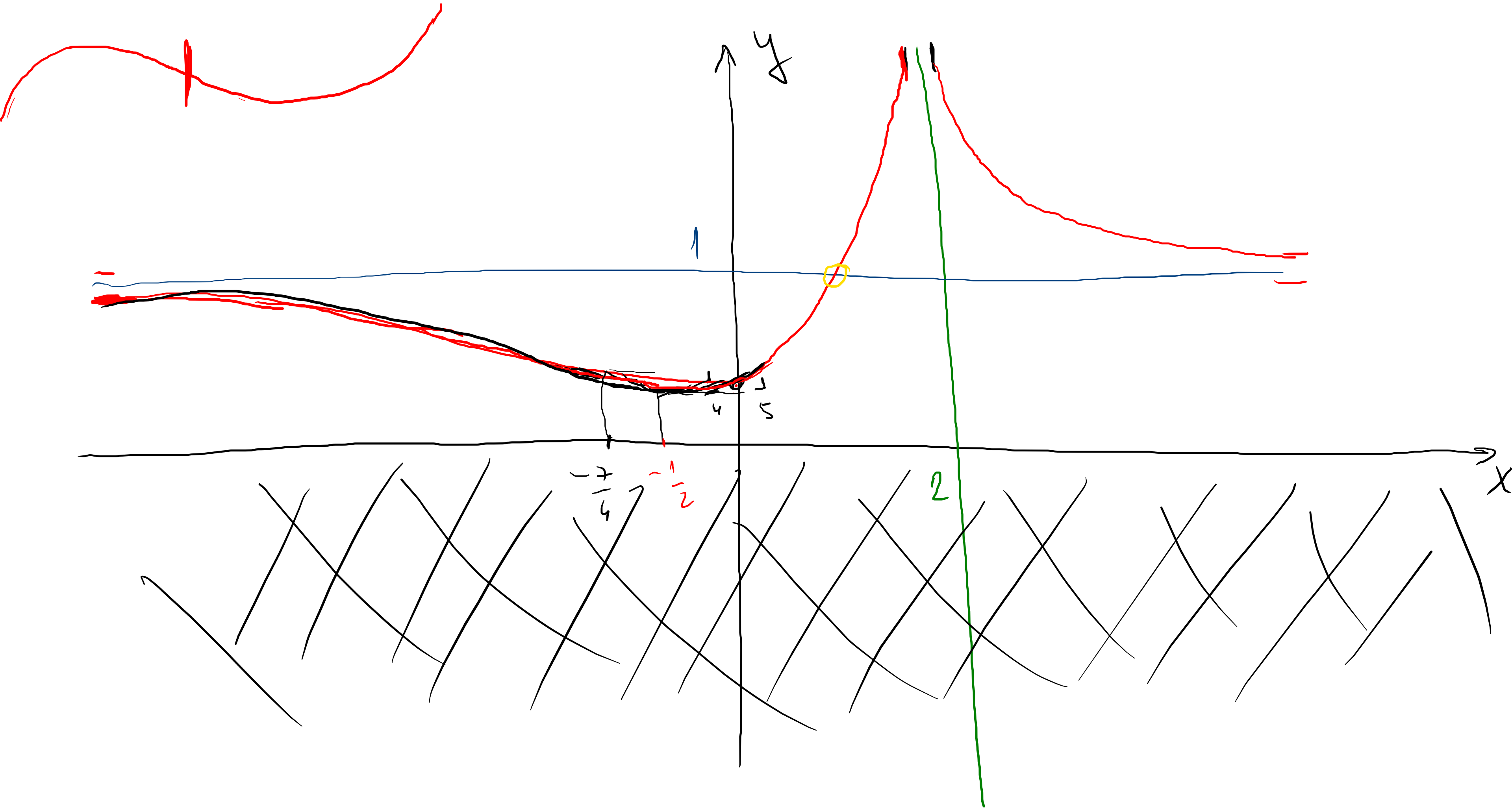
$8x+14>0$

$x > -\frac{7}{4}$

$y''>0 \quad y \cup \quad x\in\left(-\frac{7}{4}, 2\right)\cup(2,\infty)$

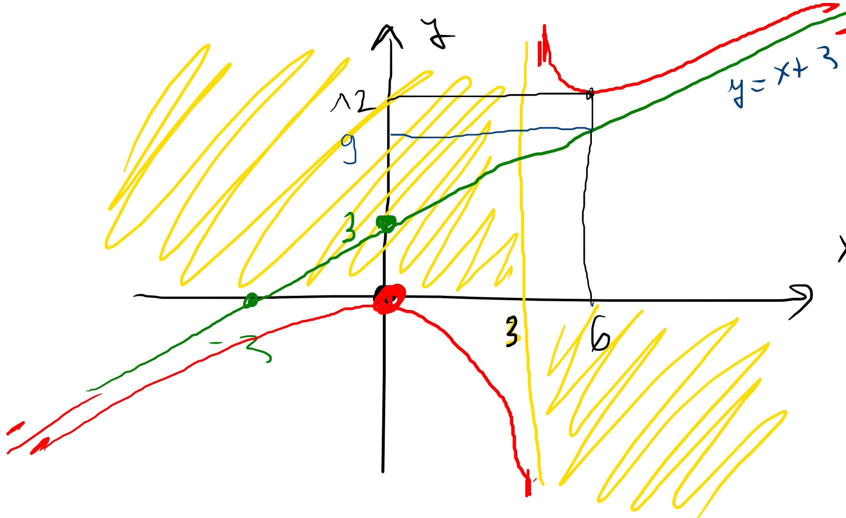
$y''<0 \quad y \cap \quad x \in \left(-\infty, -\frac{7}{4}\right)$

$x_{PT.} = -\frac{7}{4} \quad y\left(-\frac{7}{4}\right) = \dfrac{\left(-\frac{7}{2}\right)^2+1}{\left(-\frac{7}{2}-2\right)^2} = \dfrac{\frac{53}{\cancel{4}}}{\frac{81}{\cancel{4}}} = \frac{53}{81}$



y
x
1
$-\frac{7}{4}$
$-\frac{1}{2}$
2
$\frac{1}{4}$
$\frac{1}{5}$

(2) $y = \dfrac{x^2}{x-3}$



$y = \dfrac{\text{Pol } P}{\text{Pol } Q}$ — BRO, 0, ∞

$\deg(p) = \deg(q)$ – BRO
$\deg(p) < \deg(q)$ – 0
$\deg(p) > \deg(q)$ – ∞

$y = x + 3$

$x = 0 \to y = 3$

$x = -3 \to y = 0$

1, $D = \mathbb{R} \setminus \{3\}$

2, $y = 0 \quad x = 0$
$x = 0 \quad y = 0$

3, $y > 0$ $\quad \dfrac{x^2}{x-3} > 0$, $\quad x^2 > 0$

$x - 3 > 0$

$x > 3$

$y > 0 \quad x \in (3, \infty)$

$y < 0 \quad x \in (-\infty, 3)$

4, NI – NI

5, V.A.

$\lim\limits_{x \to 3^-} \dfrac{x^2}{x-3} = \dfrac{9}{0^-} = -\infty$ $\quad \boxed{x = 3}$

$\lim\limits_{x \to 3^+} \dfrac{x^2}{x-3} = \dfrac{9}{0^+} = +\infty$ $\quad$ V.A.

H.A. $\lim\limits_{x \to \pm\infty} \dfrac{x^2}{x-3} = \pm\infty$ $\quad$ NEMA

K.A. $y = kx + n$

$k = \lim\limits_{x \to \pm\infty} \dfrac{f(x)}{x} = \lim\limits_{x \to \pm\infty} \dfrac{\frac{x^2}{x-3}}{\frac{x}{1}} = \lim\limits_{x \to \pm\infty} \dfrac{x}{x-3} = 1$

$n = \lim\limits_{x \to \pm\infty} (f(x) - kx) = \lim\limits_{x \to \pm\infty} \left(\dfrac{x^2}{x-3} - x \right) = \lim\limits_{x \to \pm\infty} \dfrac{\cancel{x^2} - \cancel{x^2} + 3x}{x-3}$

$= \lim\limits_{x \to \pm\infty} \dfrac{3x}{x-3} = 3$ $\quad \boxed{y = x + 3}$ $\quad$ K.A.

$$y = \frac{x^2}{x-3}$$

6. $$y' = \frac{2x(x-3) - x^2}{(x-3)^2} = \frac{2x^2 - 6x - x^2}{(x-3)^2} = \boxed{\frac{x^2-6x}{(x-3)^2}} = \boxed{\frac{x(x-6)}{(x-3)^2}}$$

$x \neq 3$

$y' = 0 \qquad x(x-6) = 0$

$\boxed{x = 0} \lor x - 6 = 0$

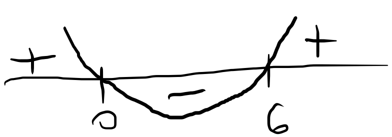
$\boxed{x = 6}$

KANDIDATI ZA E.V.

$y' > 0 \qquad \dfrac{x(x-6)}{(x-3)^2} > 0$

$(x-3)^2 > 0 \ \forall x \in D$

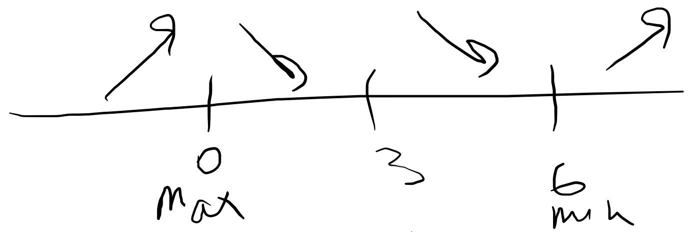
$x(x-6) > 0$



	0		6	
x	−	0 +		+
$x-6$	−		− 0	+
	+		−	+

$y' > 0 \qquad x \in (-\infty, 0) \cup (6, \infty) \qquad y \nearrow$

$y' < 0 \qquad x \in (0, 3) \cup (3, 6) \qquad y \searrow$



$x = 0 \qquad y_{max}(0) = \dfrac{0^2}{0-3} = 0$

$x = 6 \qquad y_{min}(6) = \dfrac{6^2}{6-3} = \dfrac{36}{3} = 12$

$$y' = \frac{x^2-6x}{(x-3)^2}$$

$$y'' = \frac{(2x-6)(x-3)^2 - (x^2-6x)\cdot 2(x-3)}{(x-3)^4} = \frac{\cancel{(x-3)}\left((2x-6)(x-3) - 2(x^2-6x)\right)}{(x-3)^{\cancel{4}\ 3}}$$

$$= \frac{\cancel{2x^2} - \cancel{6x} - \cancel{6x} + 18 - \cancel{2x^2} + \cancel{12x}}{(x-3)^3} = \frac{18}{(x-3)^3} \qquad x \neq 3$$

$y'' \neq 0 \quad \forall x \in D \Rightarrow$ NEMA P.T.

$$y'' > 0 \qquad \frac{18}{(x-3)^3} > 0$$

$$x - 3 > 0$$

$$x > 3$$

$y'' > 0 \quad y \cup \quad x \in (3, \infty)$

$y'' < 0 \quad y \cap \quad x \in (-\infty, 3)$

$\cap \quad 3 \quad \cup$

NIJE P.T.

$3 \notin D$

(3)

$$y = \sqrt{\sin^5 \ln \operatorname{arctg} e^{\frac{x+5}{x-7}}}$$

$\sin^5 x = (\sin x)^5$

$$y' = \frac{1}{2\sqrt{\sin^5 \ln \operatorname{arctg} e^{\frac{x+5}{x-7}}}} \cdot 5 \cdot \sin^4 \ln \operatorname{arctg} e^{\frac{x+5}{x-7}} \cdot$$

$$\cdot \cos \ln \operatorname{arctg} e^{\frac{x+5}{x-7}} \cdot \frac{1}{\operatorname{arctg} e^{\frac{x+5}{x-7}}} \cdot$$

$$\cdot \frac{1}{1+\left(e^{\frac{x+5}{x-7}}\right)^2} \cdot e^{\frac{x+5}{x-7}} \quad \frac{x-7-(x+5)}{(x-7)^2}$$

(4)

$f(x) \qquad a$

$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

$$\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)$$

$$f(x) = \begin{cases} \dfrac{\sin 2x}{x}, & x < 0 \\ a, & x = 0 \\ b\left(e^{-\frac{1}{x}} + 3\right), & x > 0 \end{cases}$$

$$\underbrace{\lim_{x \to 0^+} f(x)}_{3b} = \underbrace{\lim_{x \to 0^-} f(x)}_{2} = \underbrace{f(0)}_{a}$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} b\left(e^{-\frac{1}{x}} + 3\right) = b(0+3) = 3b$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin 2x}{2x} \, 2 = 2$$

$$3b = 2 = a \rightarrow a = 2, \; b = \frac{2}{3}$$