1

$$\boxed{y' = f\left(\frac{y}{x}\right)}$$

$$y' = \left(\frac{y}{x}\right)^2 + \left(\frac{y}{x}\right) + 2$$

$$y' = \sqrt{\frac{y}{x}} + \sin\frac{y}{x}$$

$$y' = tg\frac{y}{x} + e^{\frac{y}{x}}$$

$$x^2 y' = y^2 + x^2 \ln\frac{y}{x} \quad / : x^2$$

$$y' = \left(\frac{y}{x}\right)^2 + \ln\frac{y}{x}$$

$$y' + \frac{y}{x} = x + 2$$

$$y' + \left(\frac{y}{x}\right)^2 = \frac{y}{x} + 2$$

$$y' = \boxed{f\left(\frac{y}{x}\right)}$$

$$y' = \frac{y}{x} + a$$

$$y' = a\left(\frac{y}{x}\right)^2 + b\left(\frac{y}{x}\right) + c$$

(2)

$$y' + f(x)\,y = g(x)\,y^n$$

LINEARNA

BERNULIJEVA

$$y' + \frac{y}{x} = x+2$$

$$y' + \frac{1}{x}\,y = x+2$$

$f(x)$ $g(x)$

$$y' + y = e^x$$

$$y' + \ln x \cdot y = \sin x$$

$$x^2 y' + y = \ln x \quad / : x^2$$

$$y' + \frac{1}{x^2}\,y = \frac{\ln x}{x^2}$$

$$y' + 2xy = x^3 y^2$$

$$y' + \frac{1}{x}\,y = \ln x \cdot \sqrt[3]{y}$$

(3)

$$y' + \frac{y}{x} = (x+2)\,y^2$$

$$y = uv \Rightarrow y' = u'v + uv'$$

$$\underbrace{u'v + uv' + \frac{1}{x}uv}_{} = (x+2)\,u^2v^2$$

$$u'v + u\underbrace{\left(v' + \frac{1}{x}v\right)}_{=0} = (x+2)\,u^2v^2$$

$$v' + \frac{1}{x}v = 0$$

$$v' = -\frac{1}{x}v$$

$$\frac{dv}{dx} = -\frac{1}{x}v$$

$$dv = -\frac{1}{x}v\,dx$$

$$\frac{1}{v}dv = -\frac{1}{x}dx$$

$$\int \frac{1}{v}dv = -\int \frac{1}{x}dx$$

$$\ln v = -\ln x$$

$$v \neq -x$$

$$\ln v = \ln x^{-1}$$

$$v = x^{-1}$$

$$\boxed{v = \frac{1}{x}}$$

$$\frac{1}{x}u' = (x+2)\,u^2\frac{1}{x^2}$$

$$\frac{1}{x}\frac{du}{dx} = x+2$$

$$\frac{1}{x}du = (x+2)dx$$

$$du = x(x+2)dx$$

$$\int du = \int (x^2+2x)dx$$

$$\boxed{u = \frac{x^3}{3} + 2\frac{x^2}{2} + C}$$

$$y = uv = \left(\frac{x^3}{3} + x^2 + C\right)\cdot\frac{1}{x}$$

5. REŠITI POČETNI PROBLEM

$x^2 y' = y^2 + xy, \quad y(1) = 2$

$$x^2 y' = y^2 + xy \quad / : x^2$$

$$y' = \left(\frac{y}{x}\right)^2 + \left(\frac{y}{x}\right)$$

$\frac{y}{x} = t$

$y = tx, \quad t = t(x)$

$y' = t'x + t$

$t'x + \cancel{t} = t^2 + \cancel{t}$

$t'x = t^2$

$\frac{dt}{dx} x = t^2$

$dt \cdot x = t^2 dx$

$\frac{1}{t^2} dt = \frac{1}{x} dx$

$\int \frac{1}{t^2} dt = \int \frac{1}{x} dx$

$-\frac{1}{t} = \ln x + C$

OPŠTE REŠENJE

$\boxed{-\frac{x}{y} = \ln x + C}$

$y(1) = 2$

$x = 1 \to y = 2$

$-\frac{1}{2} = \cancel{\ln 1}^{0} + C$

$C = -\frac{1}{2}$

$\boxed{-\frac{x}{y} = \ln x - \frac{1}{2}}$ PARTIKULARNO R.

4

$$\frac{1}{x}u' = (x+2)u^2\frac{1}{x^2}$$

$$\frac{du}{dx} = (x+2)u^2\frac{1}{x}$$

$$du = (x+2)u^2\frac{1}{x}dx$$

$$\frac{1}{u^2}du = \left(1+\frac{2}{x}\right)dx$$

$$\int\frac{1}{u^2}du = \int\left(1+\frac{2}{x}\right)dx$$

$$-\frac{1}{u} = x+2\ln x + C$$

$$u = \frac{-1}{x+2\ln x+C}$$

$$y = uv$$

$$\int \frac{2x^2+9x+8}{x^2+3x+2}\,dx = \int\left(2+\frac{3x+4}{x^2+3x+2}\right)dx =$$

$$(2x^2+9x+8):(x^2+3x+2) = 2$$
$$\frac{-(2x^2+6x+4)}{3x+4}$$

$$= \int 2\,dx + \int \frac{3x+4}{x^2+3x+2}\,dx = 2x + I_1$$

$$\int \frac{3x+4}{x^2+3x+2}\,dx = \int \frac{3x+4}{(x+1)(x+2)}\,dx = \int\left(\frac{A}{x+1}+\frac{B}{x+2}\right)dx =$$

$$= A\int\frac{1}{x+1}\,dx + B\int\frac{1}{x+2}\,dx$$

$$= A\ln|x+1| + B\ln|x+2| + C$$

$$x_{1,2} = \frac{-3\pm\sqrt{9-8}}{2} = \frac{-3\pm 1}{2} \begin{cases} -1 \\ -2 \end{cases}$$

$$\frac{3x+4}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

$A, B \ldots\ldots$

$$\frac{1}{x^2(x^2+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+2}$$

1 5, $z(x,y)$

2 a 5 b 5 c 5

PARCIJALNA

RACIONALNE

IRACIONALNA

TRIGONOMETRIJSKA

3 POVRŠINA

4 DIF. JED.

$$\int \frac{x+2}{\sqrt{x^2+4x+5}}\,dx = \underbrace{A\cdot\sqrt{x^2+4x+5}} + \lambda \underbrace{\int \frac{1}{\sqrt{x^2+4x+5}}\,dx}\Big/'$$

$$\frac{x+2}{\sqrt{x^2+4x+5}} = A\,\frac{1}{2\sqrt{x^2+4x+5}}\,\overset{2(x+2)}{(2x+4)} + \lambda\,\frac{1}{\sqrt{x^2+4x+5}} \Big/ \sqrt{x^2+4x+5}$$

$$x+2 = A(x+2)+\lambda$$

$$x+2 = Ax+2A+\lambda$$

$$\boxed{A=1}$$

$$2A+\lambda=2$$

$$\boxed{\lambda=6}$$

$$\int \frac{x+8}{\sqrt{x^2+4x+5}}\,dx = \sqrt{x^2+4x+5}+6\int\frac{1}{\sqrt{x^2+4x+5}}$$

$$\int \frac{x^2+8}{\sqrt{x^2+4x+5}}\,dx = \left(\frac{1}{2}x-3\right)\sqrt{x^2+4x+5} + \frac{25}{2}\int\frac{1}{\sqrt{x^2+4x+5}}\,dx$$

$$\int \frac{x^2+8}{\sqrt{x^2+4x+5}}\,dx = \underbrace{(Ax+B)\cdot\sqrt{x^2+4x+5}} + \underbrace{\lambda\int\frac{1}{\sqrt{x^2+4x+5}}\,dx}\quad /'$$

$$\frac{x^2+8}{\sqrt{x^2+4x+5}} = (Ax+B)'\sqrt{x^2+4x+5} + (Ax+B)\left(\sqrt{x^2+4x+5}\right)' + \lambda\frac{1}{\sqrt{x^2+4x+5}}$$

$$\frac{x^2+8}{\sqrt{x^2+4x+5}} = A\sqrt{x^2+4x+5} + (Ax+B)\frac{1}{2\sqrt{x^2+4x+5}}\,2(x+2) + \lambda\frac{1}{\sqrt{x^2+4x+5}}$$

$$x^2+8 = A(x^2+4x+5) + (Ax+B)(x+2) + \lambda$$

$$x^2+8 = 2Ax^2 + 6Ax + 5A + Bx + 2B + \lambda$$

$2A = 1$

$A = \frac{1}{2}$

$6A + B = 0$

$B = -3$

$5A + 2B + \lambda = 8$

$\lambda = \frac{25}{2}$

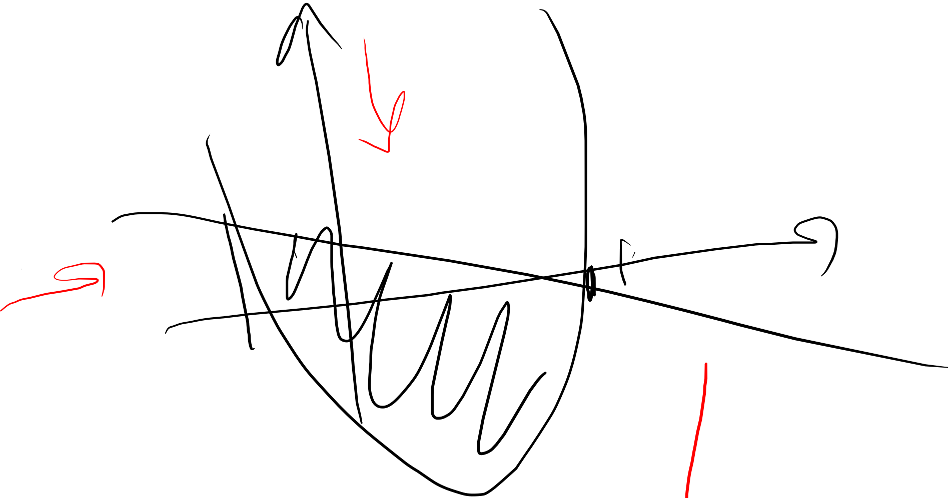
$$\int \frac{1}{\sqrt{x^2+4x+5}}\,dx = \int \frac{1}{\sqrt{(x+2)^2+1}}\,dx =$$

$$= \ln\left|x+2+\sqrt{(x+2)^2+1}\right| + C$$

$$x^2+4x+5 = \underbrace{x^2+2x\cdot 2+4}-4+5$$

$$= (x+2)^2+1$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} = \ln\left|x+\sqrt{x^2 \pm a^2}\right| + C$$



DESNI PRESEK

$$P = \int (\text{GORNJA} - \text{DONJA})\,dx$$

LEVI PRESEK