(1)

$$\boxed{tg\frac{x}{2}=t}$$

$$x = 2\,arctg\, t$$

$$dx = \frac{2dt}{1+t^2}$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$\int \frac{dx}{\sin x+\cos x+1} = \int \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}+1}$$

$$= \int \frac{\frac{2dt}{\cancel{1+t^2}}}{\frac{2t+1-\cancel{t^2}+1+\cancel{t^2}}{\cancel{1+t^2}}} = \int \frac{2dt}{2t+2}$$

$$= \int \frac{dt}{t+1} = \int \frac{du}{u} = \ln|u| + C$$

$$= \ln|t+1| + C$$

$$= \ln\left|tg\frac{x}{2}+1\right| + C$$

Smena: $t+1=u$

$dt = du$

(2)

$$\int \frac{\sin x}{\cos^2 x + 3}\,dx = \cancel{\int \frac{\frac{2t}{1+t^2}}{\left(\frac{1-t^2}{1+t^2}\right)^2 + 3} \cdot \frac{2\,dt}{1+t^2}}$$

$$\int r(\overset{t}{\sin x})\,\overset{dt}{\cos x\,dx} \rightarrow$$

smena

$\sin x = t$

$\cos x\,dx = dt$

$$\int r(\overset{t}{\cos x})\,\overset{-dt}{\sin x\,dx}$$

smena

$\cos x = t$

$-\sin x\,dx = dt$

$\sin x\,dx = -dt$

$$= \int \frac{1}{\cos^2 x + 3} \cdot \underbrace{\sin x\,dx} = -\int \frac{1}{t^2+3}\,dt$$

$$= -\frac{1}{\sqrt{3}} \operatorname{arctg} \frac{t}{\sqrt{3}} + C$$

$$= -\frac{1}{\sqrt{3}} \operatorname{arctg} \frac{\cos x}{\sqrt{3}} + C$$

smena:

$\cos x = t$

$-\sin x\,dx = dt$

$\sin x\,dx = -dt$

(3)

$$\int \frac{\cos x}{-2\cos^2 x - \sin x + 3}\, dx$$

$$= \int \frac{1}{-2\cos^2 x - \sin x + 3} \cos x\, dx = \int \frac{1}{-2(1-\sin^2 \alpha) - \sin x + 3} \cos x\, dx$$

$\sin^2\alpha + \cos^2\alpha = 1$

$$= \int \frac{1}{1 + 2\sin^2 x - \sin x} \cos x\, dx$$

смяна $\sin x = t$

$\cos x\, dx = dt$

$$= \int \frac{1}{2t^2 - t + 1}\, dt = \frac{1}{2} \int \frac{dt}{\left(t - \frac{1}{4}\right)^2 + \frac{7}{16}} =$$

$$2t^2 - t + 1 = 2\left(t^2 - \frac{1}{2}t + \frac{1}{2}\right)$$

$$= 2\left(t^2 - 2t \boxed{\tfrac{1}{4}} + \frac{1}{16} - \frac{1}{16} + \frac{1}{2}\right) = 2\left(\left(t - \frac{1}{4}\right)^2 + \frac{7}{16}\right)$$

$$t_{1,2} = \frac{1 \pm \sqrt{1-8}}{2} \notin \mathbb{R}$$

$$\cancel{\circledast} = \frac{1}{2}\int \frac{dt}{\left(t-\frac{1}{4}\right)^2+\frac{7}{16}} = \frac{1}{2}\int \frac{du}{u^2+\left(\frac{\sqrt{7}}{4}\right)^2} =$$

smeno: $t-\frac{1}{4}=u$

$dt = du$

$$= \frac{1}{\cancel{2}}\,\frac{1}{\frac{\sqrt{7}}{\cancel{4}}}\,\text{arctg}\,\frac{u}{\frac{\sqrt{7}}{4}}+C$$

$$= \frac{2}{\sqrt{7}}\,\text{arctg}\,\frac{t-\frac{1}{4}}{\frac{\sqrt{7}}{4}}+C$$

$$= \frac{2}{\sqrt{7}}\,\text{arctg}\,\frac{\sin x-\frac{1}{4}}{\frac{\sqrt{7}}{4}}+C$$