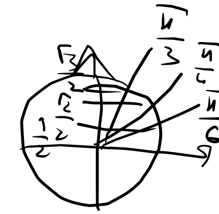
TRIGONOMETRIJA - NASTAVAK

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin(-\alpha) = -\sin\alpha$

$\cos(-\alpha) = \cos\alpha$



1) $\underbrace{\sin x + \sin 9x} = \sqrt{2}\cos 4x$

$2\sin\frac{x+9x}{2}\cos\frac{x-9x}{2} = \sqrt{2}\cos 4x$

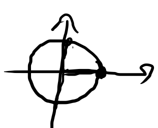
$2\sin 5x \cdot \cos(-4x) = \sqrt{2}\cos 4x$

$2\sin 5x\cos 4x = \sqrt{2}\cos 4x$ ← NIPOŠTO SKRATITI

$2\sin 5x\cos 4x - \sqrt{2}\cos 4x = 0$

$\cos 4x\,(2\sin 5x - \sqrt{2}) = 0$

$\cos 4x = 0 \quad \vee \quad 2\sin 5x - \sqrt{2} = 0$



$4x = \frac{\pi}{2} + k\pi$

$4x = \frac{\pi}{2} + 2k\pi$

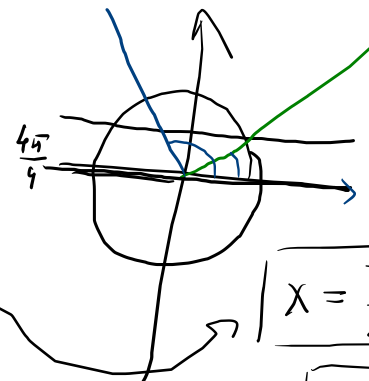
$4x = \frac{3\pi}{2} + 2k\pi$

$\boxed{x = \frac{\pi}{8} + \frac{k\pi}{4}}$ $k \in \mathbb{Z}$

$\sin 5x = \frac{\sqrt{2}}{2}$

$5x = \frac{\pi}{4} + 2k\pi$

$5x = \frac{3\pi}{4} + 2k\pi$



$\boxed{x = \frac{\pi}{20} + \frac{2k\pi}{5}}$

$\boxed{x = \frac{3\pi}{20} + \frac{2k\pi}{5}}$ $k \in \mathbb{Z}$

2) $\sin 5x \cdot \cos 3x - \sin 8x \cdot \cos 6x = 0$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\frac{1}{2}(\sin(5x+3x) + \sin(5x-3x)) - \frac{1}{2}(\sin(8x+6x) + \sin(8x-6x)) = 0$

$\frac{1}{2}(\sin 8x + \sin 2x) - \frac{1}{2}(\sin 14x + \sin 2x) = 0 \,/ \cdot 2$

$\sin 8x + \cancel{\sin 2x} - \sin 14x - \cancel{\sin 2x} = 0$

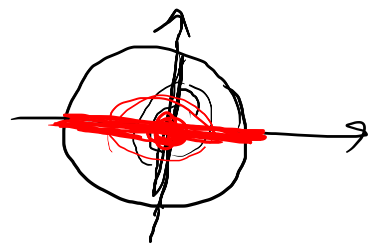
$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\sin 8x - \sin 14x = 0$

$2\cos\frac{8x+14x}{2} \cdot \sin\frac{8x-14x}{2} = 0$

$\sin(-\alpha) = -\sin\alpha$

$2\cos 11x \cdot \sin(-3x) = 0$



$-2\underbrace{\cos 11x}\cdot\underbrace{\sin 3x} = 0$

$\cos 11x = 0 \quad \vee \quad \sin 3x = 0$

$11x = \frac{\pi}{2} + k\pi \qquad 3x = k\pi$

$\boxed{x = \frac{\pi}{22} + \frac{k\pi}{11}}\ k \in \mathbb{Z} \qquad \boxed{x = \frac{k\pi}{3}}\ k \in \mathbb{Z}$

3) $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x$

$\sin\alpha \pm \cos\beta$?

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\cos(-\alpha) = \cos\alpha$

$$2\cdot\sin\frac{x+3x}{2}\cdot\cos\frac{x-3x}{2} + \sin 2x = 2\cos\frac{x+3x}{2}\cdot\cos\frac{x-3x}{2} + \cos 2x$$

$$2\sin 2x\cos(-x) + \sin 2x = 2\cos 2x\cdot\cos(-x) + \cos 2x$$

$$\sin 2x(2\cos x + 1) = \cos 2x(2\cos x + 1)$$

$$\sin 2x(2\cos x + 1) - \cos 2x(2\cos x + 1) = 0$$

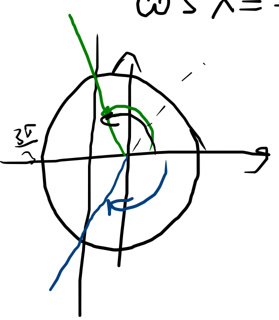
$$(2\cos x + 1)(\sin 2x - \cos 2x) = 0$$

$\frac{1}{2},\ \frac{\sqrt{2}}{2},\ \frac{\sqrt{3}}{2}$

$\frac{\pi}{6},\ \frac{\pi}{4},\ \frac{\pi}{3}$

$2\cos x + 1 = 0 \quad \vee \quad \sin 2x - \cos 2x = 0$

$\cos x = -\frac{1}{2}$

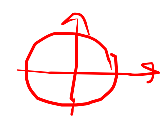


$$\left| \begin{array}{l} x = \frac{2\pi}{3} + 2k\pi \\ x = -\frac{2\pi}{3} + 2k\pi \end{array} \right.$$

$k \in \mathbb{Z}$

$\sin 2x = \cos 2x \quad |:\cos 2x \neq 0$

$2x \neq \frac{\pi}{2} + k\pi$



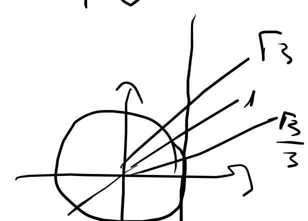
$x \neq \frac{\pi}{4} + \frac{k\pi}{2}$

$\frac{\sin 2x}{\cos 2x} = 1$

$tg\, 2x = 1$

$2x = \frac{\pi}{4} + k\pi$

$\boxed{x = \frac{\pi}{8} + \frac{k\pi}{2}}$



$k \in \mathbb{Z}$

Π ΝΑΘΝ

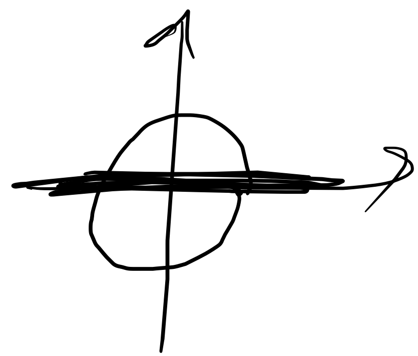
$$\sin 2x - \cos 2x = 0$$

$$\sin 2x - \sin\left(\frac{\pi}{2} - 2x\right) = 0$$

$$2\cos\frac{\cancel{2x} + \frac{\pi}{2} \cancel{- 2x}}{2}\ \sin\frac{2x - \left(\frac{\pi}{2} - 2x\right)}{2} = 0$$

$$2\overbrace{\cos\frac{\pi}{4}}^{\frac{\sqrt{2}}{2}} \cdot \sin\frac{4x - \frac{\pi}{2}}{2} = 0$$

$$\sin\left(2x - \frac{\pi}{4}\right) = 0$$



$$2x - \frac{\pi}{4} = k\pi$$

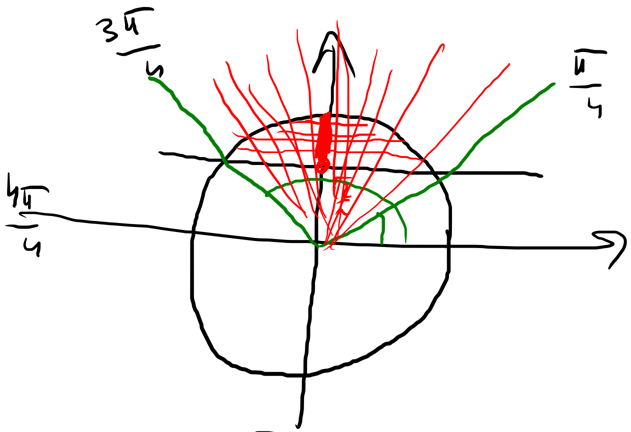
$$2x = \frac{\pi}{4} + k\pi$$

$$x = \frac{\pi}{8} + \frac{k\pi}{2}, \quad k \in \mathbb{Z}$$

④ Odrediti sva rešenja nejednačine:

a) $2\sin x - \sqrt{2} \geq 0$

$\sin x \geq \frac{\sqrt{2}}{2}$



$\sin x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4} + 2k\pi$

$x = \frac{3\pi}{4} + 2k\pi$

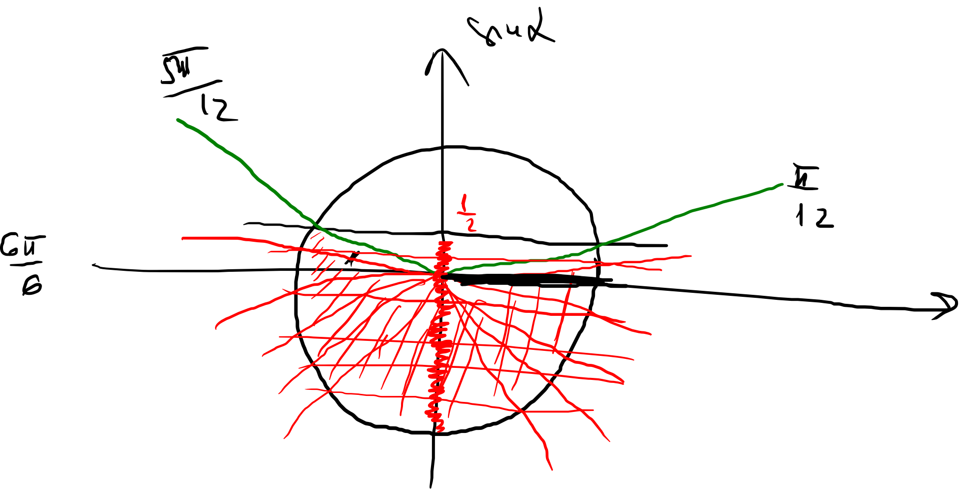
$\frac{\pi}{4} + 2k\pi \leq x \leq \frac{3\pi}{4} + 2k\pi$

$k \in \mathbb{Z}$

b)

$$\sin 2x - \frac{1}{2} < 0 \quad , \quad x \in [0, 2\pi]$$

$$\sin 2x < \frac{1}{2}$$



$$\sin 2x = \frac{1}{2}$$

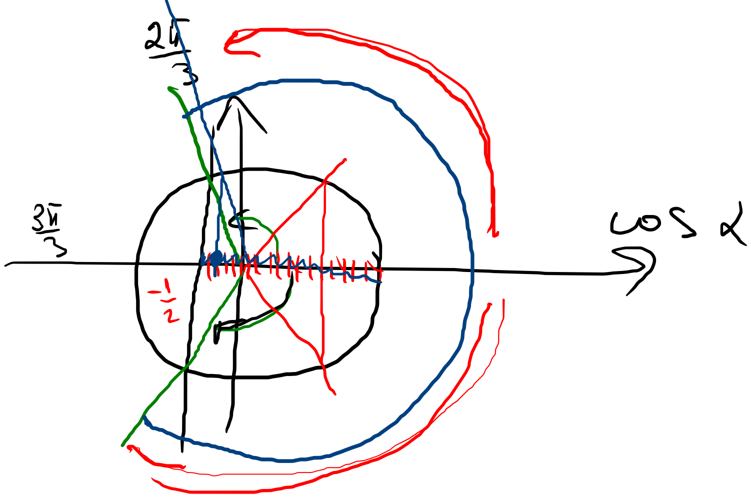
$$2x = \frac{\pi}{6} \quad \vee \quad 2x = \frac{5\pi}{6}$$

$$x = \frac{\pi}{12} \quad \vee \quad x = \frac{5\pi}{12}$$

$$\boxed{0 \le x < \frac{\pi}{12}} \quad \boxed{\frac{5\pi}{12} < x < 2\pi}$$

c) $\cos x + \frac{1}{2} > 0$

$\cos x > -\frac{1}{2}$



$0 \le x < \frac{2\pi}{3} + 2k\pi$

$\frac{4\pi}{3} + 2k\pi < x < 2k\pi$

$k \in Z$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3} + 2k\pi$ $\qquad$ $x = -\frac{2\pi}{3} + 2k\pi$

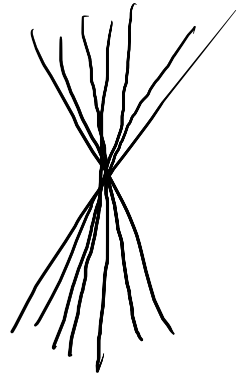
$\boxed{-\frac{2\pi}{3} + 2k\pi < x < \frac{2\pi}{3} + 2k\pi}$ $k \in Z$

d) $4\cos^2 x - 1 < 0$

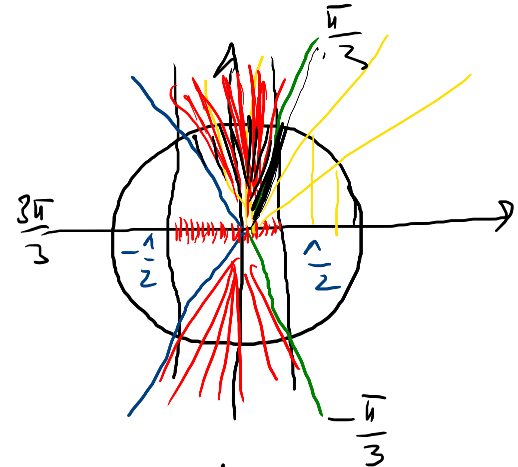
$\cos^2 x < \frac{1}{4}$

$\cos^2 x = \frac{1}{4}$

$\cos x = \pm \frac{1}{2}$



$-\frac{1}{2} < \cos x < \frac{1}{2}$



$\cos x < \frac{1}{2} \quad \wedge \quad \cos x > -\frac{1}{2}$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3} + 2k\pi$

$x = -\frac{\pi}{3} + 2k\pi$

$\cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3} + 2k\pi$

$x = -\frac{2\pi}{3} + 2k\pi$

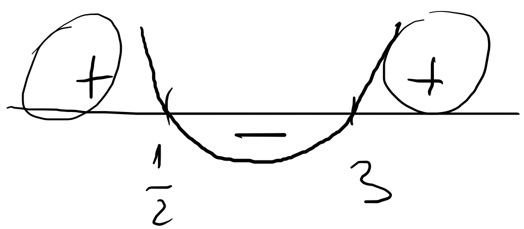
$$\boxed{\frac{\pi}{3} + k\pi < x < \frac{2\pi}{3} + k\pi}, \quad k \in Z$$

e) $2\cos^2 x - 7\cos x + 3 \geq 0$

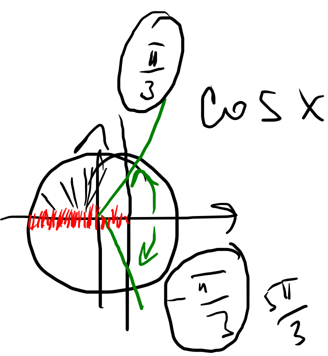
$\cos x = t$ $\qquad 2t^2 - 7t + 3 \geq 0$

$$2t^2 - 7t + 3 = 0$$

$$t_{1,2} = \frac{7 \pm \sqrt{49-24}}{4} = \frac{7 \pm 5}{4} = \begin{cases} 3 \\ \frac{1}{2} \end{cases}$$



$$t \leq \frac{1}{2} \quad \vee \quad t \geq 3$$



$\cos x \leq \frac{1}{2}$ $\qquad$ ~~$\cos x \geq 3$~~

$\cos x \in [-1, 1]$

$x = \frac{\pi}{3} + 2k\pi$

$x = -\frac{\pi}{3} + 2k\pi$

$\frac{\pi}{3} + 2k\pi \leq x \leq \pi + k\pi$

$-\pi + k\pi \leq x \leq -\frac{\pi}{3} + k\pi$

$k \in \mathbb{Z}$

$\frac{\pi}{3} + 2k\pi \leq x \leq \frac{5\pi}{3} + 2k\pi$

$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$
$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$