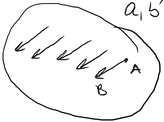
Slobodni vektori

April 27, 2024

Slobodni nenula vektor je skup svih orijentisanih duži koje su međusobno podudarne, paralelne i isto orijentisane. Svaka od tih orijentisanih duži jeste jedan predstavnik tog slobodnog vektora i zove se **vektor**.

$\vec{a}, \vec{b}, \vec{c}$



Slobodni nula vektor je skup svih nultih duži.

Slobodni vektori se obeležavaju sa $\vec{a}$, $\vec{b}$, $\vec{c}$, ...

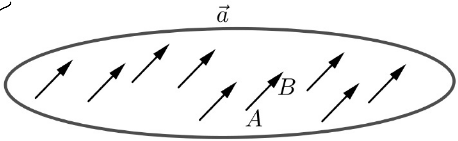
Vektor čija je početna tačka A, a krajnja B obeležava se sa $\overrightarrow{AB}$.

$\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$

Kako je svaki slobodni vektor jednoznačno određen sa bilo kojim svojim predstavnikom, tj. vektorom, uobičajeno je da se poistovete pojam slobodnog vektora i vektora i piše se $\vec{a} = \overrightarrow{AB}$.

$\vec{a} = \overrightarrow{AB}$

$\vec{a} = \overrightarrow{AB}$



Svaki vektror $\overrightarrow{AB}$, $A \neq B$ je određen svojim pravcem, smerom i intenzitetom.

Pravac vektora $\overrightarrow{AB}$, $A \neq B$ je određen pravom na kojoj leži taj vektor.

Intenzitet vektora $\overrightarrow{AB}$ je dužina duži AB i označava se sa $|\overrightarrow{AB}|$.

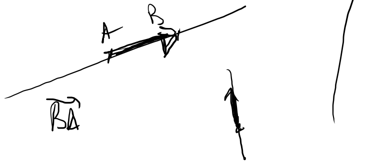
Smer vektora $\overrightarrow{AB}$, $A \neq B$ je od tačke A do tačke B.

Vektor čiji je intenzitet jednak jedan naziva se **jedinični vektor**.

Ako se $A \equiv B$, onda je $\overrightarrow{AB}$ nula vektor, u oznaci $\overrightarrow{0}$ ili samo 0. Intenzitet nula vektora je 0, a pravac i smer se ne definišu.

Nenula vektori su jednaki ako su im jednaki pravac, smer i intenzitet.

$|\vec{a}|$ $|\overrightarrow{AB}|$

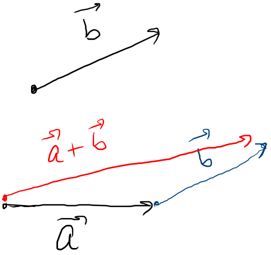


Sabiranje vektora

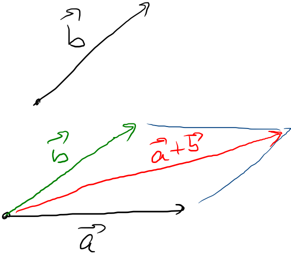
Za bilo koje tri tačke važi:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}.$$

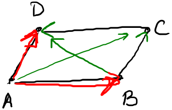
I



II



1. U paralelogramu $ABCD$ izraziti vektore $\overrightarrow{CD}$, $\overrightarrow{BC}$, $\overrightarrow{AC}$ i $\overrightarrow{BD}$ preko vektora $\vec{a} = \underset{\overrightarrow{AB}}{\cancel{\overrightarrow{CD}}}$ i $\vec{b} = \underset{\overrightarrow{AD}}{\cancel{\overrightarrow{CD}}}$.



$\vec{a} = \overrightarrow{AB}$

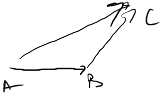
$\vec{b} = \overrightarrow{AD}$

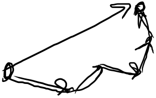
$$\overrightarrow{CD} = \overrightarrow{BA} = -\overrightarrow{AB} = -\vec{a}$$

$$\overrightarrow{BC} = \overrightarrow{AD} = \vec{b}$$

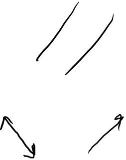
$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{a} + \vec{b}$$

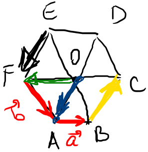
$$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD} = -\overrightarrow{AB} + \overrightarrow{AD} = -\vec{a} + \vec{b}$$





2. Neka je $ABCDEF$ pravilan šestougao, O njegov centar, $\vec{a} = \overrightarrow{AB}$ i $\vec{b} = \overrightarrow{FA}$. Izraziti preko vektora $\vec{a}$ i $\vec{b}$ vektore $\overrightarrow{OF}$, $\overrightarrow{OA}$, $\overrightarrow{BC}$ i $\overrightarrow{EF}$.



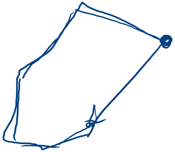


$$\overrightarrow{OF} = \overrightarrow{BA} = -\overrightarrow{AB} = -\vec{a}$$

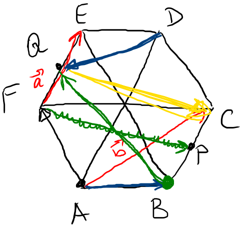
$$\overrightarrow{OA} = \overrightarrow{OF} + \overrightarrow{FA} = -\vec{a} + \vec{b}$$

$$\overrightarrow{BC} = \overrightarrow{AO} = -\overrightarrow{OA} = -(-\vec{a} + \vec{b}) = \vec{a} - \vec{b}$$

$$\overrightarrow{EF} = \overrightarrow{OA} = -\vec{a} + \vec{b}$$



3. Neka je *ABCDEF* pravilan šestougao, *P* i *Q* sredine stranica *BC* i *EF*, redom. U zavisnosti od vektora $\vec{a} = \overrightarrow{FE}$ i $\vec{b} = \overrightarrow{AC}$ izraziti vektore $\overrightarrow{AB}$, $\overrightarrow{BQ}$, $\overrightarrow{QC}$, $\overrightarrow{DQ}$ i $\overrightarrow{FP}$.



$$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB} = \overrightarrow{AC} + \overrightarrow{EF}$$
$$= \overrightarrow{AC} - \overrightarrow{FE} = \vec{b} - \vec{a}$$

$$\overrightarrow{BQ} = \overrightarrow{BA} + \overrightarrow{AF} + \overrightarrow{FQ}$$
$$= -\overrightarrow{AB} + \boxed{\overrightarrow{AF}} + \frac{1}{2}\overrightarrow{FE}$$

$$\overrightarrow{AF} = \overrightarrow{AC} + \overrightarrow{CF} = \overrightarrow{AC} + 2\overrightarrow{BA} = \overrightarrow{AC} - 2\overrightarrow{AB}$$
$$= \vec{b} - 2(\vec{b} - \vec{a}) = \vec{b} - 2\vec{b} + 2\vec{a} = 2\vec{a} - \vec{b}$$

$$= -(\vec{b} - \vec{a}) + 2\vec{a} - \vec{b} + \frac{1}{2}\vec{a}$$
$$= -\vec{b} + \vec{a} + 2\vec{a} - \vec{b} + \frac{1}{2}\vec{a} = \frac{7}{2}\vec{a} - 2\vec{b}$$

$\overrightarrow{FP}$

$\parallel$

$$\overrightarrow{QC} = \overrightarrow{QF} + \overrightarrow{FC} = \frac{1}{2}\overrightarrow{EF} + 2\overrightarrow{AB}$$
$$= -\frac{1}{2}\overrightarrow{FE} + 2\overrightarrow{AB} = -\frac{1}{2}\vec{a} + 2(\vec{b} - \vec{a})$$
$$= -\frac{1}{2}\vec{a} + 2\vec{b} - 2\vec{a} = 2\vec{b} - \frac{5}{2}\vec{a}$$

$$\overrightarrow{DQ} = \overrightarrow{DE} + \overrightarrow{EQ} = \overrightarrow{BA} + \frac{1}{2}\overrightarrow{EF} = -\overrightarrow{AB} - \frac{1}{2}\overrightarrow{FE} = -(\vec{b} - \vec{a}) - \frac{1}{2}\vec{a}$$

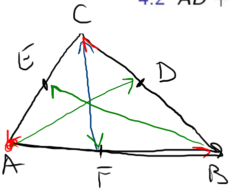
$$= -\vec{b} + \vec{a} - \frac{1}{2}\vec{a} = -\vec{b} + \frac{1}{2}\vec{a}$$

4. Ako su D, E i F redom sredine stranica BC, CA i AB trougla ABC, dokazati jednakosti:

4.1 $2\overrightarrow{AB} + 3\overrightarrow{BC} + \overrightarrow{CA} = 2\overrightarrow{FC}$;

4.2 $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = 0$.

$$\overrightarrow{AB} = \overrightarrow{AF} + \overrightarrow{FB} = 2\overrightarrow{FB}$$



4.1.

$$2\overrightarrow{AB} + 3\overrightarrow{BC} + \overrightarrow{CA} = \underbrace{\left(\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}\right)}_{0} + \overrightarrow{AB} + 2\overrightarrow{BC}$$

$$= \overrightarrow{AB} + 2\overrightarrow{BC} = 2\overrightarrow{FB} + 2\overrightarrow{BC} = 2\left(\overrightarrow{FB} + \overrightarrow{BC}\right)$$

$$= 2\overrightarrow{FC}$$

4.2.

$$\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \left(\overrightarrow{AB} + \overrightarrow{BD}\right) + \left(\overrightarrow{BA} + \overrightarrow{AE}\right) + \left(\overrightarrow{CA} + \overrightarrow{AF}\right)$$

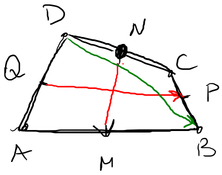
$$= \cancel{\overrightarrow{AB}} + \frac{1}{2}\overrightarrow{BC} + \cancel{\overrightarrow{BA}} + \frac{1}{2}\overrightarrow{AC} + \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB}$$

$$= \frac{1}{2}\overrightarrow{BC} + \frac{1}{2}\overrightarrow{AC} - \overrightarrow{AC} + \frac{1}{2}\overrightarrow{AB}$$

$$= \frac{1}{2}\overrightarrow{BC} - \frac{1}{2}\overrightarrow{AC} + \frac{1}{2}\overrightarrow{AB}$$

$$= \frac{1}{2}\left(\overrightarrow{BC} - \overrightarrow{AC} + \overrightarrow{AB}\right) = \frac{1}{2}\left(\overrightarrow{AC} - \overrightarrow{AC}\right) = 0$$

5. U četvorouglu $ABCD$ tačke M, P, N i Q su redom sredine stranica AB, BC, CD i DA. Dokazati da je $\overrightarrow{NM} + \overrightarrow{QP} = \overrightarrow{DB}$.



$$\overrightarrow{NM} = \cancel{\overrightarrow{NC}} + \overrightarrow{CB} + \cancel{\overrightarrow{BM}}$$
$$\overrightarrow{NM} = \cancel{\overrightarrow{ND}} + \overrightarrow{DA} + \cancel{\overrightarrow{AM}}$$

$$\boxed{2\overrightarrow{NM} = \overrightarrow{CB} + \overrightarrow{DA}}$$

$$\overrightarrow{NC} = -\overrightarrow{ND}$$

$$\overrightarrow{AM} = -\overrightarrow{BM}$$

$$\overrightarrow{QP} = \cancel{\overrightarrow{QA}} + \overrightarrow{AB} + \cancel{\overrightarrow{BP}}$$
$$\overrightarrow{QP} = \cancel{\overrightarrow{QD}} + \overrightarrow{DC} + \cancel{\overrightarrow{CP}}$$

$$\boxed{2\overrightarrow{QP} = \overrightarrow{AB} + \overrightarrow{DC}}$$

$$\overrightarrow{NM} + \overrightarrow{QP} = \frac{1}{2}\left(\overrightarrow{CB} + \overrightarrow{DA}\right) + \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{DC}\right) = \frac{1}{2}\left(\overrightarrow{CB} + \underbrace{\overrightarrow{DA} + \overrightarrow{AB}}_{\overrightarrow{DB}} + \overrightarrow{DC}\right)$$

$$= \frac{1}{2}\left(\overrightarrow{DB} + \overrightarrow{DB}\right) = \frac{1}{\cancel{2}} \cancel{2}\overrightarrow{DB} = \overrightarrow{DB}$$