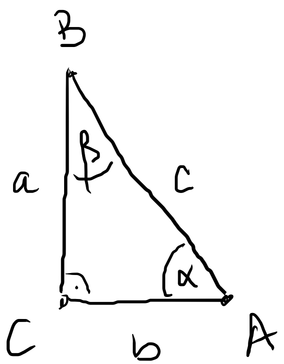
TRIGONOMETRIJA

$\boxed{\pi = 180^\circ}$



$$c^2 = a^2 + b^2$$

$$\alpha + \beta = \frac{\pi}{2} = 90^\circ \Rightarrow \beta = \frac{\pi}{2} - \alpha$$

$$\boxed{\sin\alpha} = \frac{a}{c} = \cos\beta = \boxed{\cos\left(\frac{\pi}{2} - \alpha\right)}$$

$$\boxed{\cos\alpha} = \frac{b}{c} = \sin\beta = \boxed{\sin\left(\frac{\pi}{2} - \alpha\right)}$$

$$\mathrm{tg}\,\alpha = \frac{a}{b} = \mathrm{ctg}\,\beta = \mathrm{ctg}\left(\frac{\pi}{2} - \alpha\right)$$

$$\mathrm{ctg}\,\alpha = \frac{b}{a} = \mathrm{tg}\,\beta = \mathrm{tg}\left(\frac{\pi}{2} - \alpha\right)$$

OSNOVNE RELACIJE

1) $\sin^2\alpha + \cos^2\alpha = 1$

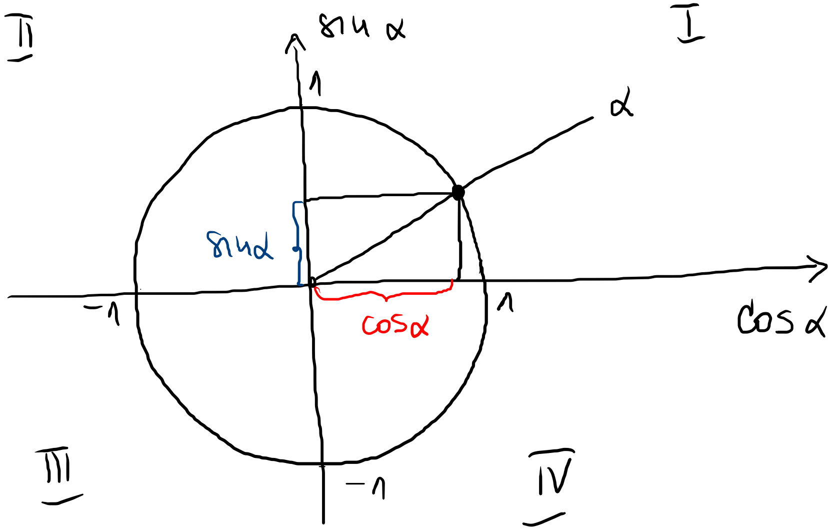
2) $\mathrm{tg}\,\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

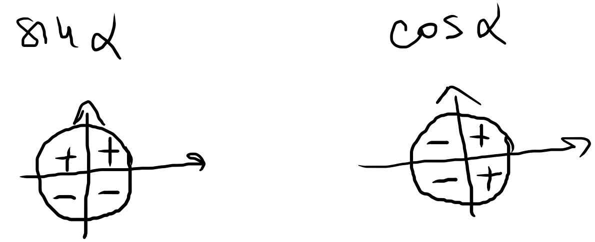
3) $\mathrm{ctg}\,\alpha = \dfrac{\cos\alpha}{\sin\alpha}$

4) $\mathrm{tg}\,\alpha \cdot \mathrm{ctg}\,\alpha = 1$

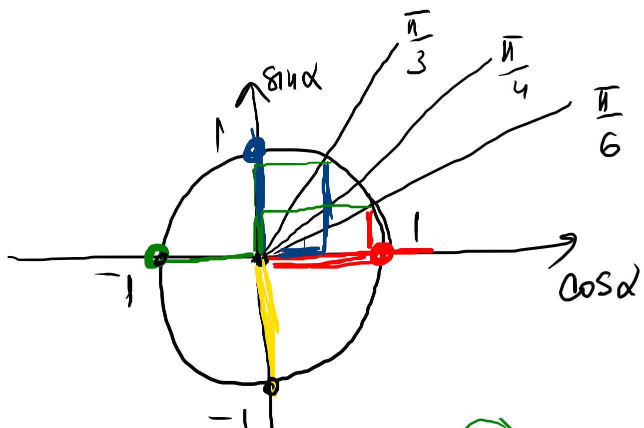
5) $-1 \leq \sin\alpha \leq 1$

6) $-1 \leq \cos\alpha \leq 1$





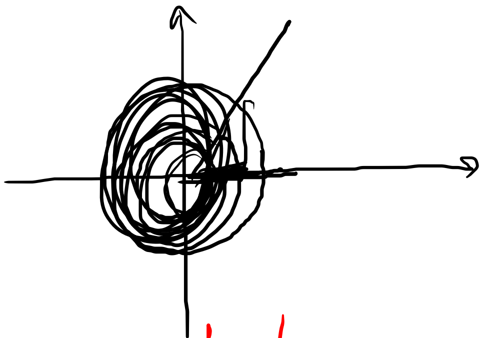
$$tg\alpha = \frac{\sin\alpha}{\cos\alpha}$$



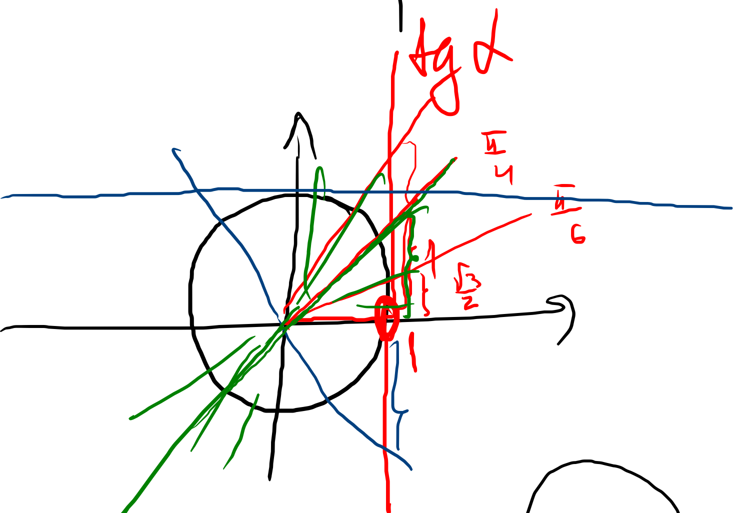
$\frac{\pi}{6}$ $\frac{\pi}{4}$ $\frac{\pi}{3}$ $\frac{1}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{\sqrt{3}}{2}$

	0	30°	45°	60°	90°	180°	270°	360°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin α	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
cos α	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	−1	0	1
tg α	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	–	0	–	0
ctg α	–	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	–	0	–

cos α: NAJVEĆI UGAO → NAJMANJU VR.
sin α: NAJVEĆI UGAO → NAJVEĆU VR.

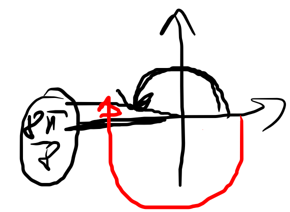


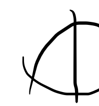
$$\cos(\alpha + 2k\pi) = \cos\alpha$$
$$\sin(\alpha + 2k\pi) = \sin\alpha$$
$$k \in \mathbb{Z}$$



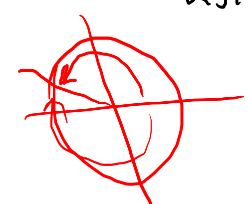
$$\operatorname{tg}(\alpha + k\pi) = \operatorname{tg}\alpha$$
$$\operatorname{ctg}(\alpha + k\pi) = \operatorname{ctg}\alpha$$

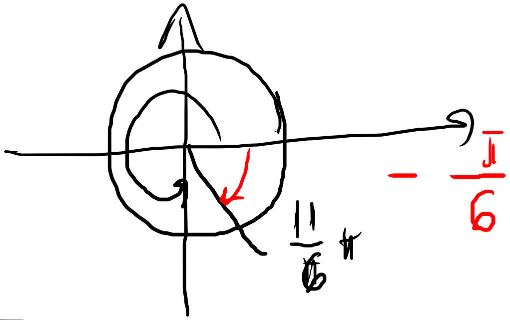
$\frac{7\pi}{8}$ $\quad$ $-\frac{9\pi}{8}$





$$2\pi = \frac{16\pi}{8}$$





$\frac{5\pi}{3}$ $\qquad -\frac{\pi}{3}$

$\frac{6\pi}{3} = 2\pi$

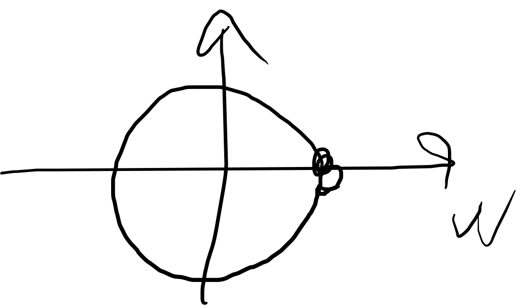
$\boxed{x \in [-\pi, \pi]}$

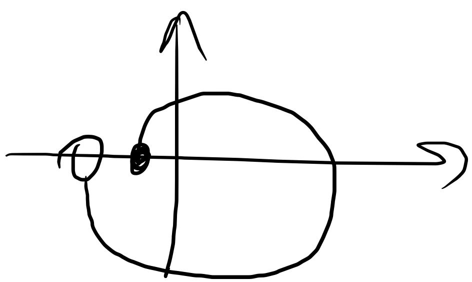
$\sin(-\alpha) = -\sin\alpha$

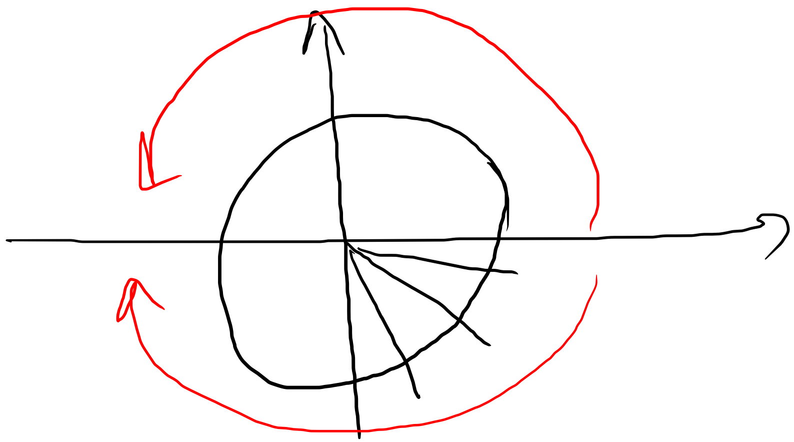
$\cos(-\alpha) = \cos\alpha$

$\mathrm{tg}(-\alpha) = -\mathrm{tg}\,\alpha$

$\mathrm{ctg}(-\alpha) = -\mathrm{ctg}\,\alpha$







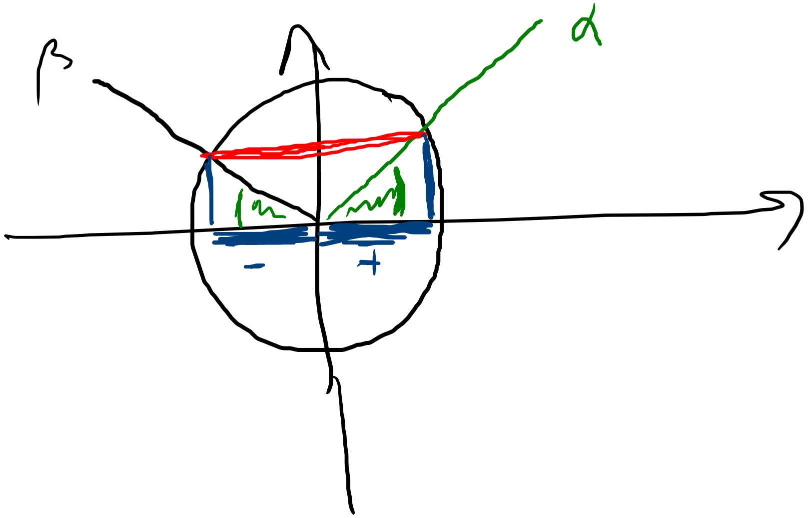
I

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\cos\alpha$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\sin\alpha$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$

IV

$$\sin\frac{5\pi}{3} = \sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\cos\frac{11\pi}{6} = \cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

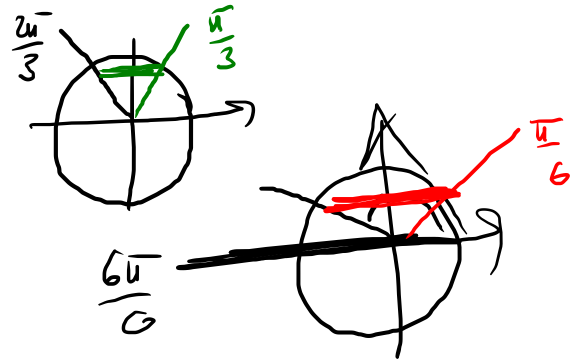


$\sin\beta = \sin\alpha$

$\cos\beta =$

$\sin(\pi-\alpha) = \sin\alpha$

$\cos(\pi-\alpha) = -\cos\alpha$

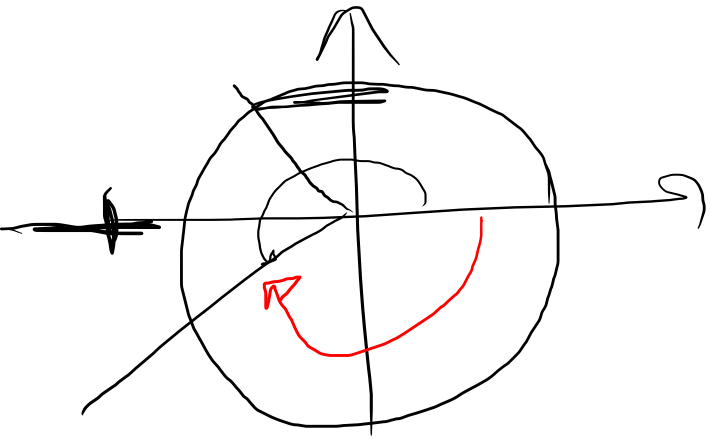


$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}$

$\cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$

$\cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$



$$\sin\frac{7\pi}{6} = \sin\left(-\frac{5\pi}{6}\right) = -\sin\frac{5\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$$

$$\cos\frac{4\pi}{3} = \cos\left(-\frac{2\pi}{3}\right) = \cos\frac{2\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

$$\pi = \frac{6\pi}{6} = \frac{3\pi}{3} = \frac{8\pi}{8}$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\boxed{\sin 2\alpha = 2\sin\alpha\cos\alpha}$$

$$\boxed{\cos 2\alpha = \cos^2\alpha - \sin^2\alpha}$$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}$$

① ODREDITI α AKO JE

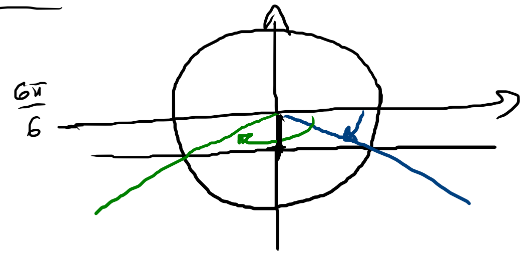
a) $\sin\alpha = -\frac{1}{2}$

b) $\cos\alpha = \frac{\sqrt{3}}{2}$

c) $tg\,\alpha = \frac{\sqrt{3}}{3}$

d) $ctg\,\alpha = -\sqrt{3}$

a)

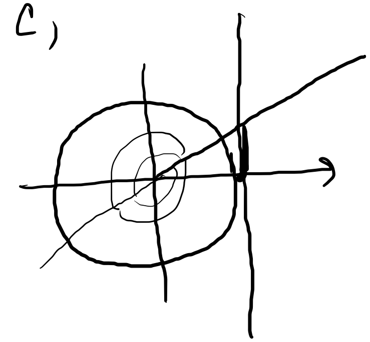


$\alpha = -\frac{\pi}{6} + 2k\pi$

$\alpha = -\frac{5\pi}{6} + 2k\pi$, $k\in \mathbb{Z}$

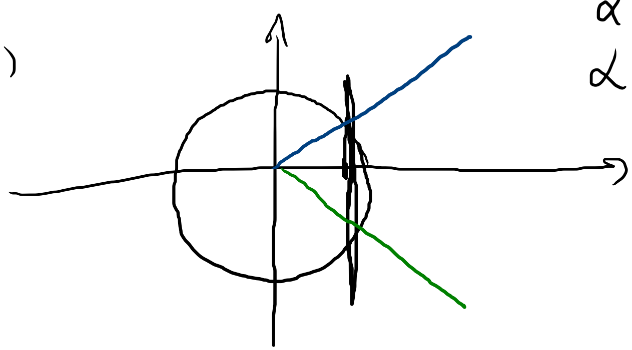
c)

$\frac{\sqrt{3}}{3}, 1, \sqrt{3}$



$\alpha = \frac{\pi}{6} + k\pi$, $k\in \mathbb{Z}$

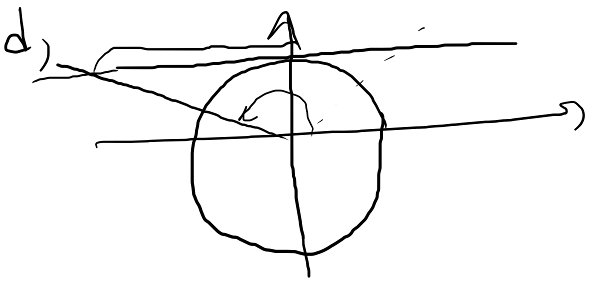
b)



$\alpha = \frac{\pi}{6} + 2k\pi$

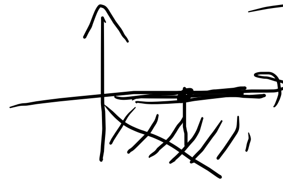
$\alpha = -\frac{\pi}{6} + 2k\pi$ $k\in \mathbb{Z}$

d)



$\alpha = \frac{5\pi}{6} + k\pi$, $k\in \mathbb{Z}$

1) AKO JE $\sin\alpha = -\frac{5}{7}$, $\alpha \in \left(\frac{3\pi}{2}, 2\pi\right)$, ODREDITI $\cos\alpha, \operatorname{tg}\alpha, \operatorname{ctg}\alpha$.



$\sin\alpha = -\frac{5}{7}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\frac{25}{49} + \cos^2\alpha = 1$

$\cos^2\alpha = \frac{24}{49}$

$\cos\alpha = \pm\frac{\sqrt{24}}{7}$

$\boxed{\cos\alpha = \frac{2\sqrt{6}}{7}}$ $\quad \cancel{\cos\alpha = -\frac{2\sqrt{6}}{7}}$

$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}$

$\operatorname{tg}\alpha = \frac{-\frac{5}{7}}{\frac{2\sqrt{6}}{7}} = -\frac{5}{2\sqrt{6}}$

$\operatorname{ctg}\alpha = \frac{1}{\operatorname{tg}\alpha} = -\frac{2\sqrt{6}}{5}$

(2) AKO JE $tg\,\alpha = \frac{9}{40}$, $\underline{\alpha \in (0, \frac{\pi}{2})}$, ODREDITI $\sin\alpha$, $\cos\alpha$, $ctg\,\alpha$.

$$tg\,\alpha = \frac{9}{40}$$

$$ctg\,\alpha = \frac{40}{9}$$

$$tg\,\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{9}{40}$$

$$\sin\alpha = \frac{9}{40}\cdot\cos\alpha$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\left(\frac{9}{40}\cos\alpha\right)^2 + \cos^2\alpha = 1$$

$$\frac{81}{1600}\cos^2\alpha + \cos^2\alpha = 1$$

$$\frac{1681}{1600}\cos^2\alpha = 1$$

$$\cos^2\alpha = \frac{1600}{1681}$$

$$\cos\alpha = \pm\frac{40}{41}$$

$$\boxed{\cos\alpha = \frac{40}{41}} \qquad \cancel{\cos\alpha = -\frac{40}{41}}$$

$$\sin\alpha = \frac{9}{40}\cdot\frac{40}{41} = \frac{9}{41}$$

3) ODREDITI $\sin(\alpha+\beta)$ AKO JE $\cos\alpha = \cos\beta = -\frac{4}{5}$

$\alpha \in \left[\frac{\pi}{2}, \pi\right)$, $\beta \in \left(\pi, \frac{3\pi}{2}\right)$

$$\sin(\alpha+\beta) = \underbrace{\sin\alpha}_{\frac{3}{5}}\underbrace{\cos\beta}_{-\frac{4}{5}} + \underbrace{\cos\alpha}_{-\frac{4}{5}}\underbrace{\sin\beta}_{-\frac{3}{5}} = \frac{3}{5}\cdot\left(-\frac{4}{5}\right) + \left(-\frac{4}{5}\right)\cdot\left(-\frac{3}{5}\right) = \frac{-12}{25} + \frac{12}{25}$$

$$= 0$$

$\cos\alpha = -\frac{4}{5}$

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha + \frac{16}{25} = 1$

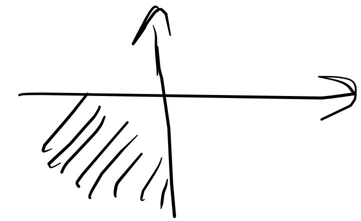
$\sin^2\alpha = 1 - \frac{16}{25}$

$\sin^2\alpha = \frac{9}{25}$

$\boxed{\sin\alpha = \frac{3}{5}}$ $\quad \cancel{\sin\alpha = -\frac{3}{5}}$

$\cos\beta = -\frac{4}{5}$

$\sin^2\beta = \frac{9}{25}$



$\cancel{\sin\beta = \frac{3}{5}}$ $\quad \boxed{\sin\beta = -\frac{3}{5}}$

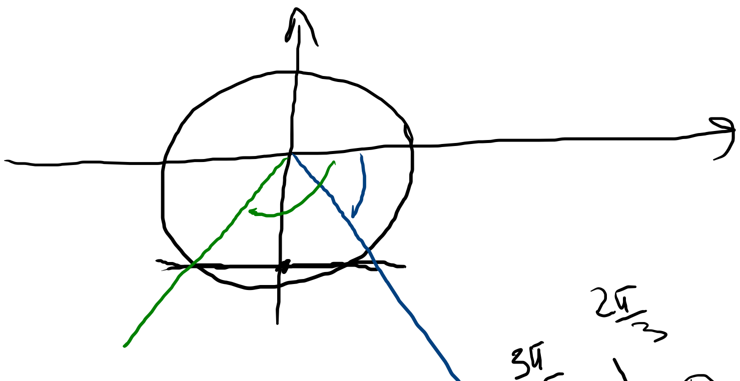
④ $2\sin x+\sqrt{3}=0$ NAPISATI SVA REŠENJA JEDNAČINE

$2\sin x=-\sqrt{3}$

$\sin x=-\frac{\sqrt{3}}{2}$

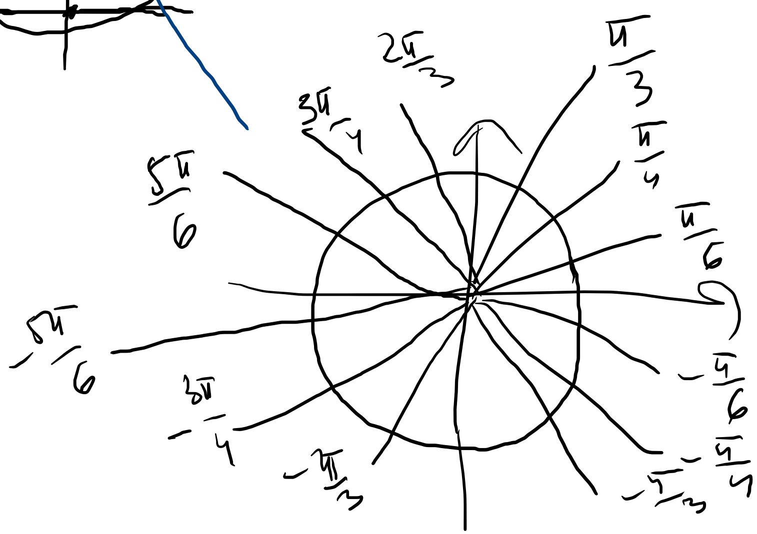
$x=-\frac{\pi}{3}+2k\pi$

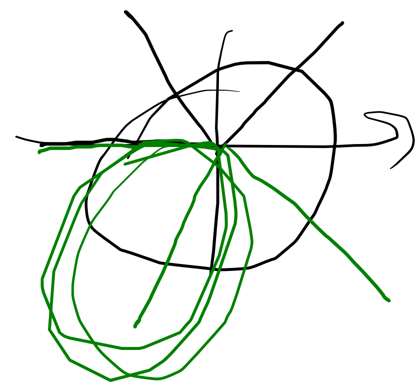
$x=-\frac{2\pi}{3}+2k\pi$, $k\in\mathbb{Z}$



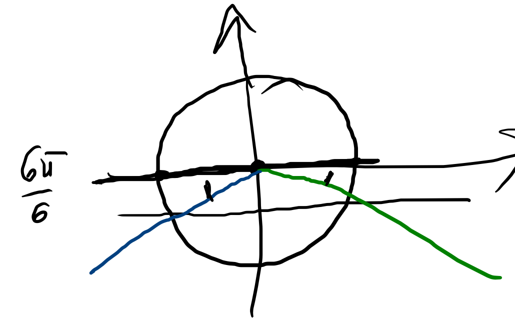
$X=\frac{5\pi}{3}+2k\pi$

$X=\frac{4\pi}{3}+2k\pi$ $k\in\mathbb{Z}$





5 ODREDITI SVA REŠENJA JEDNAČINE



$$2\sin^2 x + \sin x = 0$$

$$\sin x \cdot (2\sin x + 1) = 0$$

$\sin x = 0$ $\quad \vee \quad$ $2\sin x + 1 = 0$

$$2\sin x = -1$$

$$\sin x = -\frac{1}{2}$$

$x = 0 + 2k\pi$

$x = \pi + 2k\pi$, $\quad k \in Z$

$$\boxed{x = k\pi, \quad k \in Z}$$

$$\boxed{\begin{aligned} x &= -\frac{\pi}{6} + 2k\pi \\ x &= -\frac{5\pi}{6} + 2k\pi \end{aligned} \quad k \in Z}$$

6 Reši jednačinu:

$$\sin x = \sin 2x$$

$$\sin x - \sin 2x = 0$$

$$\sin x - 2\sin x \cos x = 0$$

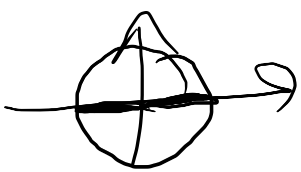
$$\sin x \cdot (1 - 2\cos x) = 0$$

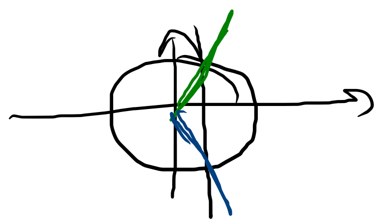
$$\sin x = 0 \quad \vee \quad 1 - 2\cos x = 0$$

$$\boxed{x = k\pi \quad k \in Z}$$

$$2\cos x = 1$$

$$\cos x = \frac{1}{2}$$





II način

$$\sin x - \sin 2x = 0$$

$$2\cos\frac{x+2x}{2} \cdot \sin\frac{x-2x}{2} = 0$$

$$2\cos\frac{3x}{2} \cdot \sin\left(\frac{-x}{2}\right) = 0$$

$$2\cos\frac{3x}{2} \cdot \left(-\sin\frac{x}{2}\right) = 0$$

$$-2\cos\frac{3x}{2} \cdot \sin\frac{x}{2} = 0$$

$$\cos\frac{3x}{2} = 0 \quad \vee$$

$$\sin\frac{x}{2} = 0$$

$$\boxed{\begin{array}{l} x = \frac{\pi}{3} + 2k\pi \\ x = -\frac{\pi}{3} + 2k\pi \end{array} \quad k \in Z}$$

7) Rešiti jednačinu (1996)

$$2\cos^2 x + 3\sin x - 3 = 0$$

$$2(1-\sin^2 x) + 3\sin x - 3 = 0$$

$$2 - 2\sin^2 x + 3\sin x - 3 = 0$$

$$-2\sin^2 x + 3\sin x - 1 = 0$$

$\sin x = t$

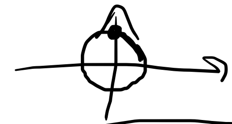
$$-2t^2 + 3t - 1 = 0$$

$$t_{1,2} = \frac{-3 \pm \sqrt{9-8}}{-4} = \frac{-3 \pm 1}{-4} \begin{cases} 1 \\ \frac{1}{2} \end{cases}$$

$\sin^2 x + \cos^2 x = 1$

$t_1 = 1$

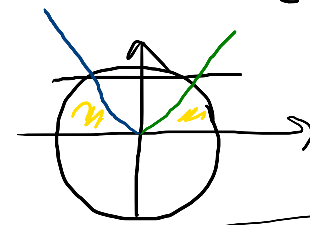
$\sin x = 1$ $\vee$



$$\boxed{x = \frac{\pi}{2} + 2k\pi}$$

$t_2 = \frac{1}{2}$

$\sin x = \frac{1}{2}$



$$x = \frac{\pi}{6} + 2k\pi$$

$$x = \frac{5\pi}{6} + 2k\pi$$

ZA DOMAĆI

$$4\sin^2 2x + 9\cos 2x - 4 = 0$$

(8) Rešiti jednačinu $\cos 2x = \sin x$

$$\cos 2x = \sin x$$

$$\cos 2x - \sin x = 0$$

$$\cos^2 x - \sin^2 x - \sin x = 0$$

$$1 - \sin^2 x - \sin^2 x - \sin x = 0$$

$$-2\sin^2 x - \sin x + 1 = 0$$

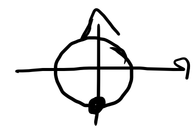
$\sin x = t$ $\qquad -2t^2 - t + 1 = 0$

$$t_{1,2} = \frac{1 \pm \sqrt{1+8}}{-4} = \frac{1 \pm 3}{-4} = \begin{cases} -1 \\ \frac{1}{2} \end{cases}$$

$t_1 = -1 \qquad t_2 = \frac{1}{2}$

$t = -1$

$\sin x = -1$



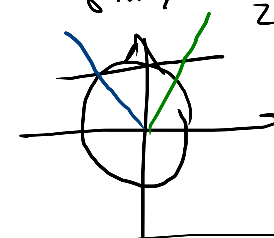
$$\boxed{x = \frac{3\pi}{2} + 2k\pi}$$

$\Updownarrow$

$$x = -\frac{\pi}{2} + 2k\pi$$

$t = \frac{1}{2}$

$\sin x = \frac{1}{2}$



$$x = \frac{\pi}{6} + 2k\pi$$

$$x = \frac{5\pi}{6} + 2k\pi$$