① $y = \dfrac{x^3 - 3x}{x^2 - 1}$

1) $x^2 - 1 \neq 0$

$x^2 \neq 1$

$x \neq \pm 1$

$D = \mathbb{R} \setminus \{\pm 1\}$
$= (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

2) $\underline{\underline{y = 0}}$ $\quad x^3 - 3x = 0$

$x(x^2 - 3) = 0$

$\underline{\underline{x = 0}} \qquad x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

3) $y = \dfrac{x(x-\sqrt{3})(x+\sqrt{3})}{(x-1)(x+1)}$

		$-\sqrt{3}$		-1		0		1		$\sqrt{3}$	
x	$-$		$-$		$-$	0	$+$		$+$		$+$
$x-\sqrt{3}$	$-$		$-$		$-$		$-$		$-$	0	$+$
$x+\sqrt{3}$	$-$	0	$+$		$+$		$+$		$+$		$+$
$x-1$	$-$		$-$		$-$		$-$	0	$+$		$+$
$x+1$	$-$		$-$	0	$+$		$+$		$+$		$+$
y	$-$		$+$		$-$		$+$		$-$		$+$

$y > 0$ ZA
$x \in (-\sqrt{3}, -1) \cup (0, 1) \cup (\sqrt{3}, +\infty)$

$y < 0$ ZA
$x \in (-\infty, -\sqrt{3}) \cup (-1, 0) \cup (1, \sqrt{3})$

4) $f(-x) = \dfrac{(-x)^3 - 3(-x)}{(-x)^2 - 1}$

$= \dfrac{-x^3 + 3x}{x^2 - 1} = -\dfrac{x^3 - 3x}{x^2 - 1}$

$= -f(x)$

NEPARNA

5) V.A.

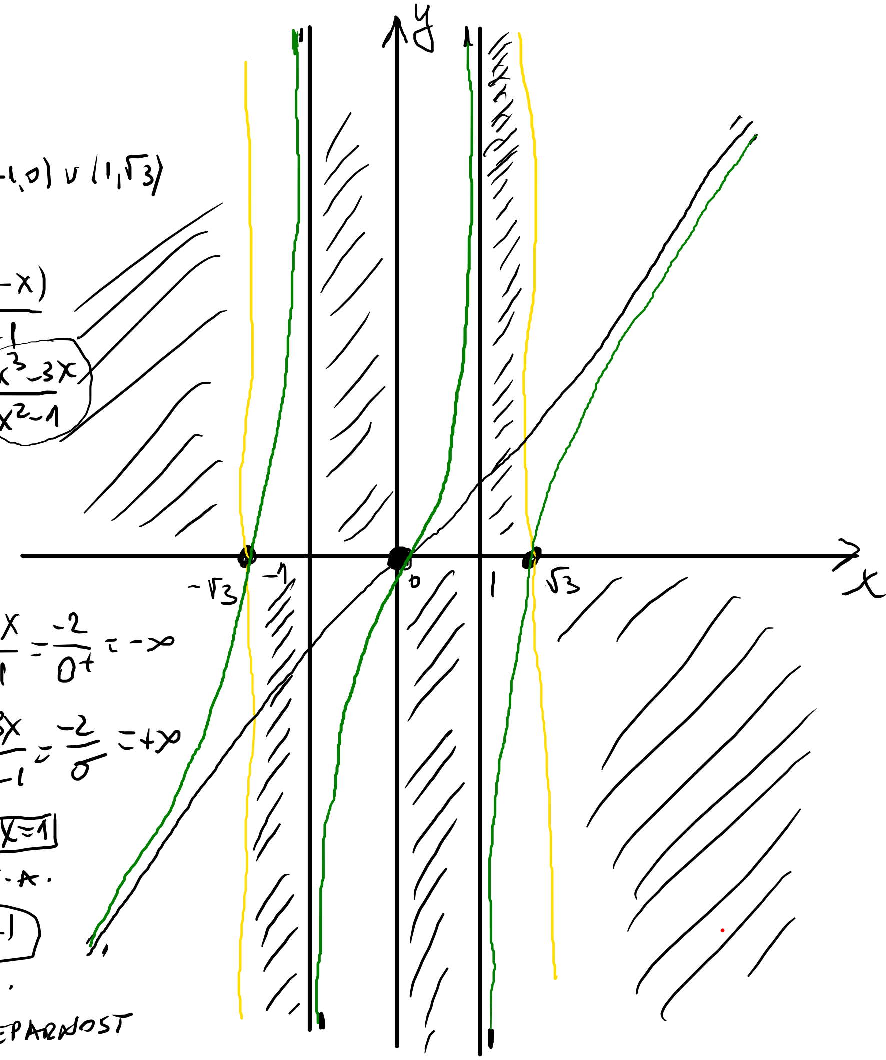
$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \dfrac{x^3 - 3x}{x^2 - 1} = \dfrac{-2}{0^+} = -\infty$

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \dfrac{x^3 - 3x}{x^2 - 1} = \dfrac{-2}{0} = +\infty$

$\boxed{x = 1}$ V.A.

$\left.\begin{array}{l} \lim\limits_{x \to -1^+} f(x) \\ \lim\limits_{x \to -1^-} f(x) \end{array}\right\}$ $\boxed{x = -1}$ V.A.

ZBOG NEPARNOST



H.A.

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{x^3-3x}{x^2-1} = \pm\infty$$ – NEMA HORIZONTALNA

K.A. $y = kx + n$

$$k = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{\frac{x^3-3x}{x^2-1}}{\frac{x}{1}} = \lim_{x\to\pm\infty} \frac{x^3-3x}{x^3-x} = 1$$

$$n = \lim_{x\to\pm\infty} (f(x) - kx) = \lim_{x\to\pm\infty} \left(\frac{x^3-3x}{x^2-1} - x\right) = \lim_{x\to\pm\infty} \frac{\cancel{x^3}-3x-\cancel{x^3}+x}{x^2-1}$$

$$= \lim_{x\to\pm\infty} \frac{-2x}{x^2-1} = 0$$

$\boxed{y = x}$ KOSA A.

6)
$$y = \frac{x^3 - 3x}{x^2 - 1}$$

$$y' = \frac{(3x^2-3)(x^2-1) - (x^3-3x)\cdot 2x}{(x^2-1)^2} = \frac{3x^4 - \cancel{3x^2} - \cancel{3x^2} + 3 - 2x^4 + \cancel{6x^2}}{(x^2-1)^2}$$

$$= \frac{x^4+3}{(x^2-1)^2}, \quad x \neq \pm 1$$

$y' \neq 0 \quad \forall x \in D \Rightarrow$ NEMA E.V.

$$y' > 0 \qquad \frac{x^4+3}{(x^2-1)^2} > 0$$

$x^4+3 > 0$

$(x^2-1)^2 > 0 \quad \forall x \in D$

$y' > 0 \quad \forall x \in D \qquad y \nearrow \quad \forall x \in D$

7. $y' = \dfrac{x^4+3}{(x^2-1)^2}$

$$y'' = \frac{4x^3(x^2-1)^2 - (x^4+3)\cdot 2(x^2-1)\cdot 2x}{(x^2-1)^4} = \frac{4x\,(x^2-1)\left(x^2(x^2-1)-(x^4+3)\right)}{(x^2-1)^{\cancel{4}\,3}}$$

$$= \frac{4x\,(\cancel{x^4}-x^2-\cancel{x^4}-3)}{(x^2-1)^3} = \frac{-4x\,(x^2+3)}{(x^2-1)^3}, \quad x \neq \pm 1$$

$y''=0$ $\boxed{x=0}$ KANDIDAT ZA P.T.

$y''>0$

$$\frac{\overset{<0}{-4}\,x\,\overset{>0}{(x^2+3)}}{\underbrace{(x^2-1)^2}_{>0\ \forall x\in D}(x^2-1)} > 0$$

$$\frac{x}{x^2-1} < 0$$

		−1		0		1	
x	−		−	0	+		+
$x-1$	−		−		−	0	+
$x+1$	−	0	+		+		+
y''	−		+		−		+
y	∩	∉D	∪	PT	∩	∉D	∪

$x=0$

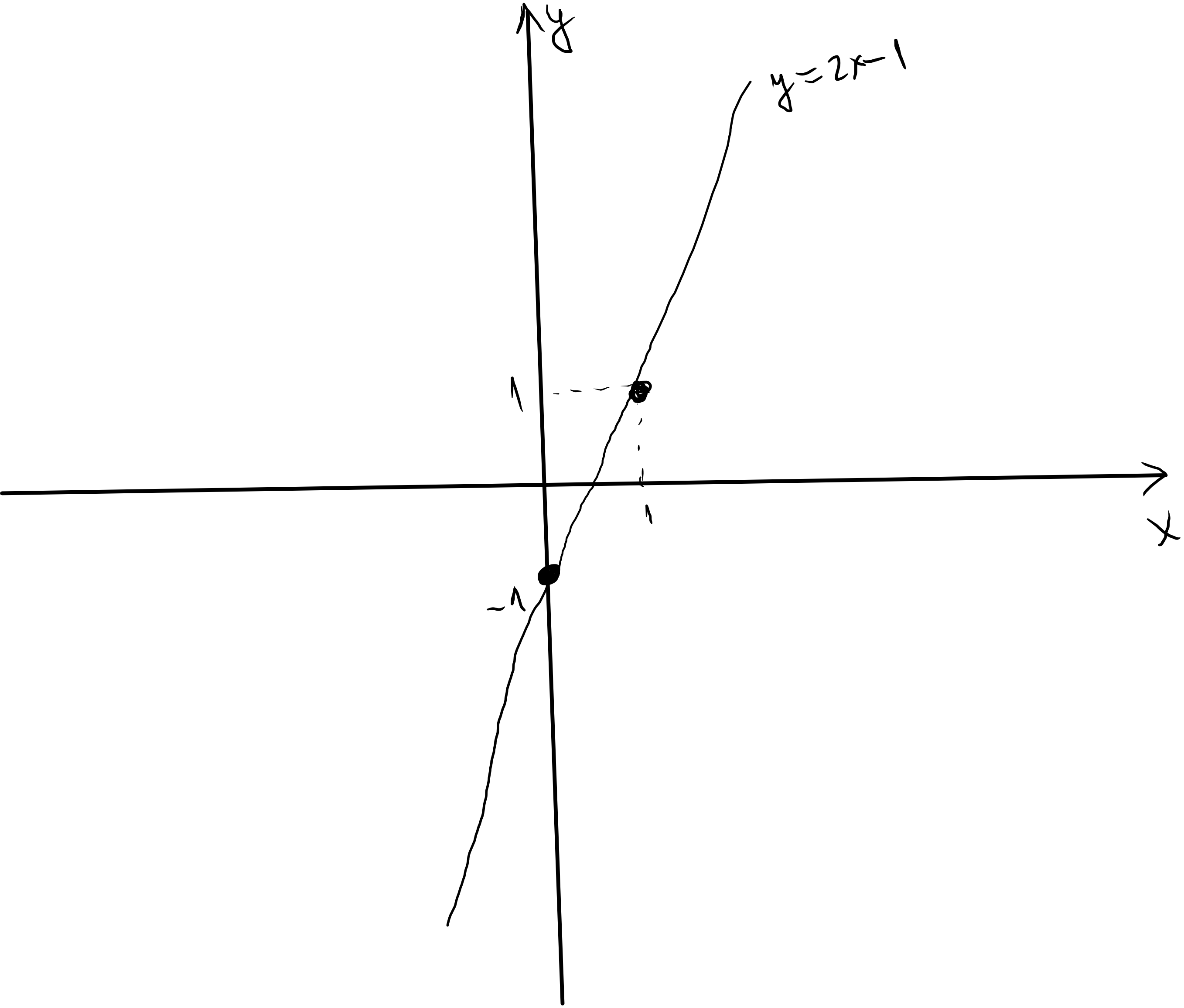
$y_{PT}(0) = \dfrac{0^3-3\cdot 0}{0^2-1} = 0$

$y''>0$ y ∪

$x \in (-1,0)\cup(1,\infty)$

$y''<0$ y ∩

$x \in (-\infty,-1)\cup(0,1)$



$\boxed{y = 2x - 1}$ K.A.

$x = 0 \quad y = -1$

$x = 1 \quad y = 1$

① a) $\sum_{n=2}^{\infty} \frac{2n+1}{\sqrt{n^4-3n^2}}$

b) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{-n^2}$

a) $a_n = \frac{2n+1}{\sqrt{n^4-3n^2}}$

$\lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum$ D

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2n+1}{\sqrt{n^4-3n^2}} = \lim_{n\to\infty} \frac{n(2+\frac{1}{n})}{n^2\sqrt{1-\frac{3}{n^2}}} = 0$

NEMAM POJMA

$\frac{2n+1}{\sqrt{n^4-3n^2}} \sim \frac{2n}{\sqrt{n^4}} = \frac{2n}{n^2} = \frac{2}{n}$

$2\sum \frac{1}{n^1}$ D $\Rightarrow \sum a_n$ D

$\sum \frac{1}{n^\alpha}$ — $\alpha > 1$ K; $\alpha \leq 1$ D

$\alpha = 1$

b) $a_n = \left(1+\frac{1}{n}\right)^{-n^2}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{-n^2} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{n \cdot \frac{1}{n}(-n^2)} = e^{\lim_{n\to\infty}(-n)} = e^{-\infty} = 0$

NEMAM POJMA

$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\left(1+\frac{1}{n}\right)^{-n^2}} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{-\frac{n^2}{n}} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{n(-1)} = e^{-1} = \frac{1}{e} < 1$

K

(2) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+n}}$

$a_n = \frac{1}{\sqrt{n^2+n}}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n}} = 0$:(

$\frac{1}{\sqrt{n^2+n}} \sim \frac{1}{\sqrt{n^2}} = \frac{1}{n}$

$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+n}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+n}} =$

$= \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}\sqrt{1+\cancel{\frac{1}{n}}}} \to 0 = 1$

$\sum \frac{1}{n}$ (D)

$\alpha = 1$

$\sum \frac{1}{n^\alpha} \begin{cases} \alpha \leq 1 & \text{(D)} \\ \alpha > 1 & \text{(K)} \end{cases}$

(3)

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{(n+1)(n+2)}}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{n \cdot 2^{n+1}}{3^n}$

a) $$a_n = \frac{1}{\sqrt{(n+1)(n+2)}} = \frac{1}{\sqrt{n^2+3n+2}} \sim \frac{1}{\sqrt{n^2}} = \frac{1}{n}$$

$$\sum \frac{1}{n} \text{ (D)} \Rightarrow \sum a_n \text{ (D)}$$

b) $$a_n = \frac{n \cdot 2^{n+1}}{3^n}$$

$$a_{n+1} = \frac{(n+1) \cdot 2^{n+2}}{3^{n+1}}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{(n+1)\cdot 2^{n+2}}{3^{n+1}}}{\frac{n \cdot 2^{n+1}}{3^n}}$$

$$= \lim_{n\to\infty} \frac{(n+1)\cdot 2^{n+2} \cdot 3^n}{n \cdot 2^{n+1} \cdot 3^{n+1}} = \lim_{n\to\infty} \frac{(n+1)}{n} \frac{4 \cdot \cancel{2^n} \cdot \cancel{3^n}}{2 \cdot \cancel{2^n} \cdot 3 \cdot \cancel{3^n}} = \frac{4}{6} \, 1 = \frac{2}{3} < 1$$

(K)

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{n \cdot 2 \cdot 2^n}{3^n}} = \lim_{n\to\infty} \frac{\overset{1}{\sqrt[n]{n}} \cdot \overset{1}{\sqrt[n]{2}} \cdot 2}{3} = \frac{2}{3} < 1$$

$$\lim_{x\to 1^+} \frac{1}{(x-1)^2} = \frac{1}{0^+} = +\infty$$

$$\lim_{x\to 1^-} \frac{1}{(x-1)^2} = \frac{1}{0^+} = +\infty$$

$$\lim_{x\to 1} \frac{1}{(x-1)^2} = +\infty$$

$$\lim_{x\to 1^+} \frac{1}{x-1} = \frac{1}{0^+} = +\infty$$

$$\lim_{x\to 1^-} \frac{1}{x-1} = \frac{1}{0^-} = -\infty$$

$\lim_{x\to 1} \frac{1}{x-1}$ - NE POSTOJI

$\lim_{x\to 5} \frac{1}{x-5} = \frac{1}{0?}$ NE POSTOJI

$$f(x) = (\sin 2x)^x$$

a) $\lim_{x\to 0} f(x)$

b) $f'(x)$

$$\lim_{x\to 0} \frac{x}{\frac{1}{\ln(\sin 2x)}} = \lim_{x\to 0} \frac{1}{?}$$

a) $\lim_{x\to 0} f(x) = \lim_{x\to 0} (\sin 2x)^x \overset{0^0}{=} A \quad / \ln$

$$\ln A = \ln \lim_{x\to 0} (\sin 2x)^x = \lim_{x\to 0} \ln (\sin 2x)^x$$

$$= \lim_{x\to 0} x \ln(\sin 2x) \overset{0(-\infty)}{=}$$

$$= \lim_{x\to 0} \frac{\ln(\sin 2x)}{\frac{1}{x}} = \lim_{x\to 0} \frac{\frac{1}{\sin 2x} \cdot \cos 2x \cdot 2}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0} \frac{2\operatorname{ctg} 2x}{-\frac{1}{x^2}} = -2 \lim_{x\to 0} x^2 \operatorname{ctg} 2x \overset{0\cdot\infty}{=}$$

$$= -2 \lim_{x\to 0} \frac{x^2}{\operatorname{tg} 2x} = -2 \lim_{x\to 0} \frac{2x}{\frac{1}{\cos^2 2x} \cdot 2} = 0$$

$$-2 \lim_{x\to 0} \frac{\cancel{2} x \cdot \cos^2 2x}{\cancel{2}} = 0$$

$$\ln A = 0 \Rightarrow A = e^0 = 1$$

b) $f(x)^{y} = (\sin 2x)^{x} \quad / \ln$

$$y = (\sin 2x)^{x}$$

$$\ln f(x)^{y} = \ln(\sin 2x)^{x} \quad /'$$

$$(x^{x})' = ~~x \cdot x^{x-1}~~$$

$$\ln f(x)^{y} = x \ln(\sin 2x) \quad /'$$

$$(x^{\alpha})' = \alpha x^{\alpha - 1}$$

$$\frac{1}{f(x)_{y}} \cdot f'(x)_{y'} = \ln(\sin 2x) + x \frac{1}{\sin 2x} \cos 2x \cdot 2$$

$$f'(x) = (\sin 2x)^{x} \left(\ln(\sin 2x) + \frac{2x \cos 2x}{\sin 2x} \right)$$

4)

$$f(x)=\begin{cases}\left(\dfrac{x^2+7}{3x^2+5}\right)^{\frac{1}{x-1}}, & x>1\\ a, & x=1\\ b\,\dfrac{x^2+x-2}{2x^2+3x-5}, & x<1\end{cases}$$

$$\boxed{\overbrace{\lim_{x\to1^+} f(x)}^{e^{-\frac{1}{2}}}=\overbrace{\lim_{x\to1^-} f(x)}^{\frac{3}{7}b}=\overbrace{f(1)}^{a}}$$

$$\lim_{x\to1^+}\left(\frac{x^2+7}{3x^2+5}\right)^{\frac{1}{x-1}} \quad 1^\infty$$

$$=\lim_{x\to1^+}\left(1+\frac{x^2+7-3x^2-5}{3x^2+5}\right)^{\frac{1}{x-1}}=$$

$$=\lim_{x\to1^+}\left(1+\frac{-2x^2+2}{3x^2+5}\right)^{\frac{3x^2+5}{-2x^2+2}\cdot\frac{-2x^2+2}{3x^2+5}\cdot\frac{1}{x-1}}$$

$$=e^{\lim_{x\to1^+}\frac{-2x^2+2}{3x^2+5}\cdot\frac{1}{x-1}}=e^{\lim_{x\to1^+}\frac{2\cancel{(1-x)}^{-1}(1+x)}{(3x^2+5)\cancel{(x-1)}}}$$

$$=e^{-\frac{4}{8}}=e^{-\frac{1}{2}}$$

$$f(1)=a$$

$$\lim_{x\to1^-} f(x)=\lim_{x\to1^-} b\,\frac{x^2+x-2}{2x^2+3x-5}=b\lim_{x\to1^-}\frac{\cancel{(x-1)}(x+2)}{2\cancel{(x-1)}\left(x+\frac{5}{2}\right)}=b\cdot\frac{3}{2\cdot\frac{7}{2}}$$

$$=b\cdot\frac{3}{7}$$

$$=b\lim_{x\to1^-}\frac{2x+1}{4x+3}=b\,\frac{3}{7}$$

$$y = e^x$$

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{e^{x+\Delta x} - e^x}{\Delta x} = \lim_{\Delta x \to 0} \frac{e^x \cdot e^{\Delta x} - e^x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{e^x (e^{\Delta x} - 1)}{\Delta x} = e^x$$

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

1)

a) $\lim\limits_{n \to \infty} \left(\dfrac{n^2+7}{n^2+2n+3} \right)^{5n}$

b) $\lim\limits_{x \to 0^+} \left(\dfrac{1}{\sin x} \right)^{x+3}$

a) $$\lim_{n \to \infty} \left(\frac{n^2+7}{n^2+2n+3} \right)^{5n} \overset{1^\infty}{=} \lim_{n \to \infty} \left(1 + \frac{\cancel{n^2}+7-\cancel{n^2}-2n-3}{n^2+2n+3} \right)^{5n} =$$

$$= \lim_{n \to \infty} \left(\left(1 + \frac{4-2n}{n^2+2n+3} \right)^{\frac{n^2+2n+3}{4-2n}} \right)^{\frac{4-2n}{n^2+2n+3} \cdot 5n}$$

$$= e^{\lim\limits_{n \to \infty} \frac{20n-10n^2}{n^2+2n+3}} = e^{-10}$$

b) $$\lim_{x\to 0^+}\left(\frac{1}{\sin x}\right)^x \overset{\infty^0}{=} A \Big|^{\ln}$$

$$\ln A = \ln \lim_{x\to 0^+}\left(\frac{1}{\sin x}\right)^x = \lim_{x\to 0^+} \ln\left(\frac{1}{\sin x}\right)^x =$$

$$= \lim_{x\to 0^+} x \ln\left(\frac{1}{\sin x}\right) \overset{0\cdot\infty}{=} \lim_{x\to 0^+} \frac{\ln\left(\frac{1}{\sin x}\right)}{\frac{1}{x}} =$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{\frac{1}{\sin x}}\cdot\frac{-\cos x}{\sin^2 x}}{-\frac{1}{x^2}} = +\lim_{x\to 0^+} x\,\frac{x^2\cdot\cos x}{\sin x} = +1\cdot 0 = 0$$

$$\hookrightarrow = +\lim_{x\to 0^+}\frac{2x\cos x + x^2\cdot(-\sin x)}{\cos x} = +1$$

a) $\begin{cases} x = 3t^3 + \sin(\ln t) - 5 \\ y = e^{3t} + (2x+3)^3 \end{cases}$

b) $\arcsin(x+y^2) + 2xy = 3\ln(x^2 y)$

---

a) $x'_t = 3 \cdot 3t^2 + \cos(\ln t) \cdot \frac{1}{t}$

$y'_t = e^{3t} \cdot 3 + 3(2x+3)^2 \cdot 2$

$\begin{cases} y'_x = \frac{y'_t}{x'_t} = \dfrac{3e^{3t} + 6(2x+3)^2}{9t^2 + \frac{\cos(\ln t)}{t}} \\ x = 3t^3 + \sin(\ln t) - 5 \end{cases}$

b) $\frac{1}{\sqrt{1-(x+y^2)^2}} \cdot (1 + 2yy') + 2(y + xy') = 3\frac{1}{x^2 y} \cdot (2xy + x^2 y')$

(x)

$y = 2x\sqrt{x+2}$

$\boxed{D = [-2, \infty)}$

1) $x + 2 \geq 0$

$x \geq -2$

2) $\boxed{y=0}$ $\boxed{x=0}$ $x+2=0$

$x=-2$

$x=0$ $y=0$

3) $y>0$ $2x\underbrace{\sqrt{x+2}}_{\geq 0}>0$

$x>0$

$y>0$ $x \in (0, +\infty)$

$y<0$ $x \in (-2, 0)$

4) NI-NI

5) V.A. NEMA zbog $D=[-2,\infty)$

H.A.

$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} 2x\sqrt{x+2} = +\infty$

NEMA

K.A.

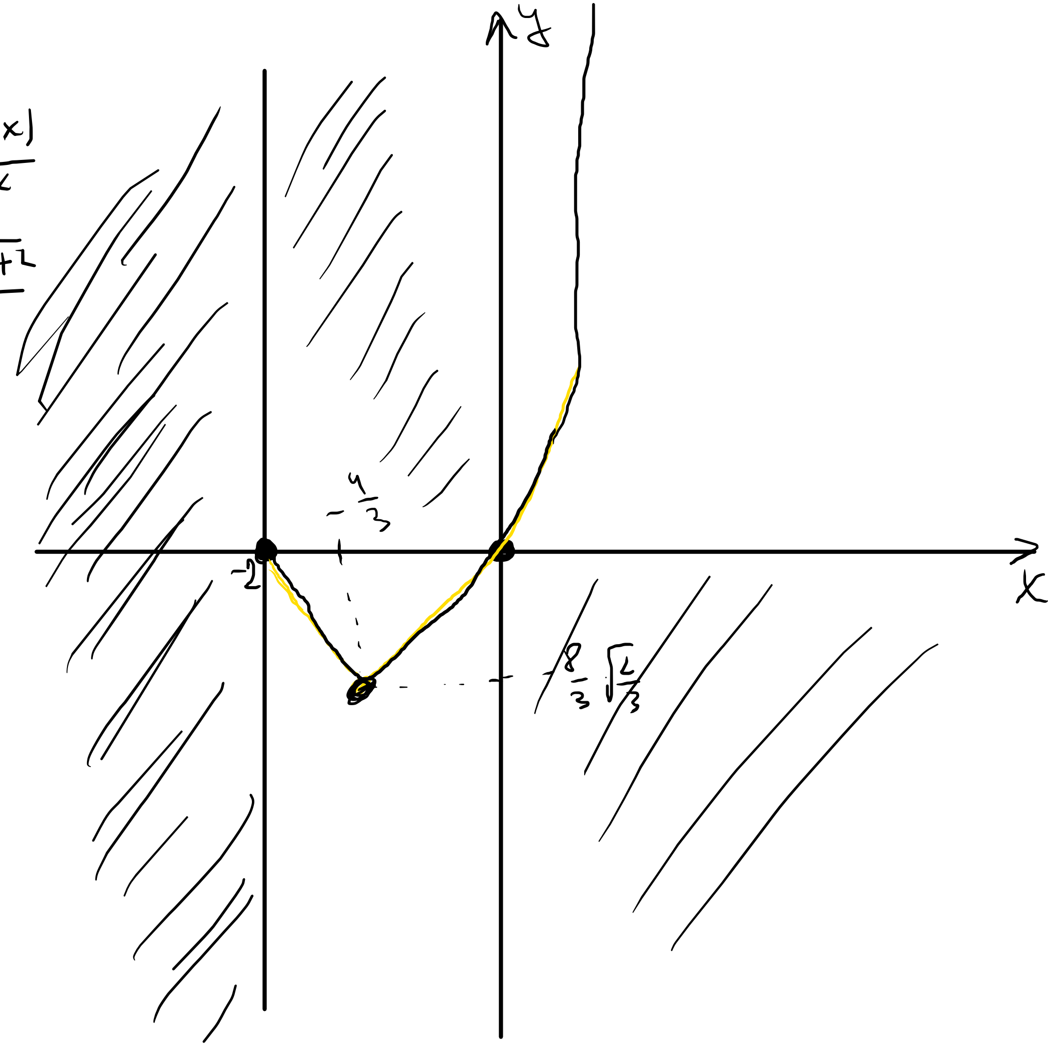
$y = kx + n$

$k = \lim_{x\to+\infty} \frac{f(x)}{x}$

$= \lim_{x\to+\infty} \frac{2x\sqrt{x+2}}{x}$

$= +\infty$

NEMA



6, $y = 2x\cdot\sqrt{x+2}$

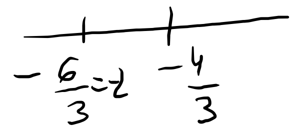
$$y' = 2\left(\underbrace{\sqrt{x+2}}_{1} + x\cdot\frac{1}{2\sqrt{x+2}}\right) = \cancel{2}\,\frac{2(x+2)+x}{\cancel{2}\sqrt{x+2}} = \frac{3x+4}{\sqrt{x+2}}$$

$\boxed{x > -2}$

$y' = 0$ $\quad 3x+4=0$

$\boxed{x = -\frac{4}{3}} \in D = [-2, \infty)$

KANDIDAT ZA E.V.



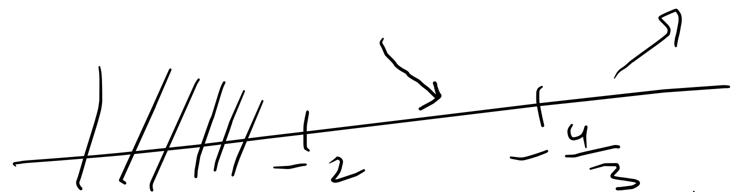
$y' > 0$ $\quad \frac{3x+4}{\sqrt{x+2}} > 0$ $\quad \sqrt{x+2} \geq 0$

$3x+4>0$

$x > -\frac{4}{3}$

$y' > 0$ $\quad x \in \left(-\frac{4}{3}, +\infty\right)$ $\quad y \nearrow$

$y' < 0$ $\quad x \in \left(-2, -\frac{4}{3}\right)$ $\quad y \searrow$



$x = -\frac{4}{3}$ LOKALNI MIN $\quad y_{min}\left(-\frac{4}{3}\right) = 2\cdot\left(-\frac{4}{3}\right)\sqrt{-\frac{4}{3}+2}$

$= -\frac{8}{3}\sqrt{\frac{2}{3}}$

7. $y = \dfrac{3x+4}{\sqrt{x+2}}$

$$y' = \frac{3\sqrt{x+2} - (3x+4)\dfrac{1}{2\sqrt{x+2}}}{x+2} = \frac{\dfrac{6(x+2) - (3x+4)}{2\sqrt{x+2}}}{x+2}$$

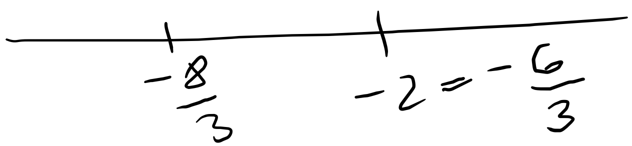
$$= \frac{3x+8}{2(x+2)\sqrt{x+2}}$$

$x > -2$

$y'' = 0$ $\quad 3x+8=0$

$x = -\frac{8}{3} \notin D$

NIJE P. T. $\quad$ NEMA P. T.



$y'' > 0 \qquad \dfrac{3x+8}{2(x+2)\sqrt{x+2}} > 0$

$3x+8 > 0$ $\quad$ $(x+2) > 0$ $\quad$ $\sqrt{x+2} \geq 0$ $\quad \forall x \in D$

$y'' > 0 \ \forall x \in D$

$y \cup$

$D = [-2, \infty)$

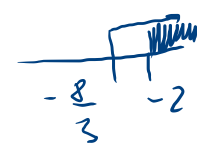
$\sqrt{x+2} \geq 0$

$x+2 > 0$

$x > -2$ ✓

$3x+8 > 0$

$x > -\frac{8}{3}$ ✓



$$y = \frac{2x}{2x^2+5x+2} = \frac{\cancel{2}x}{\cancel{2}(x+2)(x+\frac{1}{2})}$$

$$y = \frac{x^3}{x^2+x+1} \qquad x_{1,2} = \frac{-1 \pm \sqrt{-3}}{2} \notin \mathbb{R} \qquad \boxed{D = \mathbb{R}}$$

1) $2x^2+5x+2 \neq 0$

$$x_{1,2} = \frac{-5 \pm \sqrt{25-16}}{4} = \frac{-5 \pm 3}{4} = \begin{cases} -2 \\ -\frac{1}{2} \end{cases}$$

$$\boxed{D = \mathbb{R} \setminus \{-\tfrac{1}{2}, -2\} = (-\infty, -2) \cup (-2, -\tfrac{1}{2}) \cup (-\tfrac{1}{2}, +\infty)}$$

2, $y = 0 \quad x = 0$

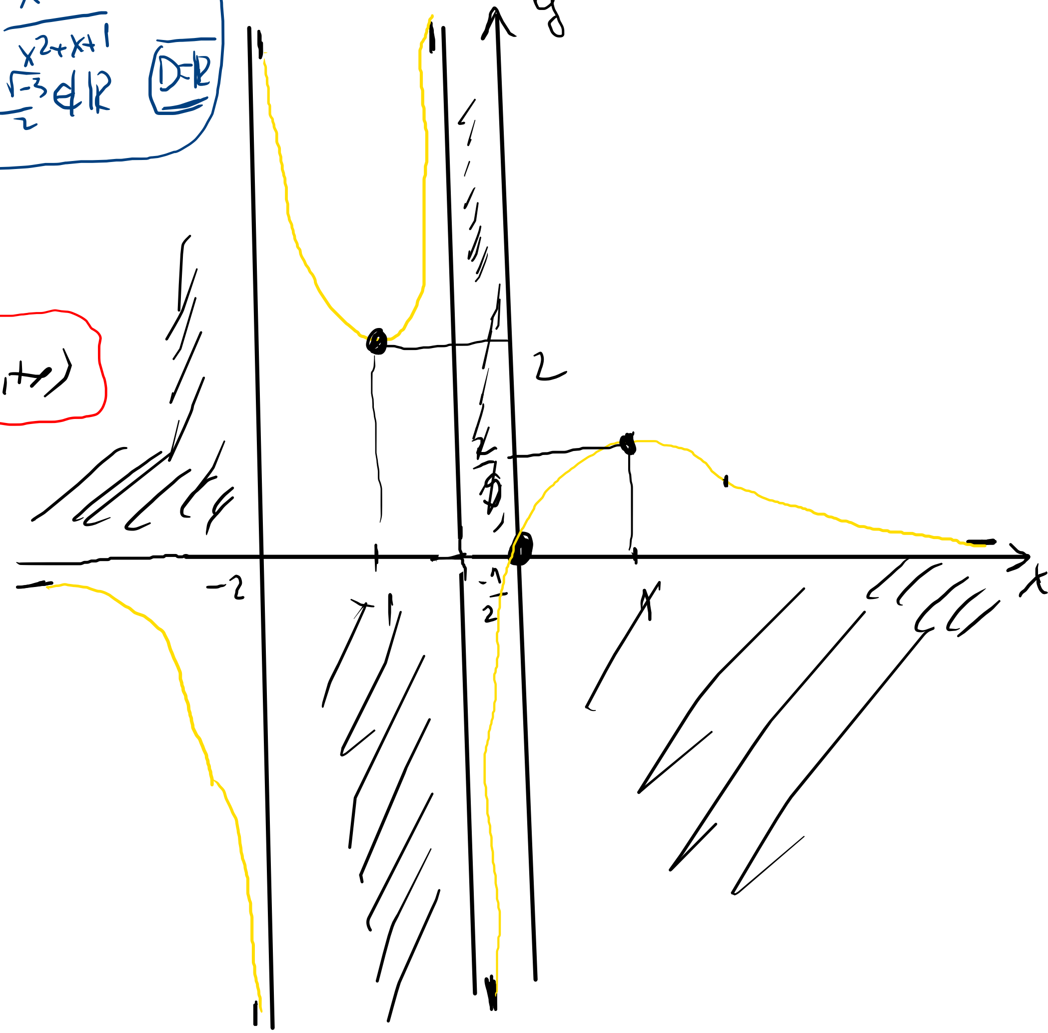
3, $y > 0 \qquad \frac{\cancel{2}x}{\cancel{2}(x+2)(x+\frac{1}{2})} > 0$

	-2	$-\frac{1}{2}$	0	
x	$-$	$-$	$-$ 0	$+$
$x+2$	$-$ 0	$+$	$+$	$+$
$x+\frac{1}{2}$	$-$	$-$ 0	$+$	$+$
	$-$	$+$	$-$	$+$

$y > 0 \quad x \in (-2, -\frac{1}{2}) \cup (0, \infty)$

$y < 0 \quad x \in (-\infty, -2) \cup (-\frac{1}{2}, 0)$

4, NI — NI



5. V.A.

$$\lim_{x\to -\frac{1}{2}^-} f(x) = \lim_{x\to -\frac{1}{2}^-} \frac{x}{(x+2)(x+\frac{1}{2})} = \frac{-\frac{1}{2}}{\frac{3}{2}\ 0^-} = +\infty \qquad \boxed{x=-\frac{1}{2}} \text{ V.A.}$$

$$\lim_{x\to -\frac{1}{2}^+} f(x) = \lim_{x\to -\frac{1}{2}^+} \frac{x}{(x+2)(x+\frac{1}{2})} = \frac{-\frac{1}{2}}{\frac{3}{2}\ 0^+} = -\infty$$

$$\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} \frac{x}{(x+2)(x+\frac{1}{2})} = \frac{-2}{0^-(-\frac{3}{2})} = -\infty \qquad \boxed{x=-2} \text{ V.A.}$$

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} \frac{x}{(x+2)(x+\frac{1}{2})} = \frac{-2}{0^+(-\frac{3}{2})} = +\infty$$

H.A. $$\lim_{x\to \pm\infty} f(x) = \lim_{x\to \pm\infty} \frac{2x}{2x^2+5x+2} = 0$$

$$\boxed{y=0} \text{ H.A.}$$

K.A. - нема

6, $y = \frac{2x}{2x^2+5x+2} = \frac{x}{(x+2)(x+\frac{1}{2})}$

$$y' = \frac{2\cdot(2x^2+5x+2) - 2x\cdot(4x+5)}{(x+2)^2(x+\frac{1}{2})^2} = \frac{2(2x^2+\cancel{5x}+2-4x^2-\cancel{5x})}{(x+2)^2(x+\frac{1}{2})^2}$$

$$= \frac{2(2-2x^2)}{(x+2)^2(x+\frac{1}{2})^2} = \frac{4(1-x)(1+x)}{(x+2)^2(x+\frac{1}{2})^2}$$

$x \neq -2,\ x \neq -\frac{1}{2}$

$y' = 0$ $\quad 1-x=0 \quad \boxed{x=1} \qquad 1+x=0 \quad \boxed{x=-1}$

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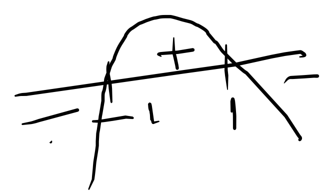
$$y' > 0 \qquad \frac{\overset{>0}{4}(1-x)(1+x)}{\underbrace{(x+2)^2(x+\frac{1}{2})^2}_{>0\ \forall x \in D}} > 0$$

$$\overbrace{(1-x)(1+x)}^{1-x^2} > 0$$

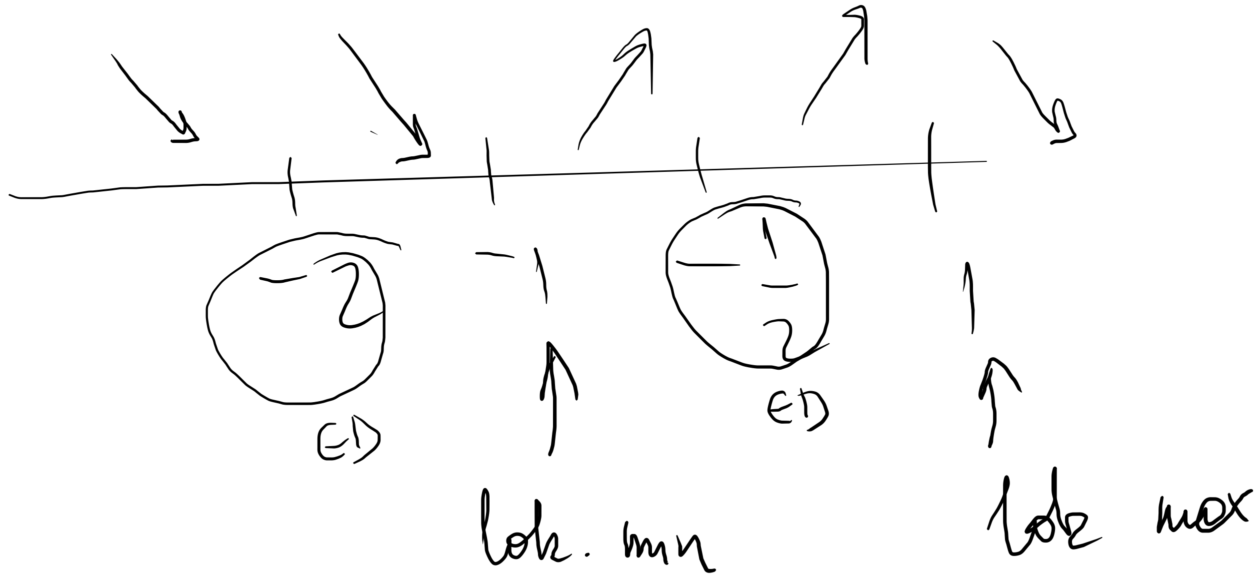


		-1		1	
$1-x$	$+$		$+$	0	$-$
$1+x$	$-$	0	$+$		$+$
	$-$		$+$		$-$

$y' > 0 \quad x \in (-1, -\frac{1}{2}) \cup (-\frac{1}{2}, 1)$ $y \nearrow$

$y' < 0 \quad x \in (-\infty, -2) \cup (-2, -1) \cup (1, \infty)$

$y \searrow$



$x=-1$ $\quad y_{min}(-1) = \dfrac{2\cdot(-1)}{2\cdot(-1)^2+5(-1)+2} = \dfrac{-2}{2-5+2} = 2$

$x=1$ $\quad y_{max}(1) = \dfrac{2\cdot 1}{2\cdot 1+5\cdot 1+2} = \dfrac{2}{9}$

7, $y' = \dfrac{4(1-x^2)}{(x+2)^2(x+\frac{1}{2})^2} = \dfrac{4(1-x^2)}{\left((x+2)(x+\frac{1}{2})\right)^2} = \dfrac{4(1-x^2)}{(x^2+\frac{5}{2}x+1)^2}$

$$y'' = \frac{-8x(x+2)^2(x+\frac{1}{2})^2 - 4(1-x^2)\, 2(x^2+\frac{5}{2}x+1)\cdot(2x+\frac{5}{2})}{(x+2)^4(x+\frac{1}{2})^4}$$