

Linearna funkcija. Linearne jednačine i nejednačine, sistemi linearnih jednačina i nejednačina

7. април 2024.

Linearna funkcija je funkcija oblika $f(x) = ax + b$, odnosno $y = ax + b$, gde su a i b realni koeficijenti. Grafik linearne funkcije je prava; broj a je koeficijent pravca prave, a broj b predstavlja presek grafika sa y -osom. U zavisnosti od koeficijenta linearne funkcije razlikujemo sledeće slučajeve:

- ▶ Ako je $a = 0$, grafik je prava linija paralelna x -osi; na slici primer ovakve funkcije je $f_1(x) = 2$ (crvena prava).
- ▶ Ako je $b = 0$, tada prava prolazi kroz koordinatni početak; takvi primeri su funkcije $f_2(x) = x$ (plava prava) i $f_5(x) = -2x$ (žuta prava).
- ▶ Za $a > 0$ linearna funkcija je rastuća. Primeri rastućih funkcija na slici su $f_2(x) = x$ (plava prava) i $f_3(x) = 3x - 1$ (zelena prava).
- ▶ Za $a < 0$ linearna funkcija je opadajuća. Opadajuće funkcije na slici su $f_4(x) = -x + 2$ (crna prava) i $f_5(x) = -2x$ (žuta prava).

- ▷ **Linearna jednačina** je oblika $ax + b = 0$, $a, b \in \mathbb{R}$, $a \neq 0$, i njeno rešenje je $x = -\frac{b}{a}$.
- ▷ **Linearna nejednačina** može biti oblika $ax + b < 0$, $ax + b > 0$, $ax + b \leq 0$, $ax + b \geq 0$.
- ▶ Neka je $a > 0$, tada iz $ax + b > 0 \implies ax > -b \implies x > -\frac{b}{a}$.
- ▶ Neka je $a < 0$, tada iz $ax + b > 0 \implies ax > -b \implies x < -\frac{b}{a}$.

Zadaci:

1. Rešiti jednačine:

$$1.1 \quad \frac{6x^2 + 9}{x(3x - 1)} - 2 = \frac{3}{x} - \frac{3}{3x - 1}$$

$$1.2 \quad \frac{3x+2}{x-1} + \frac{2x+3}{1-x} = 0 \quad \Rightarrow \quad \frac{3x+2}{x-1} + \frac{2x+3}{-(x-1)} = 0$$

$$\frac{3x+2 - (2x+3)}{x-1} = 0$$

$$\frac{x-1}{x-1} = 0$$

$$\underline{1=0}$$

NEHA RESENSE

$$\begin{aligned} & \cdot (x-1) \\ 3x+2 - 2x-3 &= 0 \\ x-1 &= 0 \\ x &= 1 \end{aligned}$$

USLOV:

$$x-1 \neq 0$$

$$\underline{x \neq 1}$$



2. Rešiti jednačine:

2.1 $|x+2| - 3 = 2x - 6$

$$|x+2| = 2x - 3$$

$$\frac{-x-2}{-2} \mid \frac{x+2}{-2}$$

$$|x+2| = \begin{cases} x+2, & x+2 \geq 0 \\ -(x+2), & x+2 < 0 \end{cases} = \begin{cases} x+2, & x \geq -2 \\ -x-2, & x < -2 \end{cases}$$

1) $x < -2$

$$-x-2 = 2x-3$$

$$-3x = -1$$

$$\left| x = \frac{1}{3} \right| > -2$$

NEHA REŠENJA

2) $x \geq -2$

$$x+2 = 2x-3$$

$$-x = -5$$

$$\boxed{x=5} > -2$$

Rešenje je $x=5$

$$x \in \{5\}$$



$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\begin{aligned} 1-x &\geq 0 & x &\leq 1 \\ -x &\geq -1 & x &\leq 1 \\ x &\geq 1 & & \end{aligned}$$

$$|3x+5| = \begin{cases} 3x+5, & 3x+5 \geq 0 \\ -(3x+5), & 3x+5 < 0 \end{cases} = \begin{cases} 3x+5, & x \geq -\frac{5}{3} \\ -3x-5, & x < -\frac{5}{3} \end{cases}$$

$$|1-x| = \begin{cases} 1-x, & 1-x \geq 0 \\ -(1-x), & 1-x < 0 \end{cases} = \begin{cases} 1-x, & x \leq 1 \\ -1+x, & x > 1 \end{cases}$$

$$2.2 \quad |x+1| + |x-1| = 4$$

$$|x+1| = \begin{cases} x+1, & x+1 \geq 0 \\ -(x+1), & x+1 < 0 \end{cases} = \begin{cases} x+1, & x \geq -1 \\ -x-1, & x < -1 \end{cases}$$

$$|x-1| = \begin{cases} x-1, & x-1 \geq 0 \\ -(x-1), & x-1 < 0 \end{cases} = \begin{cases} x-1, & x \geq 1 \\ -x+1, & x < 1 \end{cases}$$

1, $x < -1$

$$-x-1 + (-x+1) = 4$$

$$-x-x-x+1 = 4$$

$$-2x = 4$$

$$\boxed{x = -2} < -1$$

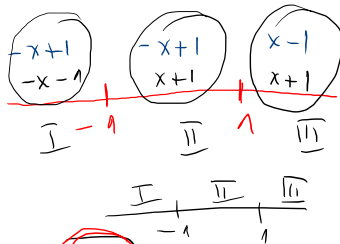
2, $-1 \leq x < 1$

$$x+1 + (-x+1) = 4$$

$$\cancel{x}+1 - \cancel{x}+1 = 4$$

$$2 = 4$$

Résulte $x \in \{2, -2\}$



3, $x \geq 1$

$$x+1 + x-1 = 4$$

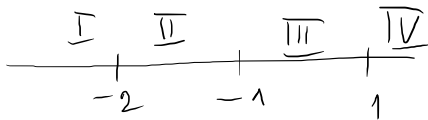
$$2x = 4$$

$$\boxed{x = 2} \geq 1$$

✓



$$|x+1| + |x-1| + |x+2| = 1$$



3. Rešiti nejednačine:

$$3.1 \quad \frac{x-3}{x-1} \geq \frac{x-5}{x-3}$$

$$\frac{x-3}{x-1} - \frac{x-5}{x-3} \geq 0$$

$$\frac{(x-3)^2 - (x-5)(x-1)}{(x-1)(x-3)} \geq 0$$

$$\frac{x^2 - 6x + 9 - (x^2 - x - 5x + 5)}{(x-1)(x-3)} \geq 0$$

$$\frac{\cancel{x^2} - \cancel{6x} + 9 - \cancel{x^2} + \cancel{6x} - 5}{(x-1)(x-3)} \geq 0$$

$$\frac{\textcircled{4} > 0}{(x-1)(x-3)} \geq 0$$

$$(x-1)(x-3) \geq 0$$

1
3

	$(-\infty, 1)$	$(1, 3)$	$[3, +\infty)$
$x-1$	-	•	+
$x-3$	-	-	•
$(x-1)(x-3)$	+	-	+

$$x \in (-\infty, 1] \cup [3, +\infty)$$

Rešenje:

$$x \in (-\infty, 1] \cup [3, +\infty)$$

USLOVI:

$$x-1 \neq 0 \quad \wedge \quad x-3 \neq 0$$

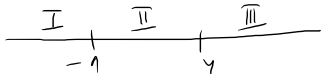
$$x \neq 1 \quad \wedge \quad x \neq 3$$



$$3.2 |x+1| + |x-4| > 7$$

$$|x+1| = \begin{cases} x+1, & x+1 \geq 0 \\ -(x+1), & x+1 < 0 \end{cases} = \begin{cases} x+1, & x \geq -1 \\ -x-1, & x < -1 \end{cases}$$

$$|x-4| = \begin{cases} x-4, & x-4 \geq 0 \\ -(x-4), & x-4 < 0 \end{cases} = \begin{cases} x-4, & x \geq 4 \\ -x+4, & x < 4 \end{cases}$$



I $x < -1$ ✓

$$-x-1 + (-x+4) > 7$$

$$-x-1-x+4 > 7$$

$$-2x+3 > 7$$

$$-2x > 4 \quad | :(-2) < 0$$

$x < -2$ ✓



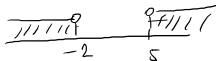
II $-1 \leq x < 4$

$$x+1 + (-x+4) > 7$$

$$x+1-x+4 > 7$$

$$5 > 7$$

⊥



Resenje:

$$x \in (-\infty, -2) \cup (5, +\infty)$$



III $x \geq 4$

$$x+1 + x-4 > 7$$

$$2x-3 > 7$$

$$2x > 10$$

$x > 5$



$x \in (5, +\infty)$

$$3.3 \quad \frac{1}{|6-x|} < \frac{2}{|3-x|} \quad / \cdot \underbrace{|6-x| \cdot |3-x|}_{\substack{\text{N02E} \quad \text{202E} \quad \text{2E}}}$$

USLOV:

$$6-x \neq 0 \wedge 3-x \neq 0$$

$$\boxed{x \neq 6} \wedge \boxed{x \neq 3}$$

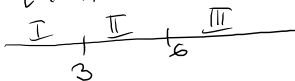
$$x \in \mathbb{D} \setminus \{3, 6\}$$

$$|3-x| < 2|6-x|$$

$$\boxed{|3-x| - 2|6-x| < 0}$$

$$|3-x| = \begin{cases} 3-x, & 3-x \geq 0 \\ -(3-x), & 3-x < 0 \end{cases} = \begin{cases} 3-x, & x \leq 3 \\ -3+x, & x > 3 \end{cases}$$

$$|6-x| = \begin{cases} 6-x, & 6-x \geq 0 \\ -(6-x), & 6-x < 0 \end{cases} = \begin{cases} 6-x & x \leq 6 \\ -6+x & x > 6 \end{cases}$$



$$\frac{1}{\cancel{|6-x|}} \cdot \cancel{|6-x|} |3-x| < \frac{2}{|3-x|} \cdot \cancel{|6-x|} \cancel{|3-x|}$$

$$|3-x| < 2|6-x|$$

$$\frac{1}{|6-x|} \Leftrightarrow \frac{2}{|3-x|}$$

$$|3-x| < 2|6-x|$$

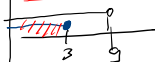
$$\text{I} \quad \boxed{x \leq 3}$$

$$3-x - 2(6-x) < 0$$

$$3-x - 12 + 2x < 0$$

$$x - 9 < 0$$

$$\boxed{x < 9}$$



$$x \in [-\infty, 3]$$

$$\text{II} \quad \boxed{3 < x \leq 6}$$

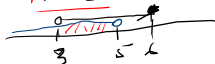
$$-3+x - 2(6-x) < 0$$

$$-3+x - 12 + 2x < 0$$

$$3x - 15 < 0$$

$$3x < 15$$

$$\boxed{x < 5}$$



$$x \in (3, 5)$$



III

$$x > 6$$

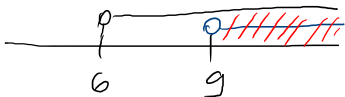
$$-3 + x - 2(-6 + x) < 0$$

$$-3 + x + 12 - 2x < 0$$

$$-x + 9 < 0$$

$$-x < -9$$

$$x > 9$$



$$x \in (9, +\infty)$$



~~Resolva~~

$$x \in (-\infty, 5) \cup (9, +\infty)$$

Zbog uslova zadatka

$$x \in (-\infty, 3) \cup (3, 5) \cup (9, +\infty)$$

$$\frac{1}{|6-x|} < \frac{1}{|3-x|}$$

I

$$x \leq 3$$

$$\frac{1}{6-x} < \frac{1}{3-x}$$

$$\frac{1}{6-x} - \frac{1}{3-x} < 0$$

⋮

⋮

⋮

$$3.4 \frac{x^2 + |x-1|}{x-3} \leq x$$

Ušlov: $x-3 \neq 0$
 $x \neq 3$

$$\frac{x^2 + |x-1|}{x-3} - x \leq 0$$

$$\frac{x^2 + |x-1| - x(x-3)}{x-3} \leq 0$$

$$\frac{x^2 + |x-1| - \cancel{x^2} + 3x}{x-3} \leq 0$$

$$\boxed{\frac{|x-1| + 3x}{x-3} \leq 0}$$

Rezultat: $x \in [-\frac{1}{2}, 3)$

$$|x-1| = \begin{cases} x-1, & x-1 \geq 0 \\ -(x-1), & x-1 < 0 \end{cases} = \begin{cases} x-1, & x \geq 1 \\ -x+1, & x < 1 \end{cases} \quad \text{I} \quad \text{II}$$

I $x < 1$

$$\frac{-x+1+3x}{x-3} \leq 0$$

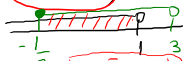
$$\boxed{\frac{2x+1}{x-3} \leq 0}$$

OPHO ZA OVO PJEZ USLOVA

	$-\frac{1}{2}$	3	
$2x+1$	-	+	+
$x-3$	-	-	+
$\frac{2x+1}{x-3}$	+	-	+

$$x \in [-\frac{1}{2}, 3)$$

$2x+1=0$
 $2x=-1$
 $x=-\frac{1}{2}$



$$x \in [-\frac{1}{2}, 1)$$

II $x \geq 1$

$$\frac{x-1+3x}{x-3} \leq 0$$

$$\frac{4x-1}{x-3} \leq 0$$

	$\frac{1}{4}$	3	
$4x-1$	-	+	+
$x-3$	-	-	+
$\frac{4x-1}{x-3}$	+	-	+

$$x \in [\frac{1}{4}, 3)$$



$$x \in [1, 3)$$



4. Rešiti sisteme jednačina:

$$4.1 \quad \begin{array}{r} 2x + y = -1 \quad / \cdot (-2) \\ 4x + 3y = 1 \quad \downarrow \end{array}$$

$$\begin{array}{r} -4x - 2y = 2 \\ 4x + 3y = 1 \quad \downarrow + \end{array}$$

$$\boxed{y = 3}$$

$$2x + 3 = -1$$

$$2x = -4$$

$$\boxed{x = -2}$$

Rešenje $\{(-2, 3)\}$

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$$x \neq 2$$

$$x \neq 3$$

4.2

$$\frac{7}{x-2} + \frac{5}{y-3} = \frac{17}{2}$$

$$\frac{4}{x-2} - \frac{1}{y-3} = 1$$

$$7a + 5b = \frac{17}{2}$$

$$4a - b = 1$$

$$b = 4a - 1$$

$$7a + 5(4a - 1) = \frac{17}{2}$$

$$7a + 20a - 5 = \frac{17}{2}$$

$$27a = \frac{17}{2} + 5$$

$$27a = \frac{27}{2}$$

$$\boxed{a = \frac{1}{2}}$$

$$\frac{1}{x-2} = a$$

$$\frac{1}{y-3} = b$$

$$b = 4 \cdot \frac{1}{2} - 1$$

$$\boxed{b = 1}$$

$$\frac{1}{x-2} = \frac{1}{2}$$

$$\frac{1}{y-3} = \frac{1}{1}$$

$$x-2=2 \Rightarrow x=4$$

$$y-3=1 \Rightarrow y=4$$

$$R_s = \{(4, 4)\}$$



5. Rešiti sisteme nejednačina:

5.1
$$\begin{cases} x + 4 < 2x - 1 \\ x - 3 < 1 + 3x \end{cases}$$

$$\rightarrow x - 2x < -1 - 4$$

$$-x < -5$$

$$\boxed{x > 5}$$



$$\begin{aligned} x - 3x - 3 &< 1 + 3x - x \\ x - 3 - 3 + 3 &< 1 + 3 \end{aligned}$$

$$\boxed{x - 3x < 1 + 3}$$

$$-2x < 4 \quad | : (-2)$$

$$x > \frac{4}{-2}$$

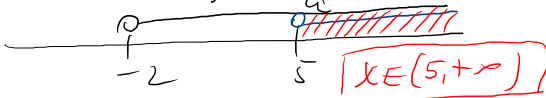
$$\boxed{x > -2}$$



$$a < 0$$

$$\underline{ax} > -b$$

$$x < -\frac{b}{a}$$



$$\underline{ax} + b > 0$$

$$a > 0 \quad ax > -b$$

$$x > -\frac{b}{a}$$

$$5.2 \quad \frac{x-1}{2} > 2 + \frac{x+1}{3}$$

$$\frac{x-2}{5} - \frac{x-10}{15} > \frac{x+1}{2} - \frac{x}{4}$$

2,5	4,15	2
1,5	2,15	2
1,5	1,15	5
1,1	1,3	3
1,1	1,1	3

(60)

$$\frac{x-1}{2} > 2 + \frac{x+1}{3} \quad | \cdot 6$$

$$\frac{x-2}{5} - \frac{x-10}{15} > \frac{x+1}{2} - \frac{x}{4} \quad | \cdot 60$$

$$3(x-1) > 12 + 2(x+1)$$

$$12(x-2) - 4(x-10) > 30(x+1) - 15x$$

$$3x - 3 > 12 + 2x + 2$$

$$12x - 24 - 4x + 40 > 30x + 30 - 15x$$

$$\boxed{x > 17}$$

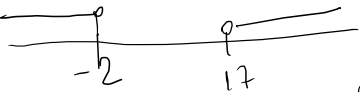
$$8x + 16 > 15x + 30$$

$$-7x > 14 \quad | : (-7) < 0$$

$$x < \frac{14}{-7}$$

$$\boxed{x < -2}$$

~~$$x < -2$$~~



NE MA RESPOSTA $R_e = \emptyset$

