Iracionalne jednačine i nejednačine.

7. април 2024.

Iracionalne jednačine su jednačine oblika $\sqrt{f(x)} = g(x)$ i one se rešavaju kvadriranjem, pri čemu mora biti $f(x) \geq 0$ i $g(x) \geq 0$.

$\sqrt{x^2+3} = -1$

$x \in \emptyset$

$-\sqrt{x^2+3} = -1$

Iracionalne nejednačine se rešavaju na više načina u zavisnosti od oblika:

1. $\sqrt{f(x)} \geq g(x) \iff$
   $(g(x) \geq 0 \;\wedge\; f(x) \geq g^2(x)) \;\vee\; (g(x) \leq 0 \;\wedge\; f(x) \geq 0)$;

2. $\sqrt{f(x)} \leq g(x) \iff g(x) \geq 0 \;\wedge\; f(x) \geq 0 \;\wedge\; f(x) \leq g^2(x)$.

Zadaci:

1. Rešiti jednačine:

   1.1 $(9 - x^2)\sqrt{x+7} = 0$

$9 - x^2 = 0 \quad \vee \quad \sqrt{x+7} = 0$

$x^2 = 9 \qquad x + 7 = 0$

$x = \pm 3 \qquad x = -7$

$\{-7, -3, 3\}$

uslov

$x + 7 \geq 0$

$x \geq -7$

$\sqrt{A}$

$A \geq 0$

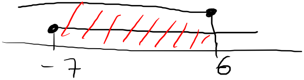
1.2 $\sqrt{x+7} = 6 - x$

1) $x+7 \geq 0$

$\boxed{x \geq -7}$

2) $6-x \geq 0$

$\boxed{x \leq 6}$



$x \in [-7, 6]$

3) $\sqrt{x+7} = 6 - x \quad /^2$

$x+7 = (6-x)^2$

$x+7 = 36 - 12x + x^2$

$x^2 - 13x + 29 = 0$

$x_{1,2} = \dfrac{13 \pm \sqrt{169-116}}{2}$

$= \dfrac{13 \pm \sqrt{53}}{2}$

$13 \cdot 13$
39
13
169

$29 \cdot 4$
116

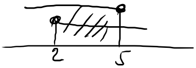
$x_1 = \dfrac{13+\sqrt{53}}{2} \notin [-7, 6]$

$\boxed{x_2 = \dfrac{13-\sqrt{53}}{2} \in [-7, 6]}$

1.3 $\sqrt{x-2}+\sqrt{5-x}=2 \quad /^2$

1) $x-2\geq 0$, 2 , $5-x\geq 0$

$x\geq 2$ $\quad$ $x\leq 5$



$x\in[2,5]$

3) $\left(\sqrt{x-2}+\sqrt{5-x}\right)^2=2^2$

$\cancel{x}-2+2\sqrt{x-2}\sqrt{5-x}+5-\cancel{x}=4$

$2\sqrt{(x-2)(5-x)}=1 \quad /^2$

$4(x-2)(5-x)=1$

$4(5x-x^2-10+2x)=1$

$-4x^2+28x-40=1$

$-4x^2+28x-41=0$

$[2,5]=\left[\frac{16}{8},\frac{40}{8}\right]$

$28\cdot 28$
224
56
784

784
-656
128

$16\cdot 41$
16
64
656

$\frac{16}{8}<\frac{17}{8}\leq\frac{40}{8}$

$$x_{1,2}=\frac{-28\pm\sqrt{784-656}}{-8}=\frac{-28\pm\sqrt{128}}{-8}$$

$$x_1=\underbrace{\frac{-28+\sqrt{128}}{-8}}_{\frac{17}{8}}\in[2,5] \checkmark$$

$$x_2=\underbrace{\frac{-28-\sqrt{128}}{-8}}_{\frac{39}{8}}\in[2,5] \checkmark$$

1.4 $\sqrt{-x^2+x+6} > 1-x$

I $\quad 1-x \geq 0 \quad \wedge \quad -x^2+x+6 > (1-x)^2$

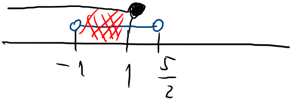
$\boxed{x \leq 1} \quad \wedge \quad -x^2+x+6 > 1-2x+x^2$

$-2x^2+3x+5>0$

$-2x^2+3x+5=0$

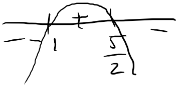
$x_{1,2} = \frac{-3 \pm \sqrt{9+40}}{-4}$

$= \frac{-3 \pm 7}{-4} = \begin{cases} -1 \\ \frac{5}{2} \end{cases}$



$x \in (-1, 1]$

$\boxed{x \in \left(-1, \frac{5}{2}\right)}$

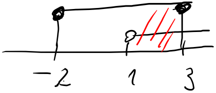


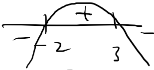
II $\quad 1-x<0 \quad \wedge \quad -x^2+x+6 \geq 0$

$x>1 \quad \wedge \quad -x^2+x+6=0$

$x_{1,2} = \frac{-1 \pm \sqrt{1+24}}{-2}$

$= \frac{-1 \pm 5}{-2} \begin{cases} -2 \\ 3 \end{cases}$





$x \in [-2, 3]$

$x \in (1, 3]$

$\boxed{x \in (-1, 3]}$

1.5 $\sqrt{x^2-5x+4} < x-3$

X-3 NE MOŽE BITI NEGATIVNO JER KOREN NE MOŽE BITI MANJI OD NEGATIVNOG IZRA

DAKLE $\boxed{x-3>0}$

DA BI KOREN BIO DEFINISAN $\boxed{x^2-5x+4 \geq 0}$.

I POD TIM USLOVIMA MOŽEMO KVADRIRATI NEJEDNAČINU, TJ.

$$\boxed{x^2-5x+4 < (x-3)^2}$$

ZNAČI REŠAVAMO:

$x-3>0 \quad \wedge \quad x^2-5x+4 \geq 0 \quad \wedge \quad x^2-5x+4 < (x-3)^2$

$\boxed{x>3}$

$x^2-5x+4=0$

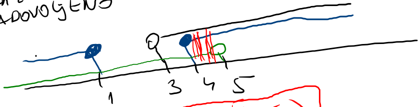
$x_{1,2} = \dfrac{5 \pm \sqrt{25-16}}{2} = \dfrac{5 \pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases}$

$\boxed{x \in (-\infty, 1] \cup [4, +\infty)}$

$\cancel{x^2}-5x+4 < \cancel{x^2}-6x+9$

$\boxed{x<5}$

TRAŽIMO PRESEK JER SVE MORA BITI ZADOVOLJENO



$\boxed{x \in [4, 5)}$

1.6 $\sqrt{\frac{x-2}{1-2x}} > -1$

KAKO JE KOREN NEKOG BROJA UVEK ≥ 0 VREDNOST OVOG KORENA JE SVAKAKO VEĆA OD -1 PA TREBA SAMO DA PROVERIMO KADA JE KOREN DEFINISAN, A TO JE ZA

$$\frac{x-2}{1-2x} \geq 0 \quad \wedge \quad 1-2x \neq 0$$

$$x \neq \frac{1}{2}$$

		$\frac{1}{2}$		2	
$x-2$	$-$		$-$	$\bullet$	$+$
$1-2x$	$+$	$\circ$	$-$		$-$
	$-$		$+$		$-$

$$\boxed{x \in \left(\frac{1}{2}, 2\right]}$$