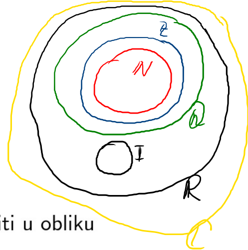
# Brojevi. Stepenovanje i korenovanje. Racionalni algebarski izrazi

7. април 2024.




Koriste se sledeće oznake za brojeve:

$\mathbb{N}$- skup prirodnih brojeva $\mathbb{N} = \{1, 2, 3 \ldots\}$, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$;

$2 = \frac{2}{1}$

$\mathbb{Z}$ - skup celih brojeva $\mathbb{Z} = \{\ldots - 3, -2, -1, 0, \underline{1, 2, 3, \ldots}\}$;

$-5 = \frac{-5}{1} = \frac{-10}{2}$

$\mathbb{Q}$ - skup racionalnih brojeva $\mathbb{Q} = \left\{ \frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{N} \right\}$;

$\mathbb{I}$- skup iracionalnih brojeva (brojeva koji se ne mogu predstaviti u obliku količnika dva cela broja); $\sqrt{2}$, $\pi$,

$\mathbb{R}$ - skup realnih brojeva $\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$;

$\mathbb{C}$ - skup kompleksnih brojeva $\mathbb{C} = \{\underline{a + bi} \mid a, b \in \mathbb{R}, \underline{i^2 = -1}\}$.

**Apsolutna vrednost broja** $a$ je: $\mid a \mid = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$.

$|5| = 5$

$|-5| = 5$

# Stepenovanje

Za $a \in \mathbb{R}$ i $n \in \mathbb{N}$ definišemo $n$-**ti stepen broja** $a$

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ puta}}$$

$$a^1 = a$$
$$a^{n+1} = a^n \cdot a$$
$$a^{-2} = \frac{1}{a^2} \qquad \frac{1}{a^3} = a^{-3}$$

- ▷ $a$ je *osnova*, a $n$ je *izložilac*.
- ▷ Ako je $a \neq 0$, onda važi $a^0 = 1$ i $\boxed{a^{-n} = \frac{1}{a^n}}$, $n \in \mathbb{N}$.

$$a^{-5} = \frac{1}{a^5}$$

**Osobine stepena:**

$a, b \in \mathbb{R}$, $n, m \in \mathbb{N}$

- $a^n \cdot a^m = a^{n+m}$
- $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$
- $(a^m)^n = a^{m \cdot n}$
- $a^n \cdot b^n = (a \cdot b)^n$
- $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$, $b \neq 0$

$$a^{-3} \cdot a^{-2} = a^{-3-2} = a^{-5}$$

$$a^3 \cdot a^5 = a^{3+5} = a^8$$
$$\frac{a^3}{a^5} = a^{3-5} = a^{-2}$$
$$a \neq 0$$
$$(a^3)^5 = a^{3 \cdot 5} = a^{15}$$

$$(a \cdot b)^6 = a^6 \cdot b^6$$
$$\left(\frac{a}{b}\right)^6 = \frac{a^6}{b^6}$$
$$b \neq 0$$

*Napomena:* Sve osobine važe i za $a, b \in \mathbb{R} \setminus \{0\}$, $n, m \in \mathbb{Z}$.

## Korenovanje:

$\boxed{\sqrt[n]{a} = a^{\frac{1}{n}}}$

- za $a \geq 0$, $a \in \mathbb{R}$ i $n \in \mathbb{N}$ $n$-**ti koren broja** $a$ je nenegativno rešenje jednačine $x^n = a$;
- za $a < 0$, $a \in \mathbb{R}$, $n \in \mathbb{N}$ i $n$ neparan broj $n$-**ti koren broja** $a$ je rešenje jednačine $x^n = a$;

## Osobine korena:

$a, b \in \mathbb{R}^+$, $n, m \in \mathbb{N}$

- $\sqrt[n]{a^m} = (\sqrt[n]{a})^m = a^{\frac{m}{n}}$
- $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
- $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- $\sqrt[n]{a} = \sqrt[n \cdot m]{a^m}$
- $a \cdot \sqrt[n]{b} = \sqrt[n]{a^n \cdot b}$

*Napomena:* Ako je $a \in \mathbb{R}$, onda važi

$$\sqrt[n]{a^n} = \begin{cases} a, & n - \text{neparno} \\ |a|, & n - \text{parno} \end{cases}$$

$a^{-n} = \frac{1}{a^n}$ $\quad \sqrt[n]{a} = a^{\frac{1}{n}}$

$\cancel{\sqrt{-5}}$ ?

$\sqrt[3]{-8} = -2$

$(-2)^3 = -8$

$\sqrt{x}, \; x \geq 0$

$\sqrt[3]{x}, \; \forall x \in \mathbb{R}$

$\sqrt{2^2} = 2$

$\sqrt{(-2)^2} = 2$

$\sqrt{a^2} = |a|$ ?

$\cancel{\sqrt{16} = \sqrt{4^2} = 4}$ ?

$\sqrt[5]{a^3} = (\sqrt[5]{a})^3 = a^{\frac{3}{5}}$

$\sqrt[3]{a \cdot b} = \sqrt[3]{a} \cdot \sqrt[3]{b}$

$e^{\pi i} \cdot e^{3\pi i} = e^{\pi i + 3\pi i} = e^{4\pi i}$

$2^{\pi} \cdot 2^4 = 2^{\pi + 4}$

$\sqrt[6]{\frac{a}{b}} = \frac{\sqrt[6]{a}}{\sqrt[6]{b}}$

$\sqrt[5]{a} = \sqrt[15]{a^3}$

$\rightarrow a^{\frac{1}{5}} = a^{\frac{3}{15}}$

$\sqrt[3]{a^4 \cdot b} = \sqrt[3]{a^4} \cdot \sqrt[3]{b}$

$= a \cdot \sqrt[3]{b}$

## Zadaci:

1. Uprostiti izraze:

1.1 $$\frac{2^{-2}+2^{0}}{\left(\frac{1}{2}\right)^{-2}-5\,(-2)^{-2}+\left(\frac{2}{3}\right)^{-2}} = \frac{\frac{1}{2^2}+1}{2^2-5\left(\frac{1}{2}\right)^2+\left(\frac{3}{2}\right)^2} = \frac{\frac{1}{4}+1}{4-\frac{5}{4}+\frac{9}{4}}$$

$$\left(\frac{1}{2}\right)^{-2} = \left(2^{-1}\right)^{-2} = 2^2$$

$$\left(\frac{1}{2}\right)^{-2} = \frac{1}{\left(\frac{1}{2}\right)^2} = 2^2$$

$$(-2)^{-2} = \left(\frac{1}{-2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$= \frac{\frac{1+4}{\cancel{4}}}{\frac{16-5+9}{\cancel{4}}} = \frac{5}{20} = \frac{1}{4}$$

$a \neq 0$
$b \neq 0$

$a^{-n} = \frac{1}{a^n}$

1.2 $\frac{2a^2}{5b^{-2}} : \frac{10a^{-3}}{6b^{-1}} = \frac{\cancel{2}a^2}{5b^{-2}} \cdot \frac{6b^{-1}}{\cancel{10}_5 a^{-3}} = \frac{6}{25} a^2 \cdot a^3 \cdot b^{-1} \cdot b^2$

$= \frac{6}{25} a^{2+3} \cdot b^{-1+2} = \frac{6}{25} a^5 \cdot b$

$= \dfrac{\frac{2a^2}{5b^{-2}}}{\frac{10a^{-3}}{6b^{-1}}} = \frac{1\cancel{2}a^2b^{-1}}{5\cancel{0}\, b^{-2}a^{-3}} = \frac{6}{25} a^{2-(-3)} \cdot b^{-1-(-2)} =$

$= \frac{6}{25} a^5 b$

$= \frac{\cancel{2}}{5} \cdot \frac{6}{\cancel{10}} \frac{a^{2-(-3)}}{b^{-2-(-1)}} = \frac{6}{25} \frac{a^5}{b^{-1}} = \frac{6}{25} a^5 b$

1.3 $\left(\frac{5x^{-5}}{2y^{-2}}\right)^{-2}\cdot\left(\frac{y^{-1}}{5x^{-1}}\right)^{-3} : \left(10x^{2}y^{-3}\right) =$

$$= \left(\frac{2y^{-2}}{5x^{-5}}\right)^{2}\cdot\left(\frac{5x^{-1}}{y^{-1}}\right)^{3}\cdot\frac{1}{10x^{2}y^{-3}}$$

$$= \frac{(2y^{-2})^{2}}{(5x^{-5})^{2}}\cdot\frac{(5x^{-1})^{3}}{(y^{-1})^{3}}\cdot\frac{1}{10x^{2}y^{-3}}$$

$$= \frac{\overset{2}{\cancel{4}}\,y^{-4}}{\underset{5^2}{\cancel{25}}\cdot x^{-10}}\cdot\frac{\overset{\cancel{8}\;5^3}{\cancel{125}}\cdot x^{-3}}{y^{-3}}\cdot\frac{1}{\underset{\cancel{2}}{\cancel{10}}\,x^{2}y^{-3}}$$

$$= 2\,y^{-4+3+3}\;x^{-3+10-2}$$

$$= 2y^{2}x^{5}$$

$$= 2y^{2}x^{5}$$

$x \neq 0$

$y \neq 0$

$$\frac{\overset{2}{\cancel{4}}\cdot\overset{\cancel{5}}{\cancel{125}}}{\cancel{25}\cdot\cancel{10}\;\cancel{2}}$$

$$1.4 \quad \left(\frac{2a^{-2}}{3ab^{-3}}\right)^{-4} : \left(\frac{4a^{-2}}{3b^{-3}}\right)^{-3} \cdot \frac{1}{12a^5b^{-2}} =$$

$$= \left(\frac{3ab^{-3}}{2a^{-2}}\right)^{4} \cdot \left(\frac{4a^{-2}}{3b^{-3}}\right)^{3} \cdot \frac{1}{12a^5b^{-2}}$$

ZABORAVILA sam ovu 3

$$(2^2)^3 = 2^6$$

$$= \frac{\cancel{3^4} \cdot a^4 \cdot b^{-12}}{2^4 \cdot a^{-8}} \cdot \frac{4^3 \cdot a^{-6}}{\cancel{3^3} \cdot b^{-9}} \cdot \frac{1}{12a^5b^{-2}}$$

$$\boxed{3}$$

$$= \frac{3 \cdot 2^{\cancel{6}}}{\cancel{2^4} \cdot 12} a^{4-6+8-5} b^{-12+9+2}$$

$$= \frac{3 \cdot 2^2}{12} a b^{-1}$$

$$= \frac{3 \cdot \cancel{4} a}{\cancel{12} b} = \frac{\cancel{3} a}{\cancel{3} b} = \frac{a}{b} \qquad a, b \neq 0$$

1.5 $\left(\left(x^{\frac{3}{4}}\right)^{2}\right)^{-\frac{1}{3}} : \left(\left(x^{-1}\right)^{\frac{1}{2}}\right)^{\frac{2}{3}} = \frac{1}{\sqrt[3]{\left(\sqrt[4]{x^3}\right)^2}} : \sqrt[3]{\left(\frac{1}{\sqrt{x}}\right)^2}$

$$= x^{\frac{\cancel{3}}{4_{2}} \cdot \cancel{2} \cdot \left(-\frac{1}{\cancel{3}}\right)} : x^{-1 \cdot \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3}}$$

$$= x^{-\frac{1}{2}} : x^{-\frac{1}{3}}$$

$$= x^{-\frac{1}{2} - \left(-\frac{1}{3}\right)}$$

$$= x^{-\frac{1}{2} + \frac{1}{3}}$$

$$= x^{\frac{-3+2}{6}}$$

$x > 0$

$$= x^{-\frac{1}{6}} = \frac{1}{\sqrt[6]{x}}$$

1.6 $\sqrt[9]{\frac{a^{17}b^3c^5}{x^8y^5}} : \sqrt[9]{\frac{a^8c^5x}{b^6y^5}}$

$$= \sqrt[9]{\frac{a^{17}\cdot b^3\cdot c^5}{x^8\cdot y^5} : \frac{a^8\cdot c^5\cdot x}{b^6\cdot y^5}}$$

$$= \sqrt[9]{\frac{a^{17}\cdot b^3\cdot \cancel{c^5}}{x^8\cdot \cancel{y^5}} \cdot \frac{b^6\cdot \cancel{y^5}}{a^8\cdot \cancel{c^5}\cdot x}}$$

$$= \sqrt[9]{\frac{a^{17-8}\ b^{3+6}}{x^{8+1}}}$$

$$= \sqrt[9]{\frac{a^9\ b^9}{x^9}}$$

$$= \frac{ab}{x} \quad , x, y, a, b \neq 0$$

# Racionalni algebarski izrazi

- kvadrat zbira: $(a+b)^2 = a^2 + 2ab + b^2$
- kvadrat razlike: $(a-b)^2 = a^2 - 2ab + b^2$
- kub zbira: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- kub razlike: $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- razlika kvadrata: $a^2 - b^2 = (a-b)(a+b)$
- razlika kubova: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- zbir kubova: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$ax + bx = (a+b)x$

$(a \pm b)^3 = (a \pm b)^2 \cdot (a \pm b)$

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

## Zadaci:

1. Skratiti razlomke i zapisati uslove pod kojima dobijena jednakost važi

1.1 $$\frac{a^2-9}{ab+3b-a-3} = \frac{(a-3)(a+3)}{3(b-1)+a(b-1)} = \frac{(a-3)\cancel{(a+3)}}{(b-1)\cancel{(3+a)}} = \frac{a-3}{b-1}$$

USLOVI: $b-1 \neq 0 \quad \wedge \quad a+3 \neq 0$

$b \neq 1 \qquad a \neq -3$

$$1.2 \quad \frac{(\overbrace{a^2+b^2-c^2}^{A})^2-(\overbrace{a^2-b^2+c^2}^{B})^2}{4ab^2-4abc}$$

$$A^2-B^2=(A-B)(A+B)$$

$$=\frac{((a^2+b^2-c^2)-(a^2-b^2+c^2))\cdot(a^2+\cancel{b^2}-\cancel{c^2}+a^2-\cancel{b^2}+\cancel{c^2})}{4ab(b-c)}$$

$$\frac{\cancel{A-B}}{\cancel{A}} \neq -B$$

$$=\frac{(\cancel{a^2}+b^2-c^2-\cancel{a^2}+b^2-c^2)\cdot 2a^2}{4ab(b-c)}$$

$$A^2+B^2$$

$$=\frac{(2b^2-2c^2)\cdot\cancel{2}a^{\cancel{2}}}{{}_2\cancel{4}\cancel{a}b(b-c)}$$

$$=\frac{\cancel{2}(b^2-c^2)\,a}{\cancel{2}b(b-c)}$$

$$=\frac{\cancel{(b-c)}(b+c)\cdot a}{b\cancel{(b-c)}}$$

$$=\frac{(b+c)a}{b}$$

USLOVI:

$b \neq 0$

$b-c \neq 0 \Rightarrow b \neq c$

$a \neq 0$

$A^3+B^3=(A+B)(A^2-AB+B^2)$

1.3 $\dfrac{a^4+8ab^3}{a^3b+6a^2b^2+12ab^3+8b^4}$

$$=\frac{a(a^3+8b^3)}{b(a^3+6a^2b+12ab^2+8b^3)}=\frac{a\left(a^3+(2b)^3\right)}{b(a^3+\underbrace{3\cdot 2a^2b}+\underbrace{3\cdot 4ab^2}+2^3b^3)}$$

$$=\frac{a(a+2b)(a^2-a\cdot 2b+(2b)^2)}{b(a^3+3a^2\cdot 2b+3\cdot a(2b)^2+(2b)^3)}=\frac{a\cancel{(a+2b)}(a^2-2ab+4b^2)}{b(a+2b)^{\cancel{3}}}$$

$A^3+3\cdot A^2\cdot B+3\cdot A\cdot B^2+B^3$

$(A+B)^3=(A+B)^2(A+B)=(A^2+2AB+B^2)(A+B)$

$=(A^3+A^2B+2A^2B+2AB^2+B^2A+B^3)$

$=(A^3+3A^2B+3AB^2+B^3)$

$$=\frac{a\cdot(a^2-2ab+4b^2)}{b(a+2b)^2}$$

$a^2+4ab+4b^2$

Uslovi:

$b\neq 0$

$a+2b\neq 0$

2. Uprostiti izraze:

2.1 $\left(\frac{x^3+2x^2-x-2}{a+1} : \frac{x^2+4x+4}{x^2-4}\right) : \frac{x^3-2x^2-x+2}{a^2+a}$

$$= \frac{x^3+2x^2-x-2}{a+1} \cdot \frac{x^2-4}{x^2+4x+4} \cdot \frac{a^2+a}{x^3-2x^2-x+2}$$

$$= \frac{2(x^2-1)+x(x^2-1)}{\cancel{a+1}} \cdot \frac{(x-2)\cancel{(x+2)}}{(x+2)^{\cancel{2}}} \cdot \frac{a\cancel{(a+1)}}{x(x^2-1)+2(-x^2+1)}$$

$$-(x^2-1)$$

$$= \frac{\cancel{(x^2-1)}\cancel{(2+x)}\cancel{(x-2)}\,a}{\cancel{(x+2)}\cancel{(x^2-1)}\cancel{(x-2)}} = a$$

USLOVIMA: $x+2 \neq 0 \wedge x^2-1 \neq 0 \wedge x-2 \neq 0 \wedge a+1 \neq 0$

$x \neq -2 \wedge x \neq \pm 1 \quad x \neq 2 \wedge a \neq -1$

$2+3\cdot(7-1)$

$=2+3\cdot 6$

$=2+18=20$

$\rightarrow 4:2\cdot 5$

$=2\cdot 5=10$

$x(x^2-1)-2(x^2-1)$

2.2 $\left(\frac{x}{y}+\frac{y}{x}\right)\cdot\frac{1}{x^2-y^2}-\left(\frac{x}{y}-\frac{y}{x}\right):(x^2-2xy+y^2)$

$$=\frac{x^2+y^2}{xy}\cdot\frac{1}{(x-y)(x+y)}-\frac{x^2-y^2}{xy}\cdot\frac{1}{(x-y)^2}$$

$$=\frac{x^2+y^2}{xy(x-y)(x+y)}-\frac{x^2-y^2}{xy(x-y)^2}$$

$$=\frac{(x^2+y^2)(x-y)-(x^2-y^2)(x+y)}{xy(x-y)^2(x+y)}$$

$(x-y)(x+y)(x+y)$

$$=\frac{(x^2+y^2)(x-y)-(x-y)(x+y)^2}{xy(x-y)^2(x+y)}$$

$$=\frac{\cancel{(x-y)}\left(x^2+y^2-(x+y)^2\right)}{xy\,(x-y)^{\cancel{2}}(x+y)}$$

$$=\frac{x^2+y^2-(x^2+2xy+y^2)}{xy(x-y)(x+y)}$$

$$=\frac{\cancel{x^2}+\cancel{y^2}-\cancel{x^2}-2xy-\cancel{y^2}}{xy(x-y)(x+y)}$$

$$=\frac{-2\cancel{xy}}{\cancel{xy}(x-y)(x+y)}=\frac{-2}{(x-y)(x+y)}$$

$$=\frac{-2}{x^2-y^2}=\frac{2}{y^2-x^2}$$

uslovi:

$x-y\neq 0 \wedge x+y\neq 0 \wedge x\neq 0 \wedge y\neq 0$

2.3 $\left(\frac{a^x}{1-a^{-x}}+\frac{a^{-x}}{1+a^{-x}}\right)-\left(\frac{a^x}{1+a^{-x}}+\frac{1}{1-a^{-x}}\right)$

$$=\frac{a^x}{1-a^{-x}}+\frac{a^{-x}}{1+a^{-x}}-\frac{a^x}{1+a^{-x}}-\frac{1}{1-a^{-x}}$$

$$=\frac{a^x-1}{1-a^{-x}}+\frac{a^{-x}-a^x}{1+a^{-x}}$$

$$=\frac{a^x-1}{1-\frac{1}{a^x}}+\frac{\frac{1}{a^x}-a^x}{1+\frac{1}{a^x}}$$

$$=\frac{a^x-1}{\frac{a^x-1}{a^x}}+\frac{\frac{1-a^{2x}}{a^x}}{\frac{a^x+1}{a^x}}$$

$$=\frac{a^x\cdot\cancel{(a^x-1)}}{\cancel{a^x-1}}+\frac{\cancel{a^x}(1-a^{2x})}{\cancel{a^x}(a^x+1)}$$

$$=a^x+\frac{1^2-(a^x)^2}{1+a^x}$$

$$=a^x+\frac{(1-a^x)\cancel{(1+a^x)}}{\cancel{1+a^x}}$$

$$=\cancel{a^x}+1-\cancel{a^x}=1$$

uslovi:

$1+a^x\neq 0 \quad\wedge\quad a^x-1\neq 0 \quad\wedge\quad a^x\neq 0$

$a^x\neq -1 \qquad\qquad a^x\neq 1$

2.4 $\left(\frac{1}{y}-\frac{1}{x}\right)\cdot\left(\frac{(x+y)^2+2y^2}{x^3-y^3}-\frac{1}{x-y}+\frac{x+y}{x^2+xy+y^2}\right)$

$$=\frac{x-y}{xy}\cdot\left(\frac{x^2+2xy+y^2+2y^2}{(x-y)(x^2+xy+y^2)}-\frac{1}{x-y}+\frac{x+y}{x^2+xy+y^2}\right)$$

$$=\frac{\cancel{x-y}}{xy}\cdot\frac{x^2+2xy+3y^2-(x^2+xy+y^2)+(x+y)(x-y)}{\cancel{(x-y)}(x^2+xy+y^2)}$$

$$=\frac{\cancel{x^2}+2xy+\cancel{3}y^2-\cancel{x^2}-xy-\cancel{y^2}+x^2-\cancel{y^2}}{xy(x^2+xy+y^2)}$$

$$=\frac{\cancel{x^2+xy+y^2}}{xy\,\cancel{(x^2+xy+y^2)}}$$

$$=\frac{1}{xy}$$

Uslovi:

$x\neq 0$, $y\neq 0$

$x\neq y$

$x^2+xy+y^2\neq 0$

2.5 $\left(\frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}-\frac{a-\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}\right):\frac{4a\sqrt{a^2-b^2}}{b^2}$

$$=\frac{(a+\sqrt{a^2-b^2})^2-(a-\sqrt{a^2-b^2})^2}{(a-\sqrt{a^2-b^2})(a+\sqrt{a^2-b^2})}\cdot\frac{b^2}{4a\sqrt{a^2-b^2}}$$

$$=\frac{(a+\cancel{\sqrt{a^2-b^2}}+a-\cancel{\sqrt{a^2-b^2}})\cdot(a+\sqrt{a^2-b^2}-(a-\sqrt{a^2-b^2}))}{a^2-(a^2-b^2)}\cdot\frac{b^2}{4a\sqrt{a^2-b^2}}$$

$$=\frac{2a(\cancel{a}+\sqrt{a^2-b^2}\cancel{-a}+\sqrt{a^2-b^2})}{\cancel{a^2}-\cancel{a^2}+b^2}\cdot\frac{b^2}{4a\sqrt{a^2-b^2}}$$

$$=\frac{\cancel{2a}\cdot\cancel{2}\cancel{\sqrt{a^2-b^2}}}{\cancel{b^2}}\cdot\frac{\cancel{b^2}}{\cancel{4a}\cancel{\sqrt{a^2-b^2}}}=1$$

Uslovi:
$b\neq 0$, $a\neq 0$
$a^2-b^2>0$

3. Pokazati da je vrednost izraza pozitivna za svako $a, b \in \mathbb{R} \setminus \{0\}$ i $|a| \neq b$

$$\left[\left(1-\left(1-\frac{b}{a}\right)^{-1}\right)^{-2}-\left(1-\left(1-\frac{a}{b}\right)^{-1}\right)^{-2}\right]\cdot(a-b)^{-3}\cdot(a+b)^{-1}$$

$$=\left[\left(1-\left(\frac{a-b}{a}\right)^{-1}\right)^{-2}-\left(1-\left(\frac{b-a}{b}\right)^{-1}\right)^{-2}\right]\cdot\frac{1}{(a-b)^3\cdot(a+b)}$$

$$=\left(\left(1-\frac{a}{a-b}\right)^{-2}-\left(1-\frac{b}{b-a}\right)^{-2}\right)\cdot\frac{1}{(a-b)^3(a+b)}$$

$$=\left(\left(\frac{\cancel{a}-b-\cancel{a}}{a-b}\right)^{-2}-\left(\frac{\cancel{b}-a-\cancel{b}}{b-a}\right)^{-2}\right)\cdot\frac{1}{(a-b)^3(a+b)}$$

$$=\left(\left(\frac{a-b}{-b}\right)^2-\left(\frac{b-a}{-a}\right)^2\right)\cdot\frac{1}{(a-b)^3(a+b)}$$

$$=\left(\frac{(a-b)^2}{b^2}-\frac{(b-a)^2}{a^2}\right)\frac{1}{(a-b)^3(a+b)}$$

$$=\frac{a^2(a-b)^2-b^2(a-b)^2}{a^2\cdot b^2}\cdot\frac{1}{(a-b)^3(a+b)}$$

$$=\frac{\cancel{(a-b)^2}(a^2-b^2)}{a^2b^2(a-b)^{\cancel{3}}(a+b)}$$

$$=\frac{\cancel{a^2-b^2}}{a^2b^2\cdot\cancel{(a^2-b^2)}}$$

$$=\frac{1}{a^2b^2}$$

$$(b-a)^2=(-(a-b))^2$$

$$=(-1)^2\cdot(a-b)^2=\cancel{(a-b)}$$