1

$$\lim_{x\to\frac{\pi}{2}} (\cos x)^{\frac{1}{\operatorname{tg} x}} \overset{0^0}{=} A \;/\; \ln$$

$$\underline{\ln A} = \ln \lim_{x\to\frac{\pi}{2}} (\cos x)^{\frac{1}{\operatorname{tg} x}} = \lim_{x\to\frac{\pi}{2}} \frac{1}{\operatorname{tg} x} \ln(\cos x)$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1}{\operatorname{tg} x}\cdot \ln(\cos x) \overset{0\cdot(-\infty)}{=} \lim_{x\to\frac{\pi}{2}} \frac{\ln(\cos x)}{\operatorname{tg} x} \overset{\frac{\infty}{\infty}}{=}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\frac{1}{\cancel{\cos x}}(-\sin x)}{\frac{1}{\cos^2 x}} = \lim_{x\to\frac{\pi}{2}} (-\sin x\cdot\cos x) = -1\cdot 0 = \underline{0}$$

$$\ln A = 0 \Rightarrow A = e^0 = 1$$

(2) $\lim\limits_{x\to\infty}\left(\frac{x^2+1}{x+2}\right)^{\frac{1}{x-3}} \overset{\infty^0}{=} A \quad / \ln$

$$\ln A = \ln \lim_{x\to\infty}\left(\frac{x^2+1}{x+2}\right)^{\frac{1}{x-3}} = \lim_{x\to\infty} \ln\left(\frac{x^2+1}{x+2}\right)^{\frac{1}{x-3}}$$

$$= \lim_{x\to\infty}\left(\frac{1}{x-3}\cdot\ln\left(\frac{x^2+1}{x+2}\right)\right) \overset{0\cdot\infty}{=} \lim_{x\to\infty}\frac{\ln\left(\frac{x^2+1}{x+2}\right)}{x-3} \overset{\frac{\infty}{\infty}}{=}$$

$$= \lim_{x\to\infty}\frac{\frac{1}{\frac{x^2+1}{\cancel{x+2}}}\cdot\frac{2x\cdot(x+2)-(x^2+1)}{(x+2)^{\cancel{2}}}}{1} = \lim_{x\to\infty}\frac{2x^2+4x-x^2-1}{(x^2+1)(x+2)}$$

$$= \lim_{x\to\infty}\frac{\overset{1}{x^2}+4x-1}{x^3+2x^2+x+2} = 0$$

$\ln A = 0 \Rightarrow A = e^0 = 1$

3

$$\lim_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-4x}\right)\cdot\frac{\sqrt{x^2+1}+\sqrt{x^2-4x}}{\sqrt{x^2+1}+\sqrt{x^2-4x}}$$

$$=\lim_{x\to\infty}\frac{\cancel{x^2}+1-(\cancel{x^2}-4x)}{\sqrt{x^2+1}+\sqrt{x^2-4x}}=\lim_{x\to\infty}\frac{1+4x}{\underbrace{\sqrt{x^2+1}+\sqrt{x^2-4x}}}$$

$$\sqrt{x^2\left(1+\frac{1}{x^2}\right)}+\sqrt{x^2\left(1-\frac{4}{x}\right)}$$

$$\underline{x}\sqrt{1+\frac{1}{x^2}}+\underline{x}\sqrt{1-\frac{4}{x}}$$

$$=\lim_{x\to\infty}\frac{\cancel{x}\left(\frac{1}{x}\xrightarrow{}0+4\right)}{\cancel{x}\left(\sqrt{1+\frac{1}{x^2}\xrightarrow{}0}+\sqrt{1-\frac{4}{x}\xrightarrow{}0}\right)}$$

$$=\frac{4}{\sqrt{1}+\sqrt{1}}=\frac{4}{2}=2$$

L.P.

$$\lim_{x\to\infty}\frac{1+4x}{\sqrt{x^2+1}+\sqrt{x^2-4x}}\overset{\frac{\infty}{\infty}}{=}\lim_{x\to\infty}\frac{4}{\frac{1}{\cancel{2}\sqrt{x^2+1}}\cdot\cancel{2}x+\frac{1}{2\sqrt{x^2-4x}}\cdot(2x-4)}$$

$$=\lim_{x\to\infty}\frac{4}{\frac{x}{\sqrt{x^2+1}}+\frac{x-2}{\sqrt{x^2-4x}}}$$

4

b) $\lim\limits_{x\to 1} \dfrac{\ln x + \sin(x^2-1)}{2x-2} \overset{\frac{0}{0}}{=} \lim\limits_{x\to 1} \dfrac{\frac{1}{x} + \cos(x^2-1)\cdot 2x}{2}$

$= \dfrac{1 + 1\cdot 2\cdot 1}{2} = \dfrac{3}{2}$

a) $\lim\limits_{x\to\infty} \left(\dfrac{x^2-x-1}{x^2-1}\right)^{\frac{5x^2+1}{7x}} \overset{1^\infty}{=} \lim\limits_{x\to\infty} \left(1 + \dfrac{x^2-x-1}{x^2-1} - 1\right)^{\frac{5x^2+1}{7x}}$

$= \lim\limits_{x\to\infty} \left(1 + \dfrac{\cancel{x^2}-x-\cancel{1}-\cancel{x^2}+\cancel{1}}{x^2-1}\right)^{\frac{5x^2+1}{7x}} = \lim\limits_{x\to\infty} \left(1+\dfrac{-x}{x^2-1}\right)^{\frac{x^2-1}{-x}\cdot\frac{-\cancel{x}}{x^2-1}\cdot\frac{5x^2+1}{7\cancel{x}}}$

$= e^{\lim\limits_{x\to\infty} \frac{-5x^2-1}{7x^2-7}} = e^{-\frac{5}{7}}$

$\left(1+\dfrac{1}{x}\right)^{x} \xrightarrow{x\to\infty} e$

1^{∞} $0^0, \infty^0$ — MOŽE L.P.

1^{∞} — MOŽE PEŠKE

$\boxed{0^0, \infty^0}$ — NE MOŽE PEŠKE

MORA L.P.

OVO JE POGREŠAN NAČIN!

$$\lim_{x\to\infty}\left(\frac{x^2+1}{x+2}\right)^{\frac{1}{x-3}} \overset{\infty^0}{=} \lim_{x\to\infty}\left(1+\frac{x^2+1}{x+2}-1\right)^{\frac{1}{x-3}}$$

$$=\lim_{x\to\infty}\left(1+\frac{x^2+1-x-2}{x+2}\right)^{\frac{1}{x-3}}=$$

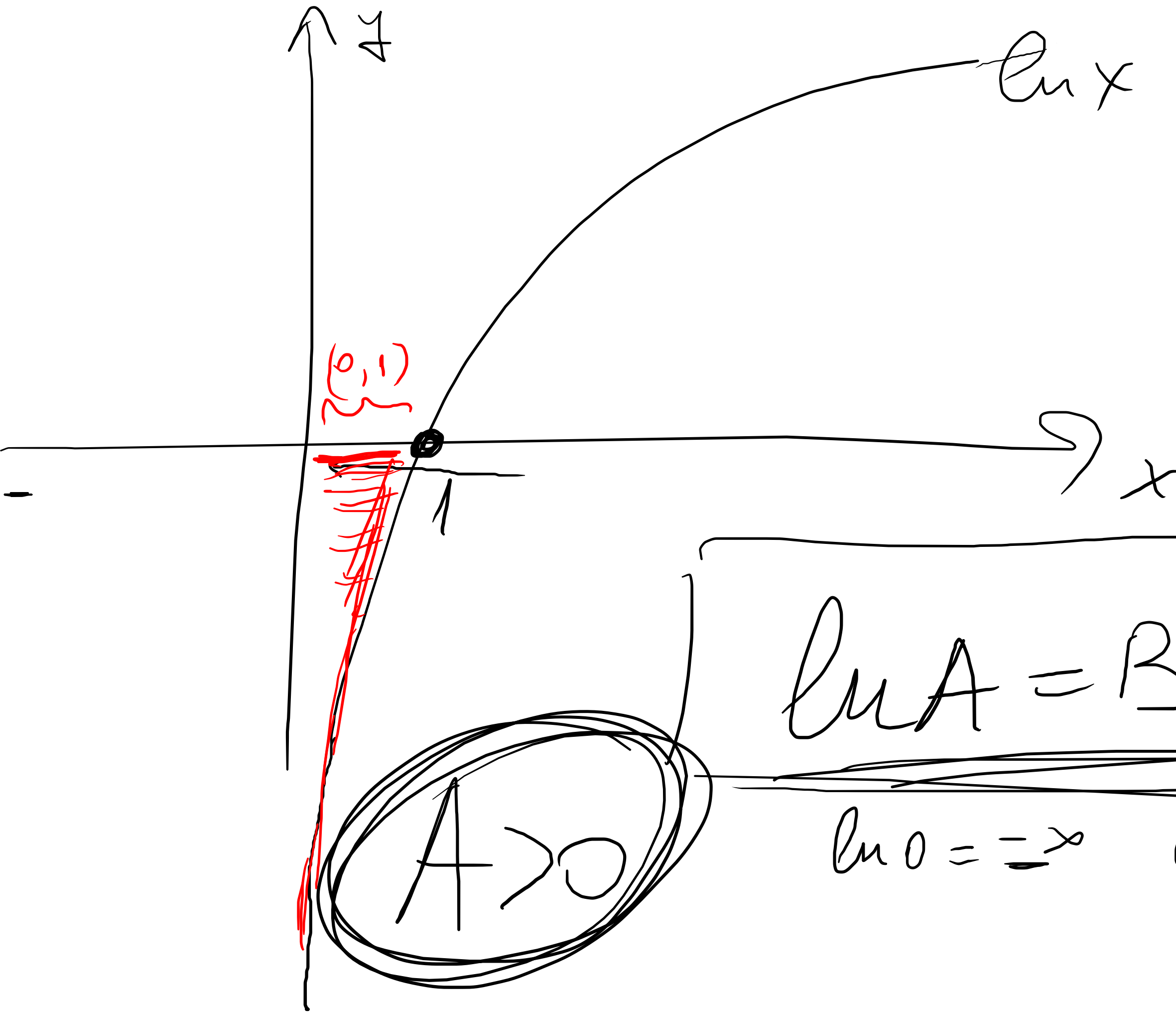
$$=\lim_{x\to\infty}\left(1+\frac{x^2-x-1}{x+2}\right)^{\frac{x+2}{x^2-x-1}\cdot\frac{x^2-x-1}{x+2}\cdot\frac{1}{x-3}}$$

∞, 0, $\cancel{e}$

$$=\cancel{e}^{\lim_{x\to\infty}\frac{x^2-x-1}{x^2-x-6}}=e^1$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=e$$

$$\cancel{\lim_{x\to\infty}\left(1+\infty\right)^{0}=e}$$



$$\boxed{\ln A = B \Leftrightarrow A = e^{B}}$$

$A > 0$

$\ln 0 = -\infty \qquad 0 = e^{-\infty}$

$$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$y = \ln \frac{x^2}{x^2-1}$$

1) $\frac{x^2}{x^2-1} > 0$, $x^2 \geq 0$ | $x^2-1 \neq 0$

$x \neq \pm 1$

$x^2-1>0$

$+ \quad -1 \quad - \quad 1 \quad +$

$$\boxed{D = (-\infty,-1) \cup (1,\infty)}$$

2, $y=0$ $\quad \ln\frac{x^2}{x^2-1} = 0$

$y \neq 0 \ \forall x \in D$ $\quad \frac{x^2}{x^2-1} = e^0 = 1$

$\cancel{x^2} = \cancel{x^2} - 1$

$0 = -1$

$\perp$

NEMA PRESEKA SA X-OSOM KAO NI SA Y

3, $y>0$

$\ln\frac{x^2}{x^2-1} > 0$

$\frac{x^2}{x^2-1} > 1 \ (=e^0)$

$\frac{x^2}{x^2-1} - 1 > 0$

$\frac{\cancel{x^2} - \cancel{x^2} + 1}{x^2-1} > 0$

$\frac{1}{x^2-1} > 0$

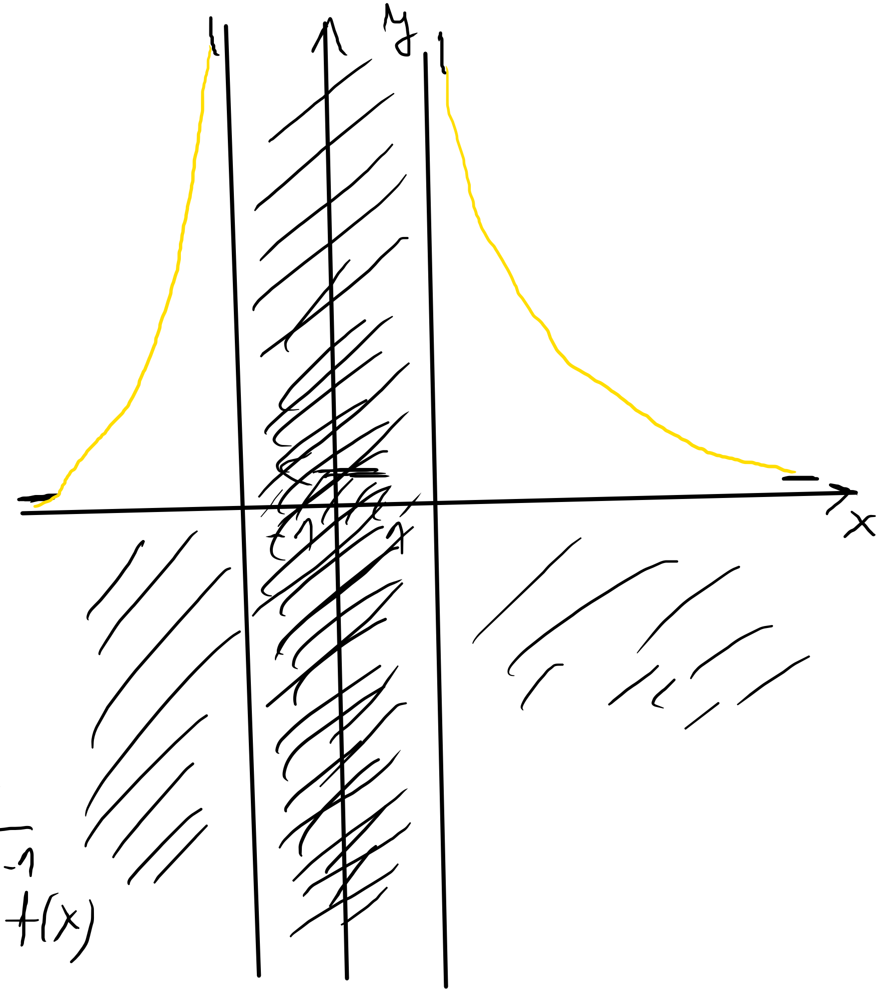
$x^2-1>0$

$\forall x \in D$

4, $f(-x) = \ln\frac{(-x)^2}{(-x)^2-1}$

$= \ln\frac{x^2}{x^2-1} = f(x)$

PARNA



4) V.A.

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} \ln\frac{x^2}{x^2-1} = \ln\frac{1}{0^+} = \ln(+\infty) = \underline{+\infty}$$

$\boxed{x=-1}$ JE V.A. SA LEVE STRANE

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \ln\frac{x^2}{x^2-1} = \ln\frac{1}{0^+} = \ln(+\infty) = +\infty$$

$\boxed{x=1}$ JE V.A. SA DESNE STRANE

H.A.

$$\lim_{x\to \pm\infty} f(x) = \lim_{x\to \pm\infty} \ln\frac{x^2}{x^2-1} = \ln 1 = 0$$

$\boxed{y=0}$ H.A.

$\ln 1 = \square \iff e^{\square} = 1$

K.A. nema jer ima H.A.

5. $y = \ln \frac{x^2}{x^2-1}$

$\boxed{D=(-\infty,-1)\cup(1,\infty)}$

$$y' = \frac{1}{\frac{x^2}{x^2-1}} \cdot \frac{2x(x^2-1) - x^2\cdot 2x}{(x^2-1)^2} = \frac{2x(\cancel{x^2}-1-\cancel{x^2})}{x^2(x^2-1)} = \frac{-2}{x(x^2-1)}$$

$x \neq 0$

$x \neq \pm 1$

$x \in \mathbb{R}\setminus\{0,\pm1\}$

$y' \neq 0 \quad \forall x \in D \Rightarrow$ NEMA E. V.

$y' > 0$ $\quad \frac{-2}{x(x^2-1)} > 0$

		-1		0		1	
x	$-$		$-$	0	$+$		$+$
$x-1$	$-$		$-$		$-$	0	$+$
$x+1$	$-$	0	$+$		$+$		$+$
-2	$-$		$-$		$-$		$-$
y'	$+$		$-$		$+$		$-$
y	$\nearrow$						$\searrow$

$y' < 0 \quad x \in (-1,0)\cup(1,\infty)$

$y \searrow \quad x \in (1,\infty)$

$y' > 0 \quad x \in (-\infty,-1)\cup(0,1)$

$y \nearrow \quad x \in (-\infty,-1)$

7) $y' = \dfrac{-2}{\underbrace{x(x^2-1)}} = -2\,(x^3-x)^{-1}$

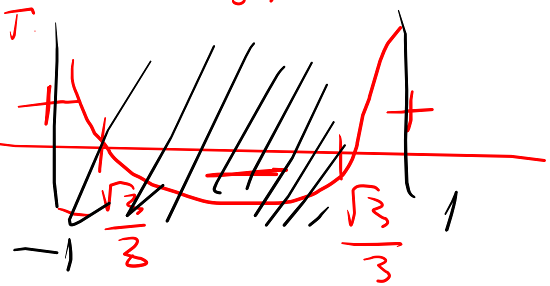
$$y'' = \frac{+2\,(x^2-1+x\cdot 2x)}{x^2\,(x^2-1)^2} = \frac{2\,(3x^2-1)}{x^2\,(x^2-1)^2}\quad , \quad x\neq 0,\ x\neq \pm 1$$

$y''=0 \qquad 3x^2-1=0 \qquad\qquad y''>0 \qquad \dfrac{\underset{>0}{2}\,(3x^2-1)}{\underbrace{x^2(x^2-1)^2}_{>0}}>0$

$x^2=\frac{1}{3} \qquad x=\pm\frac{1}{\sqrt3}\cdot\frac{\sqrt3}{\sqrt3} \qquad\qquad 3x^2-1>0$

$\cancel{\boxed{x=\pm\frac{\sqrt3}{3}}}$ NEMA P.T.

$x=-\frac{\sqrt3}{3}\notin D \qquad x=\frac{\sqrt3}{3}\notin D$



$D=(-\infty,-1)\cup(1,\infty)$

$y''>0 \quad x\in\left(-\infty,-\frac{\sqrt3}{3}\right)\cup\left(\frac{\sqrt3}{3},+\infty\right)$

$y \cup \quad \forall x\in D$

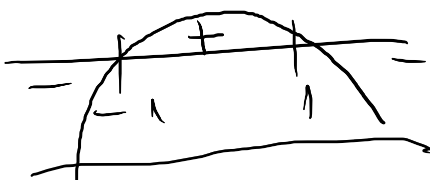
$$y = \ln \frac{x^2}{1-x^2}$$

1) D

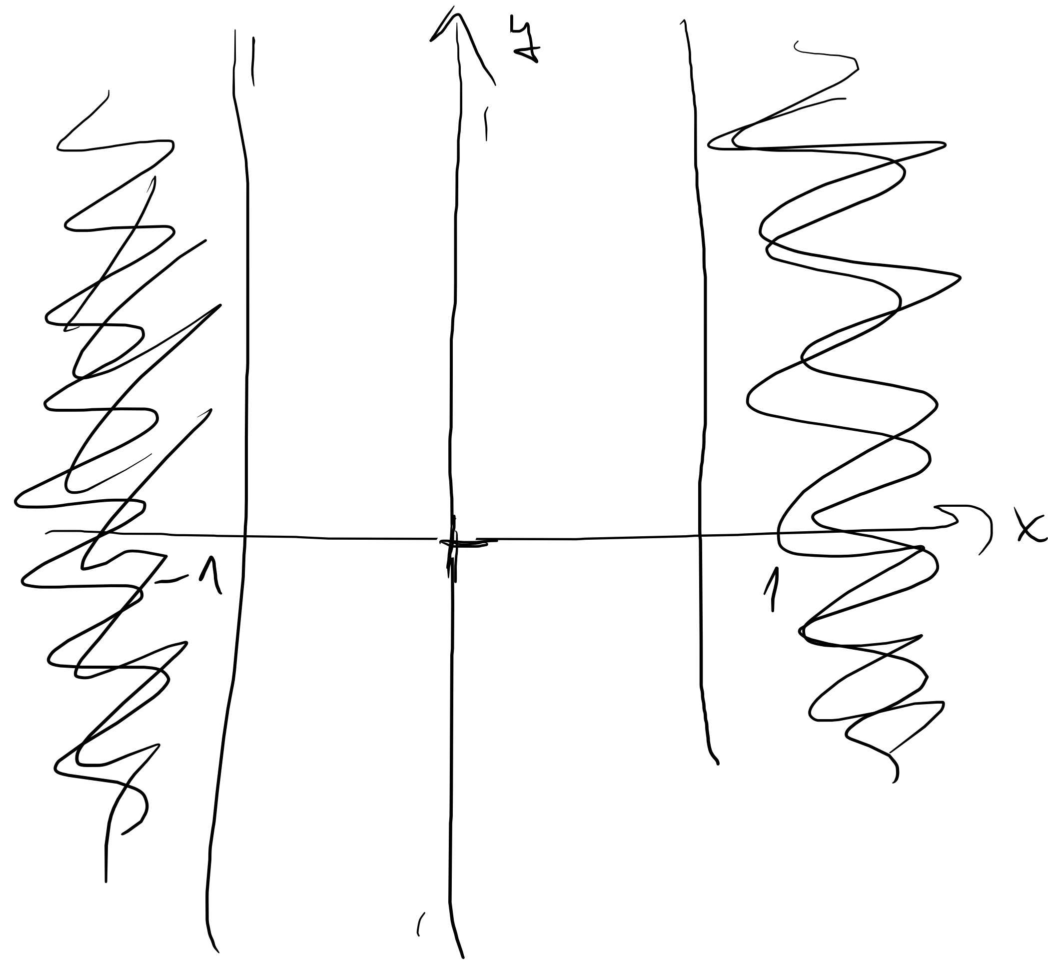
$$\frac{x^2}{1-x^2} > 0 \qquad 1-x^2 \neq 0$$

$$x \neq \pm 1$$

$$1-x^2 > 0 \qquad x \neq 0$$



$$\boxed{D = (-1, 0) \cup (0, 1)}$$



$y = f(x)$ $\qquad A(x_0, \overbrace{f(x_0)}^{y_0})$

$t: \quad y - y_0 = f'(x_0)(x - x_0)$

$n: \quad y - y_0 = -\dfrac{1}{f'(x_0)}(x - x_0)$

$y = 5x^2 + 7 \qquad A(\overset{x_0}{-1}, y_0)$

$y_0 = f(x_0) = f(-1) = 5 \cdot (-1)^2 + 7 = 12$

$A(-1, 12)$

$y' = 10x$

$y'(x_0) = f'(x_0)$

$= y'(-1) = -10$

$t: y - 12 = -10(x+1)$

$n: y - 12 = -\frac{1}{-10}(x+1)$

(x)

$$f(x)=\begin{cases} b+\dfrac{\sqrt{x+1}-1}{\sqrt[3]{x}} & , \quad x<0 \\ a & , \quad x=0 \\ (2x+e^x)^{\frac{1}{3x}} & , \quad x>0 \end{cases}$$

$f(0)=a$

$$\underbrace{\lim_{x\to 0^-} f(x)}_{b}=\underbrace{\lim_{x\to 0^+} f(x)}_{e}=\underbrace{f(0)}_{a}$$

$$\lim_{x\to 0^-} b+\lim_{x\to 0^-}\frac{\sqrt{x+1}-1}{\sqrt[3]{x}} \qquad \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\left(b+\frac{\sqrt{x+1}-1}{\sqrt[3]{x}}\right)=b+\lim_{x\to 0^-}\frac{\sqrt{x+1}-1}{\sqrt[3]{x}}\ \stackrel{\frac{0}{0}}{=}$$

$$=b+\lim_{x\to 0^-}\frac{x+\cancel{1}-\cancel{1}}{\sqrt[3]{x}\,(\sqrt{x+1}+1)}=b+\lim_{x\to 0^-}\frac{\cancel{x}}{\cancel{x}\cdot\sqrt[3]{\frac{1}{x^2}}\,(\underbrace{\sqrt{x+1}+1}_{2})}=b+0=b$$

$$\sqrt[3]{x}=\sqrt[3]{x^3\cdot\frac{1}{x^2}}=x\sqrt[3]{\frac{1}{x^2}}=x\,\frac{1}{\sqrt[3]{x^2}}$$

$$=b+\lim_{x\to 0^-}\frac{\sqrt[3]{x^2}}{\sqrt{x+1}+1}=\frac{0}{2}=0+b=b$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (2x + e^x)^{\frac{1}{3x}} \overset{1^\infty}{=}$$

$$= \lim_{x \to 0^+} \left(\left(1 + (2x + e^x - 1)\right)^{\frac{1}{2x+e^x-1}} \right)^{(2x+e^x-1) \cdot \frac{1}{3x}} =$$

$$= e^{\lim_{x \to 0^+} \frac{2x+e^x-1}{3x}} \overset{\frac{0}{0}}{=} e^{\lim_{x \to 0^+} \frac{2+e^x}{3}} = e$$

L.P.

a) $(\sin x + x^3)'$

b) $(\sin x \cdot x^3)'$

c) $\left(\dfrac{\sin x}{x^3}\right)'$

d) $\left(\sin(x^3)\right)'$