(1)

$$f(x) = \begin{cases} \dfrac{1}{1+e^{\frac{1}{x-1}}}, & x<1 \\ a, & x=1 \\ b\cdot \dfrac{x^2-x}{2x^2+x-3}, & x>1 \end{cases}$$

$a = \frac{1}{5}b = 1$

$\boxed{a=1}$ $\boxed{b=5}$

$$\boxed{\lim_{x\to 1^-} f(x)} = \boxed{\lim_{x\to 1^+} f(x)} = f(1)$$

$1 \quad \frac{1}{5}b \quad a$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{1}{1+e^{\frac{1}{x-1}}} = \text{"}\frac{1}{1+e^{\frac{1}{0^-}}} = \frac{1}{1+e^{-\infty}} = \frac{1}{1+0}\text{"} = 1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} b\cdot\frac{x^2-x}{2x^2+x-3} = b \lim_{x\to 1^+} \frac{2x-1}{4x+1} = b\cdot\frac{1}{5}$$

$f(1) = a$

(2) $y = \sqrt{x-1}$

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x-1} - \sqrt{x-1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\overset{A}{\sqrt{x+\Delta x-1}} - \overset{B}{\sqrt{x-1}}}{\Delta x} \cdot \frac{\overset{A}{\sqrt{x+\Delta x-1}} + \overset{B}{\sqrt{x-1}}}{\sqrt{x+\Delta x-1} + \sqrt{x-1}} \qquad A^2 - B^2$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x} + \Delta x \cancel{-1} - \cancel{x} + \cancel{1}}{\Delta x\left(\sqrt{x+\Delta x-1} + \sqrt{x-1}\right)} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x}\left(\sqrt{x+\cancel{\Delta x}^{\to 0}-1} + \sqrt{x-1}\right)}$$

$$= \frac{1}{\sqrt{x-1} + \sqrt{x-1}} = \frac{1}{2\sqrt{x-1}}$$

$y' = k = tg\alpha$

$y = \boxed{k}x + n$

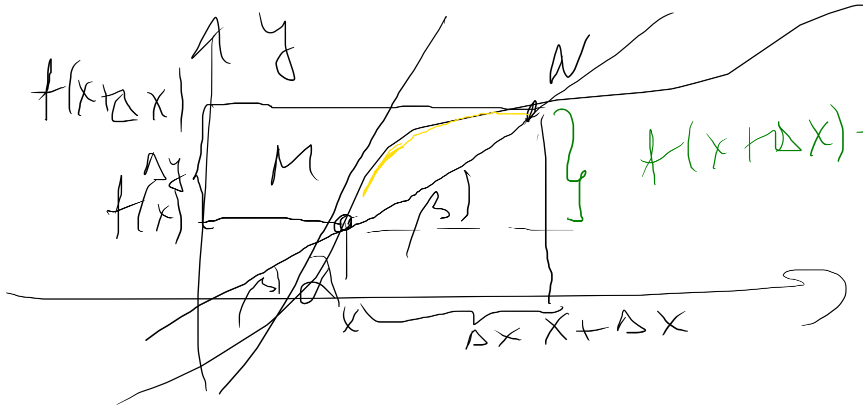
$k = f'(x_0)$

$y - y_0 = k(x - x_0)$

$A(x_0, y_0),\ k$

$t:\ y - y_0 = f'(x_0)(x - x_0)$

$n:\ y - y_0 = -\dfrac{1}{f'(x_0)}(x - x_0)$



$tg\beta = \dfrac{\Delta y}{\Delta x}$

$= \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

$\Delta x \to 0,\ N \to M$

$tg\alpha = \lim\limits_{\Delta x \to 0} tg\beta$

$= \lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = y'$

③

$y = \sin x + 4x - 5$ $\quad$ $y(0) = \sin 0 + 4 \cdot 0 - 5$

APSCISA $\quad x = 0$ $\quad$ $= -5$

$A(\underset{x_0}{0}, \underset{y_0}{-5})$

$y'(0) = ?$

$y' = \cos x + 4$

$y'(0) = \cos 0 + 4 = 1 + 4 = 5$

$t: \quad y - y_0 = f'(x_0)(x - x_0)$

$y - (-5) = 5(x - 0)$

$y + 5 = 5x$

$\cancel{5x + y = -5}$

$A(x_0, y_0) \quad t: \quad y - y_0 = f'(x_0)(x - x_0)$

$n: \quad -\frac{1}{f'(x_0)}$

$n: \quad y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$

$y + 5 = -\frac{1}{5}(x - 0)$

$5 \cdot y + 25 = -x \cancel{+0}$

$|5y + x = 25|$

4

$$\sum_{n=1}^{\infty} \frac{1}{n+1} \cdot \left(\frac{1}{2}\right)^n$$

$$a_n = \frac{1}{n+1} \cdot \left(\frac{1}{2}\right)^n > \left(\frac{1}{2}\right)^n$$

$$\left|\frac{1}{2}\right| < 1 \quad K$$

$$n \geq 1$$

$$n+1 \geq 2$$

$$\frac{1}{n+1} \geq \frac{1}{2} > 0$$

5

$$\sum \frac{2n-1}{\sqrt[3]{n^7+n^4}}$$

$$a_n = \frac{2n-1}{\sqrt[3]{n^7+n^4}} \sim \frac{n}{\sqrt[3]{n^7}} = \frac{n}{n^{\frac{7}{3}}} = n^{1-\frac{7}{3}} = n^{-\frac{4}{3}}$$

$$= \frac{1}{\sqrt[3]{n^4}} = \frac{1}{n^{4/3}}$$

$$\alpha = \frac{4}{3} > 1$$

K

$\frac{0}{0}, \frac{\infty}{\infty}$ $\qquad \lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}$

$0 \cdot \infty$ $\qquad \lim_{x \to x_0} \underbrace{f(x)}_{\to 0} \underbrace{g(x)}_{\to \infty} = \lim_{x \to x_0} \frac{f(x)}{\frac{1}{g(x)}}$

$\infty - \infty$ $\qquad \lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = \lim_{x \to 0} \frac{x - \sin x}{x \sin x}$

$$= \lim_{x \to 0} \frac{1 - \cos x}{\sin x + x \cos x} \overset{\frac{0}{0}}{=} \lim_{x \to 0} \frac{\sin x}{\cos x + \cos x + x(-\sin x)}$$

$$= \frac{0}{1 + 1 + 0} = 0$$

$1^{\infty}, 0^{0}, \infty^{0}$

$$\lim_{x \to x_0} f(x)^{g(x)} = A \quad \Big/ \ln$$

$$\ln A = \ln \lim_{x \to x_0} f(x)^{g(x)} = \lim_{x \to x_0} \ln f(x)^{g(x)}$$

$$= \lim_{x \to x_0} \underbrace{g(x) \cdot \ln f(x)}_{0 \cdot \infty}$$

1

$$\begin{cases} \lim\limits_{n\to\infty} a_n \neq 0 \to D \\ \lim\limits_{n\to\infty} a_n = 0 \to \text{POJMA NEMAM} \end{cases}$$

I

II

$a_n \sim b_n$

$\sum \frac{1}{n^\alpha}$

$\sum 2^n$

III

$\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n}$

IV

$\lim\limits_{n\to\infty} \sqrt[n]{a_n}$

$L < 1$ K

$L > 1$ D

$L = 1$ NEMAM POJMA