1. (10 bodova)

   (a) Odrediti realan parametar a tako da polinom $p(x) = 4x^5 + ax^4 + 6x^3 + ax^2 + 2x$, pri deljenju sa polinomom $x + 1$ daje ostatak -4.

   (b) Rastaviti na zbir parcijalnih razlomaka racionalnu funkciju $r(x) = \frac{4x^2+5x+5}{x^3+3x^2+5x+3}$.

2. (5 bodova) Rastaviti na zbir parcijalnih razlomaka racionalnu funkciju $r(x) = \dfrac{3x^2 + 7}{x^3 + x^2 + 9x + 9}$.

3. (15 bodova) Dat je sistem linearnih jednačina

$$\begin{array}{rcrcrcl} 2ax & - & 4y & & & = & 2b \\ 3x & - & 3y & - & (a-4)z & = & 0 \\ x & + & 3y & + & az & = & 4ab \end{array} \quad , \qquad a, b \in \mathbb{R}.$$

   (a) U zavisnosti od realnih parametara a i b diskutovati prirodu rešenja datog sistema i rešiti ga u slučaju neodredjenosti.

   (b) Za $a = -2$ i $b = 0$ dokazati da je skup rešenja datog sistema potprostor vektorskog prostora $\mathbb{R}^3$, i odrediti jednu njegovu bazu.

   (c) Za $a = 0$ i $b = 1$ rešiti dati sistem matričnom metodom.

4. (8 bodova) Dati su vektori $\overrightarrow{a} = 2\overrightarrow{m} - \overrightarrow{n}$ i $\overrightarrow{b} = -\overrightarrow{m} + \alpha\overrightarrow{n}$, $\alpha \in \mathbb{R}$. gde je $|\overrightarrow{m}| = 3$, $|\overrightarrow{n}| = 2$ i $\angle(\overrightarrow{m}, \overrightarrow{n}) = \frac{\pi}{3}$.

   (a) Odrediti realan parametar α tako da vektori $\vec{a}$ i $\vec{b}$ budu uzajamno normalni.

   (b) Za $\alpha = 5$ izračunati površinu trougla određenog vektorima $\overrightarrow{a}$ i $\overrightarrow{b}$.

5. (6 bodova) Vektorski prostor V je generisan skupom vektora $A = \{a, b, c, d, \}$ gde su

$$a = (1, 2, 0, 1)\,, \quad b = (3, 1, 2, 0)\,, \quad c = (5, 4, 2, 2) \quad \text{i} \quad d = (0, 1, -2, 3)\,.$$

   Odrediti dimenziju i jedan podskup skupa A koji je baza prostora V.

5)

$$M = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 2 & 1 & 4 & 1 \\ 0 & 2 & 2 & -2 \\ 1 & 0 & 2 & 3 \end{bmatrix} \begin{matrix} -2 \\ \\ \\ -1 \end{matrix} \sim \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & -5 & -6 & 1 \\ 0 & 2 & 2 & -2 \\ 0 & -3 & -3 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & -5 & -6 \\ 0 & -2 & 2 & 2 \\ 0 & 3 & -3 & 3 \end{bmatrix} \begin{matrix} \\ 2 \\ \\ -3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & -5 & -6 \\ 0 & 0 & -8 & -10 \\ 0 & 0 & 12 & 21 \end{bmatrix} \begin{matrix} \\ \\ /:2 \\ /:3 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & -5 & -6 \\ 0 & 0 & -4 & -5 \\ 0 & 0 & 4 & 7 \end{bmatrix} \begin{matrix} \\ \\ \\ + \end{matrix} \sim$$

$$\sim \begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & -5 & -6 \\ 0 & 0 & -4 & -5 \\ 0 & 0 & 0 & 2 \end{bmatrix} \Rightarrow \text{rang}(M) = 4$$

$$\Rightarrow \dim(V) = 4$$

BAZA $\{(1,2,0,1),(3,1,2,0),(5,4,2,2),(0,1,-2,3)\}$

4) $\vec{a} = 2\vec{m} - \vec{n}$

$\vec{b} = -\vec{m} + \alpha\vec{n}$

$|\vec{m}| = 2$

$|\vec{n}| = 3$

$\measuredangle(\vec{m}, \vec{n}) = \frac{\pi}{3}$

a) $\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$

$$\vec{a} \cdot \vec{b} = (2\vec{m} - \vec{n}) \cdot (-\vec{m} + \alpha\vec{n})$$

$$= -2\,\vec{m} \cdot \vec{m} + 2\alpha\,\vec{m} \cdot \vec{n} + \vec{n} \cdot \vec{m} - \alpha\,\vec{n} \cdot \vec{n}$$

$$= -2\,|\vec{m}|^2 + (2\alpha + 1)\,\vec{m} \cdot \vec{n} - \alpha\,|\vec{n}|^2$$

$$= -2 \cdot 4 + (2\alpha + 1)\,|\vec{m}| \cdot |\vec{n}| \cdot \cos\measuredangle(\vec{m}, \vec{n}) - \alpha \cdot 9$$

$$= -8 + (2\alpha + 1) \cdot 2 \cdot 3 \cdot \cos\frac{\pi}{3} - 9\alpha$$

$$= -8 + (2\alpha + 1) \cdot \cancelto{3}{6}\,\frac{1}{\cancel{2}} - 9\alpha$$

$$= -8 + 6\alpha + 3 - 9\alpha$$

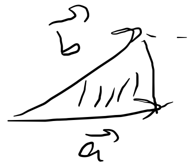
$$= -5 - 3\alpha \implies -5 - 3\alpha = 0$$

$$\alpha = -\frac{5}{3}$$

b) $\alpha = 5$

$\vec{a} = 2\vec{m} - \vec{n}$

$\vec{b} = -\vec{m} + 5\vec{n}$



$$P_\Delta = \frac{1}{2}\,|\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (2\vec{m} - \vec{n}) \times (-\vec{m} + 5\vec{n})$$

$$= -2\cancelto{0}{\vec{m} \times \vec{m}} + 10\,\vec{m} \times \vec{n} + \vec{n} \times \vec{m} - 5\cancelto{0}{\vec{n} \times \vec{n}}$$

$$= 10\,\vec{m} \times \vec{n} - \vec{m} \times \vec{n} = 9\,\vec{m} \times \vec{n}$$

$$P_\Delta = \frac{1}{2}\,|9\,\vec{m} \times \vec{n}| = \frac{9}{2}\,|\vec{m} \times \vec{n}| = \frac{9}{2}\,|\vec{m}|\,|\vec{n}|\,\sin\measuredangle(\vec{m}, \vec{n}) = \frac{9}{\cancel{2}} \cdot \cancel{2} \cdot 3 \cdot \sin\frac{\pi}{3} = 27 \cdot \frac{\sqrt{3}}{2}$$

3

c) $2ax - 4y = 2b$

$3x - 3y - (a-4)z = 0$

$x + 3y + az = 4ab$

$$D_s = \begin{vmatrix} 2a & -4 & 0 \\ 3 & -3 & -(a-4) \\ 1 & 3 & a \end{vmatrix} \begin{matrix} 2a & -4 \\ 3 & -3 \\ 1 & 3 \end{matrix}$$

$= -\cancel{6a^2} + 4a - 16 + \cancel{6a^2} - 24a + 12a$

$= -8a - 16 = -8(a+2)$

I $D_s \neq 0$ ZA $a \neq -2 \to$ sistem je određen

II $D_s = 0$ ZA $a = -2 \Rightarrow$ sistem je nemoguć, ili neodređen

$a = -2$

$-4x - 4y = 2b \quad /:2$

$3x - 3y + 6z = 0 \quad /:3$

$x + 3y - 2z = -8b$

$x - y + 2z = 0$

$-2x - 2y = b$

$x + 3y - 2z = -8b$

$x - y + 2z = 0$

$-4y + 4z = b$

$4y - 4z = -8b$

$x - y + 2z = 0$

$-4y + 4z = b$

$0 = -7b$

$b \neq 0$

$R_s = \emptyset$

$b = 0$

$x - y + 2z = 0$

$-4y + 4z = 0 \quad /:-4$

$x - y = -2z$

$y = z$

1x neodređen

$z = t, \ t \in \mathbb{R}$

$y = t$

$x = -t$

$R_s = \{(-t, t, t) \mid t \in \mathbb{R}\}$

b, $a = -2, b = 0$

$-4x - 4y = 0$

$3x - 3y + 4z = 0$

$x + 3y - 2z = 0$

$R_s = \{(-t, t, t) \mid t \in \mathbb{R}\}$

$= \{t(-1,1,1) \mid t \in \mathbb{R}\}$

$= L\{(-1,1,1)\}$

PO TEOREMI L je potprostor od $\mathbb{R}^3$

$(-1,1,1)$ – GENERIRA R_s

$(-1,1,1)$ – LIN. NEZ.

$\{(-1,1,1)\}$ BAZA

c) $a = 0 \quad b = 1$

$$\begin{aligned} -4y &= 2 \\ 3x - 3y + 4z &= 0 \\ x + 3y &= 0 \end{aligned}$$

$$A = \begin{bmatrix} 0 & -4 & 0 \\ 3 & -3 & 4 \\ 1 & 3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$AX = B \mid A^{-1} \cdot$ s leve strane

$$\underbrace{A^{-1}A}_{E}X = A^{-1}B$$

$$\boxed{X = A^{-1}B}$$

$$\det(A) = \begin{vmatrix} 0 & -4 & 0 \\ 3 & -3 & 4 \\ 1 & 3 & 0 \end{vmatrix} \begin{matrix} 0 & -4 \\ 3 & -3 \\ 1 & 3 \end{matrix} = -16 \neq 0$$

$$\operatorname{adj}(A) = \begin{bmatrix} +\begin{vmatrix} -3 & 4 \\ 3 & 0 \end{vmatrix} & -\begin{vmatrix} 3 & 4 \\ 1 & 0 \end{vmatrix} & +\begin{vmatrix} 3 & 3 \\ 1 & 3 \end{vmatrix} \\ -\begin{vmatrix} -4 & 0 \\ 3 & 0 \end{vmatrix} & +\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 0 & -4 \\ 1 & 3 \end{vmatrix} \\ +\begin{vmatrix} -4 & 0 \\ -3 & 4 \end{vmatrix} & -\begin{vmatrix} 0 & 0 \\ 3 & 4 \end{vmatrix} & +\begin{vmatrix} 0 & -4 \\ 3 & -3 \end{vmatrix} \end{bmatrix}^{T}$$

$$= \begin{bmatrix} -12 & 4 & 12 \\ 0 & 0 & -4 \\ -16 & 0 & 12 \end{bmatrix}^{T} = \begin{bmatrix} -12 & 0 & -16 \\ 4 & 0 & 0 \\ 12 & -4 & 12 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) = \frac{1}{-16} \begin{bmatrix} -12 & 0 & -16 \\ 4 & 0 & 0 \\ 12 & -4 & 12 \end{bmatrix} = -\frac{1}{4} \begin{bmatrix} -3 & 0 & -4 \\ 1 & 0 & 0 \\ 3 & -1 & 3 \end{bmatrix}$$

$$X = A^{-1}B = -\frac{1}{4} \underbrace{\begin{bmatrix} -3 & 0 & -4 \\ 1 & 0 & 0 \\ 3 & -1 & 3 \end{bmatrix}}_{3\times 3} \underbrace{\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}}_{3\times 1} = -\frac{1}{4} \begin{bmatrix} -6 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ -\frac{2}{4} \\ -\frac{3}{2} \end{bmatrix}$$

$$Rs = \left\{ \left(\frac{3}{2}, -\frac{2}{4}, -\frac{3}{2} \right) \right\} \checkmark$$

ZAOKRUŽI . . .

a) $(x+1)(x-3)(x+5)(x-7)$

b) $(x+1)(x^2+4)(x-3)$

c) $(x+1)(x-i)(x+i)(x-7)$

nad $\mathbb{R}$ a, b

nad $\mathbb{C}$ a, c

nad $\mathbb{R}$ i $\mathbb{C}$ a)

(2) $r(x) = \dfrac{3x^2+7}{x^3+x^2+9x+9}$

DODATAK

$$r_1(x) = \frac{x^4+4x^3+9x^2+9x+7}{x^3+x^2+9x+9}$$

NE PRAVA RAC. F.

$$(x^4+4x^3+9x^2+9x+7) : (x^3+x^2+9x+9) = x$$
$$-(x^4+x^3+9x^2+9x)$$
$$3x^3 \quad +7$$

$$r_1(x) = x + \frac{3x^3+7}{x^3+x^2+9x+9}$$

$$x^3+x^2+9x+9 = x^2(x+1)+9(x+1)$$

$K|$ 1, −1, 0, 3, −3, 3i, −3i

$= (x+1)(x^2+9)$ − nad $\mathbb{R}$

nad $\mathbb{C}$ → $= (x+1)(x-3i)(x+3i)$

$k|9 \Rightarrow \{\pm1, \pm3, \pm9\}$

$m|1 \Rightarrow m \in \{1\}$

$\frac{k}{m} \in \{\pm1, \pm3, \pm9\}$

	1	1	9	9
−1	1	0	9	0

$\underbrace{1 \quad 0 \quad 9}_{x^2+9}$

$$r(x) = \frac{3x^2+7}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9} \Big/ \cdot (x+1)(x^2+9)$$

$A = 1$, $B = 2$, $C = -2$

$$3x^2+7 = A(x^2+9)+(Bx+C)(x+1)$$
$$3x^2+7 = Ax^2+9A+Bx^2+Bx+Cx+C$$
$$3x^2+7 = (A+B)x^2 + (B+C)x + 9A+C$$

$A+B=3 \rightarrow A = 3-B$

$B+C=0 \rightarrow C=-B$

$9A+C=7 \qquad 27-9B-B=7 \qquad -10B=-20 \qquad B=2 \quad A=1 \quad C=-2$

1

a) $\dfrac{p(x)}{x+1}$

BEZUOV STAV

OSTATAK JE $\boxed{p(-1)}$

OSTATAK $\boxed{-4}$

$p(-1) = -4$

$$p(-1) = 4\cdot(-1)^5 + a(-1)^4 + 6(-1)^3 + a(-1)^2 + 2(-1)$$
$$= -4 + a - 6 + a - 2$$
$$= 2a - 12$$

$$2a - 12 = -4$$
$$2a = 8$$
$$\boxed{a = 4}$$

DODATAK NA ZADATAK:

FAKTORISATI $p(x)$ NA $\mathbb{R}$ I $\mathbb{C}$

OVO NE MOŽEMO DA URADIMO

$$p(x) = 4\cdot x^5 + 4x^4 + 6x^3 + 4x^2 + 2x$$
$$= 2x(2x^4 + 2x^3 + 3x^2 + 2x + 1)$$

$k \mid 1 \Rightarrow k \in \{\pm 1\}$

$m \mid 2 \Rightarrow m \in \{1, 2\}$

$\dfrac{k}{m} \in \{\pm 1, \pm \frac{1}{2}\}$

	2	2	3	2	1
-1	2	0	3	-1	2
$-\frac{1}{2}$	2	1	$\frac{5}{2}$	$\frac{3}{4}$	$\frac{5}{8}$

NEMA RACIONALNE KORENE OSIM 0

PA MI NE ZNAMO DA GA FAKTORIŠEMO

b) $r(x) = \frac{4x^2+5x+5}{x^3+3x^2+5x+3}$ - JESTE PRAVA RACIONALNA F.

$x^3+3x^2+5x+3 = (x+1)(x^2+2x+3)$

$k|3 \Rightarrow k \in \{\pm1, \pm3\}$

$m|1 \Rightarrow m \in \{1\}$

$\frac{k}{m} \in \{\pm1, \pm3\}$

	1	3	5	3
−1	1	2	3	0

$\underbrace{1 \quad 2 \quad 3}_{x^2+2x+3}$

$x_{1,2} = \frac{-2\pm\sqrt{4-12}}{2} \notin \mathbb{R}$

$= \frac{-2\pm 2\sqrt{2}i}{2} = -1\pm\sqrt{2}i$

$p(x) = x^3+3x^2+5x+3 = (x+1)(x-(-1+\sqrt{2}i))(x-(-1-\sqrt{2}i))$ nad $\mathbb{C}$

$r(x) = \frac{4x^2+5x+5}{(x+1)(x^2+2x+3)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+2x+3}$ $\Big/ \cdot (x+1)(x^2+2x+3)$

$4x^2+5x+5 = A(x^2+2x+3)+(Bx+C)(x+1)$

$4x^2+5x+5 = Ax^2+2Ax+3A+Bx^2+Bx+Cx+C$

$4x^2+5x+5 = (A+B)x^2+(2A+B+C)x+3A+C$

$A+B=4 \rightarrow B=4-A$

$2A+B+C=5$

$3A+C=5 \rightarrow C=5-3A$

$2A+4-A+5-3A=5$

$-2A=-4$

$\boxed{A=2}$ $\boxed{B=2}$

$\boxed{C=-1}$

$r(x) = \frac{2}{x+1} + \frac{2x-1}{x^2+2x+3}$

$$2x + 3y = 5$$
$$-x + 4y = 2$$

KRAMER.

$$D_s = \begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} = 8 + 3 = 11 \neq 0$$

$$D_x = \begin{vmatrix} 5 & 3 \\ 2 & 4 \end{vmatrix} = 20 - 6 = 14$$

$$D_y = \begin{vmatrix} 2 & 5 \\ -1 & 2 \end{vmatrix} = 4 - (-5) = 9$$

$$x = \frac{D_x}{D_s} = \frac{14}{11}$$

$$y = \frac{D_y}{D_s} = \frac{9}{11}$$

$$R_S = \left\{ \left(\frac{14}{11}, \frac{9}{11} \right) \right\}$$

MATRICNA M.

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$AX = B \;/ A^{-1}$ so leve strane

$$\underbrace{A^{-1}A}_{E} X = A^{-1} B$$

$$X = A^{-1} B$$

$$\det(A) = \begin{vmatrix} 2 & 3 \\ -1 & 4 \end{vmatrix} = 8 + 3 = 11 \neq 0$$

$$\operatorname{adj}(A) = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) = \frac{1}{11} \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}$$

$$\boxed{\begin{aligned} X &= A^{-1} B \\ &= \frac{1}{11} \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} \end{aligned}} = \frac{1}{11} \begin{bmatrix} 14 \\ 9 \end{bmatrix}$$