(1)

$$\left\{\begin{aligned} b_1 &= (3,0,0) \\ b_2 &= (0,3,0) \\ b_3 &= (0,0,3) \end{aligned}\right.$$

$b_4 = (5,7,9)$

$\alpha b_1 + \beta b_2 + \gamma b_3 + \delta b_4$

$\mathbb{R}^3$

$\dim(\mathbb{R}^3) = 3$

IMA 4 VEKTORA PA NISU LIN. NEZ. U $\mathbb{R}^3$

jesu lin. nez u $\mathbb{R}^3$ koji je $\dim(\mathbb{R}^3) = 3$

$\Rightarrow$ ova tri bi bili baze u $\mathbb{R}^3$ do su hemi

$\Rightarrow$ oni generatori

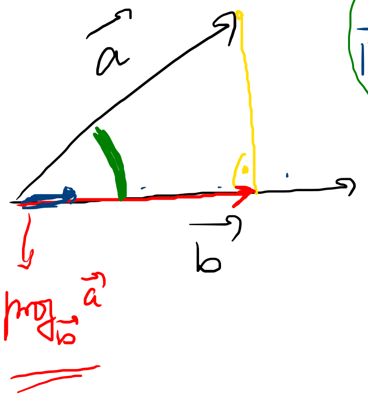
$$(x,y,z) = \frac{1}{3}x(3,0,0) + \frac{1}{3}y(0,3,0) + \frac{1}{3}z(0,0,3)$$

$\rightarrow$ $(8,10,9) = b_1 + b_2 + b_4$

$\rightarrow$ $(8,10,9) = \frac{8}{3}(3,0,0) + \frac{10}{3}(0,3,0) + \frac{9}{3}(0,0,3)$

$= \frac{8}{3}b_1 + \frac{10}{3}b_2 + \frac{9}{3}b_3$

② PROJEKCIJA $\vec a$ NA $\vec b$



$\frac{1}{|\vec b|}\vec b = \frac{\vec b}{|\vec b|}$ – jedinični vektor, pravac i smer kao $\vec b$

$$\left|\frac{\vec b}{|\vec b|}\right| = \frac{1}{\cancel{|\vec b|}}\cancel{|\vec b|} = 1$$

$$proj_{\vec b}\vec a = \boxed{|proj_{\vec b}\vec a|}\,\frac{\vec b}{|\vec b|} = \underbrace{|\vec a|\cdot\cos\angle(\vec a,\vec b)}\cdot\frac{\vec b}{|\vec b|}$$

$$= \cancel{|\vec a|}\,\frac{\vec a\cdot\vec b}{\cancel{|\vec a|}\,|\vec b|}\cdot\frac{\vec b}{|\vec b|} = \frac{\vec a\cdot\vec b}{|\vec b|^2}\vec b$$

$$\cos\angle(\vec a,\vec b) = \frac{|proj_{\vec b}\vec a|}{|\vec a|} \Rightarrow |proj_{\vec b}\vec a| = |\vec a|\cdot\cos\angle(\vec a,\vec b)$$

$$\vec a\cdot\vec b = |\vec a||\vec b|\cdot\cos\angle(\vec a,\vec b)$$

$$\cos\angle(\vec a,\vec b) = \frac{\vec a\cdot\vec b}{|\vec a||\vec b|}$$

$\vec b = (2,1,3)$

$|\vec b| = \sqrt{4+1+9} = \sqrt{14}$

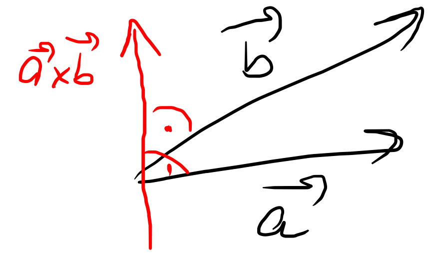
$\frac{1}{\sqrt{14}}\vec b =$

$$\left|\frac{1}{\sqrt{14}}\vec b\right| = \frac{1}{\sqrt{14}}|\vec b| = \frac{1}{\cancel{\sqrt{14}}}\cancel{\sqrt{14}} = 1$$

$\vec{a} \times \vec{b}$ – VEKTOR

1) $|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin \sphericalangle(\vec{a}, \vec{b})$

2) $\vec{a} \times \vec{b} \perp \vec{a}$, $\vec{a} \times \vec{b} \perp \vec{b}$



3) pravilo desne ruke

KO NAPIŠE

$$\underbrace{\vec{a} \times \vec{b}}_{\text{VEKTOR}} = \overbrace{|\vec{a}|\,|\vec{b}| \cdot \sin \sphericalangle(\vec{a}, \vec{b})}^{\text{BROJ}}$$

TAJ JE PAO!

$$r(x) = \frac{x^4-2x^3+2x^2-5x+5}{x^3-2x^2+2x-4} = x + \boxed{\frac{-x+5}{x^3-2x^2+2x-4}}$$

$$(x^4-2x^3+2x^2-5x+5) : \underbrace{(x^3-2x^2+2x-4)} = \underbrace{x}_{\text{KOLIČNIK}}$$

$$\frac{-(x^4-2x^3+2x^2-4x)}{\underbrace{-x+5}_{\text{OSTATAK}}}$$

$$x^4-2x^3+2x^2-5x+5 = (x^3-2x^2+2x-4)\cdot x + (-x+5)$$

$$r(x) = \frac{(x^3-2x^2+2x-4)\cdot x + (-x+5)}{x^3-2x^2+2x-4} = \frac{\cancel{(x^3-2x^2+2x-4)}x}{\cancel{x^3-2x^2+2x-4}} + \frac{-x+5}{x^3-2x^2+2x-4}$$

$$= x + \frac{-x+5}{x^3-2x^2+2x-4}$$

$$\frac{-x+5}{x^3-2x^2+2x-4} = \underbrace{\frac{A}{x^3} + \frac{B}{-2x^2} + \frac{C}{2x} + \frac{D}{-4}}_{\text{PAO !}}$$

$x^3-2x^2+2x-4=$

$k \mid (-4) \Rightarrow k \in \{\pm 1, \pm 2, \pm 4\}$

$m \mid 1 \Rightarrow m \in \{1\}$

$\frac{k}{m} \in \{\pm 1, \pm 2, \pm 4\}$

	1	-2	2	-4
~~1~~	~~1~~	~~-1~~	~~1~~	~~-3~~
~~-1~~	~~1~~	~~-3~~	~~5~~	~~-9~~
2	1	0	4	0

$\underbrace{1 \quad 0 \quad 4}_{x^2+4}$

$p(x) = (x-2)(x-2i)(x+2i)$ — nad $\mathbb{C}$

$= (x-2)(x^2+4)$ — nad $\mathbb{R}$

NULE su: $2, \pm 2i$

$x^2+4=0$

$x^2=-4$

$x=\pm 2i$

$$\frac{-x+5}{x^3-2x^2+2x-4} = \frac{-x+5}{(x-2)(x^2+4)}$$

$a = b + c \quad / 2$

$2a = 2b + 2c$

$$\frac{-x+5}{(x-2)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+4} \quad / (x-2)(x^2+4)$$

$$\frac{-x+5}{\cancel{(x-2)(x^2+4)}} \cdot \cancel{(x-2)(x^2+4)} = \frac{A}{\cancel{x-2}}\cancel{(x-2)}(x^2+4) + \frac{Bx+C}{\cancel{x^2+4}}(x-2)\cancel{(x^2+4)}$$

$$-x+5 = A(x^2+4) + (Bx+C)(x-2)$$

$$-x+5 = Ax^2 + 4A + Bx^2 - 2Bx + Cx - 2C$$

$$-x+5 = (A+B)x^2 + (-2B+C)x + 4A - 2C$$

$A + B = 0$

$-2B + C = -1$

$4A - 2C = 5$

$B = -A$

$2A + C = -1 \quad \cdot(-2)$

$4A - 2C = 5$

$-4C = 7$

$C = -\frac{7}{4}$

$2A = -1 + \frac{7}{4}$

$A = \frac{3}{8}$

$B = -\frac{3}{8}$

$$r(x) = x + \frac{\frac{3}{8}}{x-2} + \frac{-\frac{3}{8}x - \frac{7}{4}}{x^2+4}$$

(*)

$f:\mathbb{R}^2 \to \mathbb{R}^2$

$f(x,y) = (x^2-y,\ x+y)$

$x,y \to ax+by, \quad a,b \in \mathbb{R}$

$x,y,z \to ax+by+cz, \quad a,b,c \in \mathbb{R}$

$f:\mathbb{R}^3 \to \mathbb{R}^2$

$f(x,y,z) = (3x+5y-1,\ 6z+2x-3y)$

$f(x,y,z) = (3x+\ln y,\ 2x+3z)$

+0y+

$f(x,y,z) = (3x+\underbrace{\ln 2}_{\text{BİR SAYI}}\cdot y,\ 2x+3z)$

$\downarrow$

$2x+0y+3z$

$f(x,y,z) = (x,y,1)$

$f(x,y,z) = (x,y,0)$

$f : \mathbb{R}^2 \to \mathbb{R}^3$ — RIGHE

COLONNE

$$f(x,y) = (ax + y^b, \ln \boxed{a} \cdot x, y) = (ax + y, \ln a \cdot x, y)$$

$\boxed{b=1}$ $a \in \mathbb{R}^+$

$\ln a$ è def. se $\boxed{a > 0}$

$$M = \begin{bmatrix} a & 1 \\ \ln a & 0 \\ 0 & 1 \end{bmatrix}$$

$f: \mathbb{R}^2 \to \mathbb{R}^3$

$$f(x,y) = \left(2^{bx+1} \cancel{b}\, y,\ \sin(ax),\ bx\right)$$

$\boxed{b=0}$ $\boxed{a=0}$

$$= (2y, 0, 0)$$

$$\begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

rang $= 1$

$f: \mathbb{R}^3 \to \mathbb{R}^2$

$$f(x,y,z) = \left(\frac{ax+b}{bx+a} + y, \ \sin(bx) + az \right)$$

$\boxed{b=0}$ ✓

$$\frac{\cancel{a}x}{\cancel{a}} + y, \quad \boxed{a \neq 0} \ ✓$$

$$= (x+y, az)$$

$$M = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & a \end{bmatrix} \Rightarrow \text{rang}(M) = 2$$

$\Rightarrow \text{rang}(f) = 2$

$a \neq 0$

$\vec{a} = \vec{m} + 2\vec{n}$

$|\vec{a}|^2 = \vec{a} \cdot \vec{a}$

$= (\vec{m} + 2\vec{n})(\vec{m} + 2\vec{n})$

$|\vec{a}| = \sqrt{1^2 + 2^2}$

$\vec{a} = \vec{i} + \vec{j}$

$= (1, 2, 0)$

$\sqrt{a^2+b^2} \neq a+b$

$\dfrac{\dfrac{p(x)}{x-a}}{p(a)}$

$(x+3)(x+5)(x+7)$

$(x-1)(x^2-1) = (x-1)(x-1)(x+1)$

$(x-1)(x^2+1)$

$(x^3+1)(x-1)$

$a \neq 3$ . . . . oди

$a = 3$ — — нет

$\boxed{a=3}$

$R_S = \{1, 2, 3\}$

$$\cancel{\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 1 & 2 \\ 4 & 5 \end{matrix}}$$

$$= \vec{i} \begin{vmatrix} 2 & 6 \\ 5 & 3 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$$

$$A = \frac{1}{\det(A)} \operatorname{adj}(A) = -\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$-\frac{1}{2}$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2$$

$$\operatorname{adj}(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -2 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 0 & 2 \\ 0 & -2 & 1 \\ 1 & -1 & 1 \end{vmatrix} \begin{matrix} 1 & 0 \\ 0 & -2 \\ 1 & -1 \end{matrix}$$

$$= -2 + 4 + 1 = 3 \neq 0$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) = \frac{1}{3} \begin{bmatrix} -1 & -2 & 4 \\ 1 & -1 & -1 \\ 2 & 1 & -2 \end{bmatrix}$$

$$\operatorname{adj}(A) = \begin{bmatrix} +\begin{vmatrix} -2 & 1 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 0 & -2 \\ 1 & -1 \end{vmatrix} \\ -\begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} \\ +\begin{vmatrix} 0 & 2 \\ -2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix} \end{bmatrix}^T$$

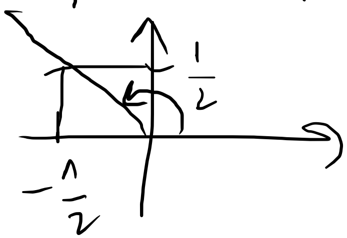
$$= \begin{bmatrix} -1 & 1 & 2 \\ -2 & -1 & +1 \\ 4 & -1 & -2 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & -2 & 4 \\ 1 & -1 & -1 \\ 2 & 1 & -2 \end{bmatrix}$$

$$\sqrt[4]{4i\left(\frac{-2+i}{3+i}\right)^6+\frac{\sqrt{3}}{2}i}$$

$$\frac{-2+i}{3+i}\cdot\frac{3-i}{3-i}=\frac{-6+2i+3i+1}{9+1}=\frac{-5+5i}{10}=-\frac{1}{2}+\frac{1}{2}i=\frac{\sqrt{2}}{2}e^{\frac{3\pi}{4}i}$$

$$\left|-\frac{1}{2}+\frac{1}{2}i\right|=\sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{1}{4}}=\frac{\sqrt{2}}{2}$$



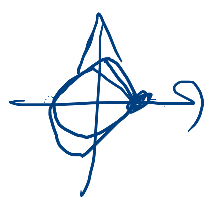
$$\varphi=\frac{3\pi}{4}$$

$$\left(\frac{\sqrt{2}}{2}e^{\frac{3\pi}{4}i}\right)^6=\frac{\cancel{2}^{3}}{\cancel{2^6}}e^{\frac{9\cancel{18}\pi}{\cancel{4}2}i}$$

$$=\frac{1}{8}e^{\frac{\pi}{2}i}=\frac{1}{8}\left(\cancel{\cos\frac{\pi}{2}}^{0}+i\,\cancel{\sin\frac{\pi}{2}}^{1}\right)=\frac{1}{8}i$$

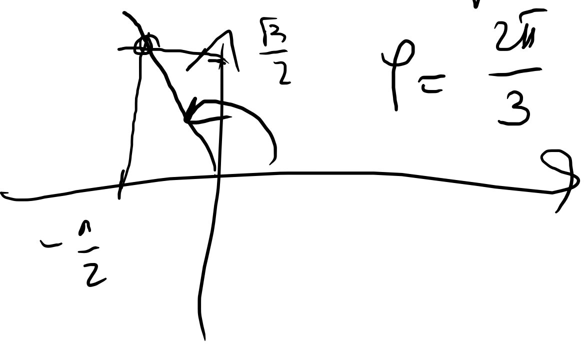
$$\sqrt[4]{4i\cdot\frac{1}{\cancel{8}2}i+\frac{\sqrt{3}}{2}i}=\sqrt[4]{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}$$

$$\frac{9\pi}{2}=\frac{8\pi}{2}+\frac{\pi}{2}=4\pi+\frac{\pi}{2}$$



$$\sqrt[4]{-\frac{1}{2}+\frac{\sqrt{3}}{2}i} = \sqrt[4]{e^{\frac{2\pi}{3}i}} = e^{\frac{\frac{2\pi}{3}+2k\pi}{4}i} \qquad k=0,1,2,3$$

$$\left|-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right| = \sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = 1$$



$$\varphi = \frac{2\pi}{3}$$

$k=0$ $\quad e^{\frac{\pi}{6}i} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}+\frac{1}{2}i$

$k=1$ $\quad e^{\frac{2\pi}{3}i} = \cos\frac{2\pi}{3}i + i\sin\frac{2\pi}{3} = -\frac{1}{2}+\frac{\sqrt{3}}{2}i$

$k=2$

$k=3$

k) $(z-\sqrt{3}+2i)^6 = a$

$x+y=3$

$x=1 \quad y=2$

$x+y=a$

$(1,2)$

$a=1+2$

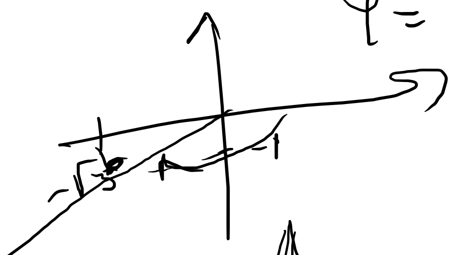
$z_1 = -3i$

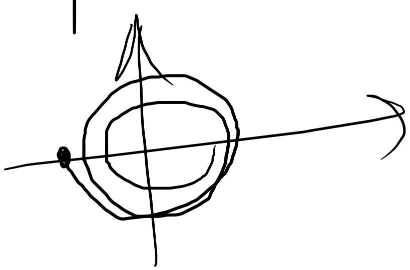
$a = (-3i-\sqrt{3}+2i)^6 = (-\sqrt{3}-i)^6 = \left(2e^{-\frac{5\pi}{6}i}\right)^6$

$= 2^6 e^{-5\pi i} = \boxed{2^6 e^{\pi i}}$

$|-\sqrt{3}-i| = \sqrt{(-\sqrt{3})^2+(-1)^2} = 2$

$\varphi = -\frac{5\pi}{6}$





$(z-\sqrt{3}+2i)^6 = 2^6 e^{\pi i}$

$z-\sqrt{3}+2i = \sqrt[6]{2^6 e^{\pi i}}$

$= 2e^{\frac{\pi+2k\pi}{6}i}$

$k=0,1,2,3,4,5$

$k=0$

$z-\sqrt{3}+2i = 2e^{\frac{\pi}{6}i}$

$z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) + \sqrt{3} - 2i$

$$\begin{array}{c|c|c|c} p(x) & & & \\ x+1 & x-1 & x-2 & (x+1)(x-1)(x-2) \\ \hline 2 & 3 & -1 & \end{array}$$

$7 : 2 = 3$

1

$7 = 2 \cdot 3 + 1$

$p(-1) = 2$

$p(1) = 3$

$p(2) = -1$

$p(x) : (x+1)(x-1)(x-2) = S(x)$

$r(x)$

$\deg(r(x)) < \deg((x+1)(x-1)(x-2)) = 3$

$r(x) = ax^2 + bx + C$

$a - b + c = 2$

$a + b + c = 3$

$4a + 2b + c = -1$

$$p(x) = \underbrace{(x+1)}_{-1+1=0}\underbrace{(x-1)}_{1-1=0}\underbrace{(x-2)}_{2-2=0} \circ S(x) + ax^2 + bx + C$$