① $r(x) = \dfrac{x^4-2x^3+2x^2-5x+5}{x^3-2x^2+2x-4} = \boxed{x} + \dfrac{-x+5}{x^3-2x^2+2x-4}$!?

$(x^4-2x^3+2x^2-5x+5) : \underbrace{(x^3-2x^2+2x-4)} = \boxed{x}$

$-(x^4-2x^3+2x^2-4x)$

$\underbrace{-x+5}$

$x^3-2x^2+2x-4 = (x-2)(x^2+2)$

$k \mid (-4) \Rightarrow k \in \{\pm 1, \pm 2, \pm 4\}$

$m \mid 1 \Rightarrow m \in \{1\}$

$\frac{k}{m} \in \{\pm 1, \pm 2, \pm 4\}$

	1	-2	2	-4
~~1~~	~~1~~	~~-1~~	~~1~~	~~-3~~
~~-1~~	~~1~~	~~-3~~	~~5~~	~~-9~~
2	1	0	2	$\boxed{0}$

$\underbrace{1 \quad 0 \quad 2}_{x^2+2}$

$\dfrac{-x+5}{(x-2)(x^2+2)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+2} \Big/ (x-2)(x^2+2)$

$-x+5 = A(x^2+2) + (Bx+C)(x-2)$

$-x+5 = Ax^2+2A+Bx^2-2Bx+Cx-2C$

$-x+5 = (A+B)x^2 + (-2B+C)x + 2A-2C$

$A+B=0 \Rightarrow B=-A$

$-2B+C=-1$

$2A-2C=5$

$2A+C=-1$

$2A-2C=5 \quad / \cdot (-1)$

$-3C=6$

$C=-2$

$A=\frac{1}{2} \quad B=-\frac{1}{2}$

$r(x) = x + \dfrac{\frac{1}{2}}{x-2} + \dfrac{-\frac{1}{2}x-2}{x^2+2}$

$$\frac{1}{(x-2)^3(x^2+4)^2\,x^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3} + \frac{Dx+E}{(x^2+4)} + \frac{Mx+F}{(x^2+4)^2} + \frac{I}{x} + \frac{J}{x^2}$$

$$\frac{1}{x^2} = \frac{A}{x} + \frac{B}{x^2}$$

$x-0$ $\quad (x-0)^2$

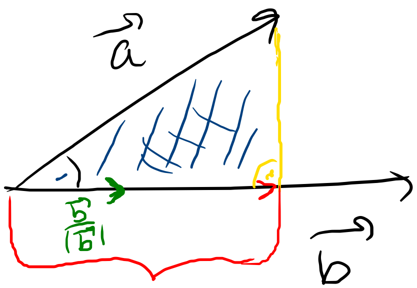
$$(x-0)^2$$

(2) $\vec{a} = \vec{i} + \vec{j} + 2\vec{k} = (1,1,2)$

$\vec{b} = \vec{i} - \vec{j} + 4\vec{k} = (1,-1,4)$

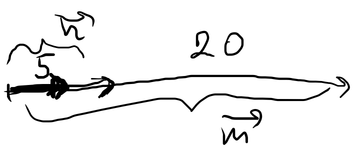
$pr_{\vec{b}}\vec{a}$?

$pr_{\vec{n}}\vec{m} = \dfrac{\vec{m}\cdot\vec{n}}{|\vec{n}|^2}\cdot\vec{n}$



$pr_{\vec{b}}\vec{a}$

$\left|\dfrac{\vec{m}}{|\vec{m}|}\right| = 1$



$\cancel{\vec{n} = \frac{\vec{m}}{4}}$

$pr_{\vec{b}}\vec{a} = \boxed{|pr_{\vec{b}}\vec{a}|}\dfrac{\vec{b}}{|\vec{b}|}$

$\cos\sphericalangle(\vec{a},\vec{b}) = \dfrac{|pr_{\vec{b}}\vec{a}|}{|\vec{a}|}$

$\Rightarrow |pr_{\vec{b}}\vec{a}| = |\vec{a}|\cos\sphericalangle(\vec{a},\vec{b})$

$= \cancel{|\vec{a}|}\cdot\dfrac{\vec{a}\cdot\vec{b}}{\cancel{|\vec{a}|}\cdot|\vec{b}|}$

$pr_{\vec{b}}\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\cdot\dfrac{\vec{b}}{|\vec{b}|} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\sphericalangle(\vec{a},\vec{b})$

$\cos\sphericalangle(\vec{a},\vec{b}) = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\vec{a}\cdot\vec{b} = (1,1,2)\cdot(1,-1,4)$

$= 1+(-1)+8 = 8$

$|\vec{b}|^2 = 1^2 + (-1)^2 + 4^2$

$= 1+1+16 = 18$

$pr_{\vec{b}}\vec{a} = \dfrac{8}{18}(1,-1,4)$

(3) $\mathbb{R}^3$

$x+y+z=0$

$y+z=0$

~~a)~~ $U=\{(x,y,z)\in\mathbb{R}^3 \mid x+y+z=5\}$

b) $U=\{(x,y,z)\in\mathbb{R}^3 \mid x+y+z=0,\ y+z=0\}$

~~c)~~ $U=\{(x,y,z)\in\mathbb{R}^3 \mid x^2+y+z=0\}$

$(0,0,0)\in\mathbb{R}^3$

HOMOGENI LIN. SISTEMI

$$+\begin{cases}(1,-1,0)\in U & 1^2-1+0=0\\ (-1,-1,0)\in U & (-1)^2-1+0=0\end{cases}$$

$(0,-2,0)\notin U \quad 0-2+0\neq 0$

~~d)~~ $U=\{(x,y,z)\in\mathbb{R}^3 \mid x\cdot y=0\}$

$x\cdot y=0$ → $x=0$ / $y=0$

$(0,5,0)$, $(3,0,0)$

$0\cdot 5=0$

$3\cdot 0=0$

$$U = \{(x,y,z) \in \mathbb{R}^3 \mid xy = 0\}$$

$$\left.\begin{array}{l}(0,5,0)\\(3,0,0)\end{array}\right\} +$$

$$\overline{}$$

$$(3,5,0) \qquad 3 \cdot 5 \neq 0$$

решими сис систем и добием решение

$$R_S = \{(t+2m,\ 3t-m,\ m) \mid t, m \in \mathbb{R}\}$$

a, b

$\alpha a + \beta b$

$2a+3b$

$5a-6b$

$(a,b,c) + (a_2, b_2, c_2) = (a+a_2,\ b+b_2,\ c+c_2)$

$\alpha(a,b,c) = (\alpha a, \alpha b, \alpha c)$

$$= \{(t,\ 3t,\ 0) + (2m,\ -m,\ m) \mid t, m \in \mathbb{R}$$

$$= \{t(1,3,0) + m(2,-1,1) \mid t, m \in \mathbb{R}\}$$

$= L\{(1,3,0), (2,-1,1)\}$ – све могуће линеарне комбинације ова два вектора

линеал

L – линеал је потпростор од $\mathbb{R}^{n}$ ^{3}

$\{(1,3,0), (2,-1,1)\}$ – generatorni, linearno nezavisni $\Rightarrow$ BAZA

$$\begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}_{\underline{\underline{2\times 3}}} + \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}_{\underline{\underline{2\times 3}}}$$

$$= \begin{bmatrix} 1+7 & 2+8 & 3+9 \\ 4+10 & 5+11 & 6+12 \end{bmatrix}_{2\times 3}$$

$$\begin{aligned} ax + (a-1)y + z &= 1 \\ x - y + az &= a \\ -ay + az &= 2 \end{aligned}$$

$$D_s = \begin{vmatrix} a & a-1 & 1 \\ 1 & -1 & a \\ 0 & -a & a \end{vmatrix} \begin{matrix} a & a-1 \\ 1 & -1 \\ 0 & -a \end{matrix}$$

$$= -a^2 - a + a^3 - a(a-1)$$

$$= -a^2 - \cancel{a} + a^3 - a^2 + \cancel{a}$$

$$= a^3 - 2a^2$$

$$= a^2 \cdot (a-2)$$

I $D_s \neq 0$ ZA $a \neq 0$, $a \neq 2$

$\Rightarrow$ SISTEM ODREĐEN

II $D_s = 0$ ZA $a = 0$ ILI $a = 2$

$\rightarrow$ SISTEM NEMOGUĆ ILI NEODREĐEN

NIKAKO DALJE DETERMINANTE
MORA GAUS !

$\boxed{a=0}$

$$\begin{aligned} -y + z &= 1 \\ x - y &= 0 \\ 0 &= 2 \end{aligned}$$

NEMOGUĆ

$R_s = \emptyset$

$\boxed{a=2}$

$$\begin{aligned} 2x + y + z &= 1 \\ x - y + 2z &= 2 \\ -2y + 2z &= 2 \quad / : 2 \end{aligned}$$

$$\begin{aligned} x - y + 2z &= 2 \quad \cdot(-2) \\ \boxed{2x} + y + z &= 1 \\ -y + z &= 1 \end{aligned}$$

$$\begin{aligned} x - y + 2z &= 2 \\ 3 : / \; 3y - 3z &= -3 \\ -y + z &= 1 \end{aligned}$$

$$\begin{aligned} x - y + 2z &= 2 \\ y - z &= -1 \\ -y + z &= 1 \end{aligned}$$

$$\begin{aligned} x - y + 2z &= 2 \\ y - z &= 1 \\ 0 &= 0 \end{aligned}$$

$$x - y + 2z = 2$$
$$y - z = 1$$

$$x - y = 2 - 2z$$
$$y = 1 + z$$

1 x neodreden

$z = t,\quad t \in \mathbb{R}$

$y = 1 + t$

$x = 2 - 2t + 1 + t$

$x = 3 - t$

$$R_s = \{(3-t, 1+t, t) \mid t \in \mathbb{R}\}$$

$(3, 1, 0)$

$(2, 2, 1)$

$(4, 0, -1)$

$\vdots$

$$x+y+z=3$$
$$0=0$$
$$0=0$$

$$x+y+z=3$$
$$\boxed{x=3-y-z}$$

2x neodređen

$$z=t,\ t\in\mathbb{R}$$
$$y=m,\ m\in\mathbb{R}$$
$$x=3-m-t$$

$$R_S=\{(3-m-t,\ m,\ t)\mid m,t\in\mathbb{R}\}$$

(α)

$$\begin{cases} x + z = 1 \\ x - y + z = 1 \\ -y + z = 2 \end{cases}$$

$$\boxed{AX = B} \quad / A^{-1} \text{ sa leve}$$

$$\underbrace{A^{-1}A}_{E}X = A^{-1}B$$

$$\boxed{X = A^{-1}B}$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x + z \\ x - y + z \\ -y + z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{cases} x + z = 1 \\ x - y + z = 1 \\ -y + z = 2 \end{cases}$$

$$\underbrace{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}}_{3 \times 2} \cdot \underbrace{\begin{bmatrix} -1 & 2 \\ -3 & 1 \end{bmatrix}}_{2 \times 2} =$$

$$= \underbrace{\begin{bmatrix} 1\cdot(-1)+2\cdot(-3) & 1\cdot 2+2\cdot 1 \\ 3\cdot(-1)+4\cdot(-3) & 3\cdot 2+4\cdot 1 \\ 5\cdot(-1)+6\cdot(-3) & 5\cdot 2+6\cdot 1 \end{bmatrix}}_{3 \times 2}$$

$ax+by+cz$, $a,b,c\in\mathbb{R}$

~~a)~~ $f:\mathbb{R}^2\to\mathbb{R}^3$, $f(x,y)=(x+5y+6,\ x,\ 3y+x)$

~~b)~~ $f:\mathbb{R}^3\to\mathbb{R}^2$, $f(x,y,z)=(x+y,\ z+\ln x)$

c) $f:\mathbb{R}^3\to\mathbb{R}^2$, $f(x,y,z)=(x+y,\ z+\ln 5\cdot x)$

d) $f:\mathbb{R}^2\to\mathbb{R}^2$, $f(x,y)=(x+3y,\ \underset{0x+0y}{\underset{\parallel}{0}})$

~~e)~~ $f:\mathbb{R}^2\to\mathbb{R}^2$, $f(x,y)=(x+3y,\ 7)$

~~f)~~ $f:\mathbb{R}^2\to\mathbb{R}^2$, $f(x,y)=(x-3y,\ x^2+y)$

g) $f:\mathbb{R}^2\to\mathbb{R}^2$, $f(x,y)=(x-3y,\ \sqrt{5}\,x+y)$

~~β)~~ $f:\mathbb{R}^2 \to \mathbb{R}^2$, $f(x,y) = (x, συν5)$

$\uparrow$ ✓ $1\cdot x + 0\cdot y$

[illegible]

$συν5 = \_\_\, x + \_\_\, y$

---

a, b, c

$\{\underbrace{\alpha a + \beta b + \gamma c} \mid \alpha, \beta, \gamma \in \mathbb{R}\}$

$\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$

$(1,1,1)$
$(1,1,0)$
$(1,0,0)$

$(a,b,c) = c(1,1,1) - c(1,1,0) + b(1,1,0) - b(1,0,0) + a(1,0,0)$

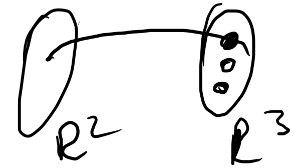
$\mathbb{R}^2$

$$\left.\begin{matrix}(1,0)\\(0,1)\end{matrix}\right\} \qquad \vec{a} = a_1(1,0) + a_2(0,1)$$

$$\left.\begin{matrix}(1,1)\\(2,2)\end{matrix}\right\} \qquad \underline{\underline{(2,2) = 2(1,1)}}$$

$$\left.\begin{matrix}(1,0)\\(0,1)\\(1,2)\\(5,7)\end{matrix}\right\} \qquad \begin{matrix}(1,1)\\(2,2) = 2(1,1)\\(3,3) = 3(1,1)\\(4,4) = 4(1,1)\end{matrix}$$

$f:\mathbb{R}^2 \to \mathbb{R}^3, \quad f(x,y) = (x+y,\ 2x+2y,\ 3x+3y)$



$\ker(f) = \{(x,y)\in\mathbb{R}^2 \mid f(x,y)=0\}$

$f(x,y)=0$

$(x+y,\ 2x+2y,\ 3x+3y)=0$

$$\begin{aligned} x+y &= 0 \quad -2 \\ 2x+2y &= 0 \\ 3x+3y &= 0 \quad -3 \end{aligned}$$

$x+y=0$ $\quad y=t,\ t\in\mathbb{R}$

$x=-y$ $\quad x=-t$

$\ker(f) = \{(-t,t) \mid t\in\mathbb{R}\} = \{t(-1,1) \mid t\in\mathbb{R}\}$

$\operatorname{Img}(f) = \{(a,b,c)\in\mathbb{R}^3 \mid \exists (x,y)\in\mathbb{R}^2\ f(x,y)=(a,b,c)\}$

$f(x,y)=(a,b,c)$

$(x+y,\ 2x+2y,\ 3x+3y)=(a,b,c)$

$$\begin{aligned} x+y &= a \quad -2 \\ 2x+2y &= b \\ 3x+3y &= c \quad -3 \end{aligned}$$

$x+y=a$

$0=b-2a \Rightarrow b$

$0=c-3a$

$\operatorname{Img}(f)=\{$

$f: \mathbb{R}^2 \to \mathbb{R}^3$

$$x+y=a$$
$$0=b-2a \Rightarrow b=2a$$
$$0=c-3a \Rightarrow c=3a$$

$$\operatorname{Img}(f)=\{(a,b,c)\in\mathbb{R}^3 \mid \underline{\exists (x,y)\in\mathbb{R}^2},\ f(x,y)=(a,b,c)\}$$
$$=\{(a,2a,3a) \mid a\in\mathbb{R}\}$$
$$=\{a(\underline{1,2,3}) \mid a\in\mathbb{R}\}$$