

$$\begin{aligned} x - ay + 3z &= -4 \\ (a-2)x + y - 3z &= a-9 \\ -ax - y + 4z &= 3 \end{aligned}$$

$$D_5 = \begin{vmatrix} 1 & -a & 3 & 1 & -a \\ a-2 & 1 & -3 & a-2 & 1 \\ -a & -1 & 4 & -a & -1 \end{vmatrix}$$

$$\begin{aligned} &= 4 - 3a^2 - 3(a-2) + 3a - 3 + 4a(a-2) \\ &= 4 - 3a^2 - 3a + 6 + 3a - 3 + 4a^2 - 8a \\ &= a^2 - 8a + 7 \end{aligned}$$

$$a_{1,2} = \frac{8 \pm \sqrt{64 - 28}}{2} = \frac{8 \pm 6}{2}$$

$$a_1 = 7 \quad a_2 = 1$$

I $D_5 \neq 0$ ZA $a \neq 7$, $a \neq 1 \Rightarrow$ ODREBEN SIS.
 II $D_5 = 0$ ZA $a = 7$, $a = 1 \Rightarrow$ NEODREBEN ILI NEMOGUC

DAGE NE MOZE KRAMER MORO PREKO DETERMINANTI
 GAUS

$$a = 7$$

$$\begin{aligned} x - 7y + 3z &= -4 \\ 5x + y - 3z &= -2 \quad \leftarrow -5 \\ -7x - y + 4z &= 3 \quad \leftarrow +7 \end{aligned}$$

$$\begin{aligned} x - 7y + 3z &= -4 \\ 36y - 18z &= 18 \quad | :18 \\ -50y + 25z &= -25 \quad | :25 \end{aligned}$$

$$\begin{aligned} x - 7y + 3z &= -4 \\ 2y - z &= 1 \\ +2y + z &= -1 \quad \leftarrow + \end{aligned}$$

$$R_5 = \left\{ 1 - \frac{1}{2} + \frac{1}{2}t, \frac{1}{2} + \frac{1}{2}t \right\} \quad t \in \mathbb{R}$$

$$\begin{aligned} x - 7y + 3z &= -4 \\ 2y - z &= 1 \\ 0 &= 0 \quad \checkmark \end{aligned}$$

$$\begin{aligned} x - 7y &= -4 - 3z \\ 2y &= 1 + z \end{aligned}$$

1x neodreden sistem

$$z = t, \quad t \in \mathbb{R}$$

$$y = \frac{1+t}{2}$$

$$x = -4 - 3t + 7 \frac{1+t}{2}$$

$$x = -\frac{1}{2} + \frac{1}{2}t$$

$$a=1$$

$$\begin{array}{r} x - y + 3z = 4 \\ -x + y - 3z = -8 \\ -x - y + 4z = 3 \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} +$$

$$x - y + 3z = -4$$

$$0 = -12$$

↙ несовместно

$$R_s = \emptyset$$

$$r(x) = \frac{x^4 - 4x^3 + 9x^2 - 6x + 12}{(x-2)(x^2 - 2x + 5)} = x + \frac{4x + 12}{(x-2)(x^2 - 2x + 5)}$$

$$\begin{array}{r} (x^4 - 4x^3 + 9x^2 - 6x + 12) : (x^3 - 4x^2 + 9x - 10) = \underline{x} \\ -(x^4 - 4x^3 + 9x^2 - 10x) \\ \hline 4x + 12 \end{array}$$

$$x_{1,2} = \frac{2 \pm \sqrt{4-20}}{2} \notin \mathbb{R}$$

$$= \frac{2 \pm 4i}{2} = 1 \pm 2i$$

$$(x-2)(x^2 - 2x + 5) = x^3 - 2x^2 + 5x - 2x^2 + 4x - 10$$

$$= \underline{x^3 - 4x^2 + 9x - 10}$$

НАРОМЕНА: $k | 10 \Rightarrow k \in \{\pm 1, \pm 2, \pm 5, \pm 10\}$
 $m | 1 \Rightarrow m \in \{1\}$
 $\frac{k}{m} \in \{\pm 1, \pm 2, \pm 5, \pm 10\}$
 $x^3 - 4x^2 + 9x - 10 = (x-2)(x^2 - 2x + 5)$

	1	-4	9	-10
1	1	-3	6	-4
1	1	-5	14	-24
2	1	-2	5	0

$x^2 - 2x + 5$
 НЕМА РЕАЛНЕ РОЛЕ

РОЛЕ
 2, $1 + 2i$
 $1 - 2i$
 $p(x) = (x-2)(x-(1+2i))(x-(1-2i))$
 mod \mathbb{C}

$$\frac{4x+12}{(x-2)(x^2-2x+5)} = \frac{A}{x-2} + \frac{Bx+C}{x^2-2x+5} \quad / ((x-2)(x^2-2x+5))$$

$$4x+12 = A(x^2-2x+5) + (Bx+C)(x-2)$$

$$4x+12 = \underline{Ax^2} - \underline{2Ax} + 5A + \underline{Bx^2} - \underline{2Bx} + \underline{Cx} - 2C$$

$$4x+12 = (A+B)x^2 + (-2A-2B+C)x + 5A-2C$$

$$A+B = 0$$

$$-2A-2B+C = 4$$

$$5A-2C = 12$$

→ A, B, C

$$f(x) = x + \frac{\textcircled{A}}{x-2} + \frac{\textcircled{B}x + \textcircled{C}}{x^2-2x+5}$$

$$\frac{1}{(x+1)^2 (x^2+3)^3} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+3} + \frac{Ex+F}{(x^2+3)^2} + \frac{Gx+H}{(x^2+3)^3}$$

$$\frac{1}{(x+1)(x^2+3)x^2} = \frac{A}{x+1} + \frac{Bx+C}{x^2+3} + \frac{D}{x} + \frac{E}{x^2}$$

$$\begin{array}{c} \uparrow \\ (x-0)^2 \end{array}$$

$$\frac{1}{x^2} = \frac{A}{x} + \frac{B}{x^2}$$

(*)

$$\vec{a} = \vec{m} + \vec{n}$$

$$\vec{b} = 2\vec{m} - 3\vec{n}$$

$$|\vec{m}| = 5$$

$$|\vec{n}| = 7$$

$$\angle(\vec{m}, \vec{n}) = \frac{2\pi}{3}$$

$$P_D = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (\vec{m} + \vec{n}) \times (2\vec{m} - 3\vec{n})$$

$$= 2\vec{m} \times \vec{m} - 3\vec{m} \times \vec{n} + 2\vec{n} \times \vec{m} - 3\vec{n} \times \vec{n}$$

$$= -3\vec{m} \times \vec{n} - 2\vec{m} \times \vec{n}$$

$$= -5\vec{m} \times \vec{n}$$

→ absolute value

$$P_D = |\vec{a} \times \vec{b}| = |-5\vec{m} \times \vec{n}| = | -5 | |\vec{m} \times \vec{n}|$$

$$= 5 |\vec{m}| |\vec{n}| \sin \angle(\vec{m}, \vec{n}) = 5 \cdot 5 \cdot 7 \cdot \sin \frac{2\pi}{3}$$

$$\vec{p} \times \vec{p} = \vec{0}$$

$$\vec{p} \cdot \vec{p} = |\vec{p}|^2$$

$$\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k} = (2, 3, -1)$$

$$\vec{b} = -3\vec{i} + 4\vec{j} = (-3, 0, 4)$$

$$P_{\square} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -1 \\ -3 & 0 & 4 \end{vmatrix} = \vec{i} \begin{vmatrix} 3 & -1 \\ 0 & 4 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & -1 \\ -3 & 4 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & 3 \\ -3 & 0 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 3 & -1 \\ 0 & 4 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & -1 \\ -3 & 4 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & 3 \\ -3 & 0 \end{vmatrix} = 12\vec{i} - 5\vec{j} + 9\vec{k}$$
$$= (12, -5, 9)$$

$$P_{\square} = |(12, -5, 9)| = \sqrt{12^2 + (-5)^2 + 9^2}$$

$$\star \vec{a} = 2\vec{i} + 3\vec{j} - 5\vec{k} = (2, 3, -5)$$

$$\vec{b} = 3\vec{i} + 3\vec{j} = (3, 3, 0)$$

-100
P500 DOVA

$$2\vec{a} - 6\vec{b} = 2(2, 3, -5) - 6(3, 3, 0) = (4, 6, -10) + (-18, -18, 0) = (-14, -12, -10)$$

(6, 9, 0) = $\vec{a} \cdot \vec{b} = (2, 3, -5) \cdot (3, 3, 0) = 2 \cdot 3 + 3 \cdot 3 + (-5) \cdot 0 = 6 + 9 = 15$

$$|\vec{b}| = \sqrt{3^2 + 3^2 + 0^2} = \sqrt{18} = 2\sqrt{3}$$

$$|\vec{a}| = \sqrt{4 + 9 + 25} = \sqrt{38}$$

cos $\angle(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{15}{\sqrt{38} \cdot 2\sqrt{3}}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -5 \\ 3 & 3 & 0 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 3 & -5 \\ 3 & 0 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & -5 \\ 3 & 0 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} = 15\vec{i} - 17\vec{j} - 3\vec{k} = (15, -17, -3) = -3(-5, 17, 1)$$

$$\begin{bmatrix} 2 & 3 & 4 \\ -2 & 0 & -1 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}_{2 \times 2} = \text{✓}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}_{2 \times 2} \begin{bmatrix} 2 \\ -2 \end{bmatrix}_{2 \times 1} \begin{bmatrix} 3 \\ 0 \end{bmatrix}_{2 \times 1} \begin{bmatrix} 4 \\ -1 \end{bmatrix}_{2 \times 1} =$$

$$\begin{bmatrix} 1 \cdot 2 + 2 \cdot (-2) & 1 \cdot 3 + 2 \cdot 0 & 1 \cdot 4 + 2 \cdot (-1) \\ 0 \cdot 2 + (-1) \cdot (-2) & 0 \cdot 3 + (-1) \cdot 0 & 0 \cdot 4 + (-1) \cdot (-1) \end{bmatrix}_{2 \times 3}$$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A) = \frac{1}{1} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$$

$$\operatorname{adj}(A) = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$