Polinomi

November 30, 2021

5. Za koje realne vrednosti parametra a je polinom $p(x) = x^3 + ax^2 + 3x - 5$ deljiv polinomom $x + 1$?

p(x) je deljiv so x+1 ako je ostatak pri njihovom deljenju 0.

Bezu:

$$\frac{p(x)}{x+1} \qquad p(-1)$$

$$p(-1) = 0$$

$$(-1)^3 + a \cdot 1^2 - 3 \cdot 1 - 5 = 0$$

$$a - 9 = 0$$

$$a = 9$$

6. Odrediti koeficijente a, b i c polinoma $p(x) = x^3 + ax^2 + bx + c$ tako da bude deljiv polinomima $x - 1$ i $x + 2$, a da pri deljenju sa polinomom $x - 4$ daje ostatak 18.

$$\frac{p(x)}{x-1} \quad p(1) \quad \boxed{p(1)=0}$$

$$\frac{p(x)}{x+2} \quad p(-2) \quad \boxed{p(-2)=0}$$

$$\frac{p(x)}{x-4} \quad p(4) \quad \boxed{p(4)=18}$$

$$p(1) = 1+a+b+c \Rightarrow 1+a+b+c=0$$

$$p(-2) = -8+4a-2b+c \Rightarrow -8+4a-2b+c=0$$

$$p(4) = 64+16a+4b+c \Rightarrow 64+16a+4b+c=18$$

7. Ostatak pri deljenju polinoma $p(x)$ sa $x-1$ je -5, a sa $x-2$ je -25. Koliki je ostatak pri deljenju polinoma $p(x)$ sa $(x-1)(x-2)$?

$$\frac{p(x)}{x-1} \qquad p(1)$$

$$\boxed{p(1) = -5}$$

$$\frac{p(x)}{x-2} \qquad p(2)$$

$$\boxed{p(2) = -25}$$

$$\frac{p(x)}{(x-1)(x-2)} = x^2-3x+6$$

NE MOŽE REŠAVATI STAV

$$p(x) : (x-1)(x-2) = s(x)$$

$$r(x)$$

$$\deg(r) < \deg((x-1)(x-2)) = 2$$

$$\boxed{r(x) = Ax+B}$$

$$7 : 2 = 3$$
$$1$$
$$7 = 2\cdot 3 + 1$$

$$\boxed{p(x) = (x-1)(x-2)\cdot s(x) + Ax+B}$$

$$p(1) = \underbrace{0\cdot(-1)\cdot s(1)}_{0} + A+B$$

$$\boxed{A+B = -5}$$

$$p(2) = \underbrace{1\cdot 0\cdot s(2)}_{0} + 2A+B$$

$$\boxed{2A+B = -25}$$

12. Naći sve nule polinoma $p(x)$, a zatim ga faktorisati nad poljima $\mathbb{R}$ i $\mathbb{C}$:

12.1 $p(x) = 3x^5 + 8x^4 - 10x^2 - 3x + 2$;

$k \mid 2 \Rightarrow k \in \{\pm 1, \pm 2\}$

$m \mid 3 \Rightarrow m \in \{1, 3\}$

$\frac{k}{m} \in \{\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}\}$

	3	8	0	-10	-3	2
1	3	11	11	1	-2	$\boxed{0}$
~~1~~	~~3~~	~~14~~	~~25~~	~~26~~	~~24~~	
-1	3	8	3	-2	$\boxed{0}$	
-1	3	5	-2	$\boxed{0}$		

$3x^2 + 5x - 2 = 0$

$x_{1,2} = \frac{-5 \pm \sqrt{25+24}}{6}$

$= \frac{-5 \pm 7}{6}$ — -2, $\frac{1}{3}$

SVE NULE SU:

$1, -2, \frac{1}{3}$ – 1x nule

-1 – 2x nule

5 – NULA

$p(x) = 3(x-1)(x+2)\left(x-\frac{1}{3}\right)(x+1)^2$

nad $\mathbb{C}$, $\mathbb{R}$

12.4 $p(x) = x^5 + x^4 - 6x^3 - 4x^2 + 8x = x(\underbrace{x^4 + x^3 - 6x^2 - 4x + 8})$

$k \mid 8 \Rightarrow k \in \{\pm 1, \pm 2, \pm 4, \pm 8\}$

$m \mid 1 \Rightarrow m \in \{1\}$

$\frac{k}{m} = k \in \{\pm 1, \pm 2, \pm 4, \pm 8\}$

	1	1	−6	−4	8
1	1	2	−4	−8	$\boxed{0}$
~~1~~	~~1~~	~~3~~	~~−1~~	~~−9~~	
~~−1~~	~~1~~	~~1~~	~~−5~~	~~−3~~	
2	1	4	4	$\boxed{0}$	

$\underbrace{1 \quad 4 \quad 4}$

$x^2 + 4x + 4 = (x+2)^2$

nule:

0, 1, 2 – jednostruke

−2 – dvostruka

$p(x) = x(x-1)(x-2)(x+2)^2$

nad $\mathbb{R}$ i $\mathbb{C}$

NORMIRAN $\Rightarrow \boxed{a_n = 1}$

NULE SU: $2, -3, 1$ – JEDNOSTRUKE

4 – DVOSTRUKA

$\pm i$ – JEDNOSTRUKA

$$p(x) = (x-2)(x+3)(x-1)(x-4)^2(x-i)(x+i) \quad \text{nad } \mathbb{C}$$

$$= (x-2)(x+3)(x-1)(x-4)^2(x^2+1) \quad \text{nad } \mathbb{R}$$

$$x^2+1=0$$

$$q(x) = (x+3)(x^2+9) \quad \text{nad } \mathbb{R}$$

$$= (x+3)(x+3i)(x-3i) \quad \text{nad } \mathbb{C}$$

$$x^2+9=0$$
$$x^2=-9$$
$$x=\pm 3i$$