

Relacije

October 6, 2021

Uređen par elementa a i b , u oznaci (a, b) je $(a, b) = \{\{a\}, \{a, b\}\}$, $(b, a) = \{\{b\}, \{b, a\}\}$
gde je a prva komponenta, a b druga komponenta uređenog para.

Napomena: $(b, a) = \{\{b\}, \{a, b\}\}$ pa za $a \neq b \Rightarrow (a, b) \neq (b, a)$.

$(a, b) = (c, d) \Leftrightarrow a = c \wedge b = d$.

Dekartov proizvod skupova A i B je skup svih uređenih parova čija je prva komponenta iz skupa A , a druga komponenta iz skupa B , tj.

$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$.

Primer: $A = \{1, 2, 3\}$, $B = \{x, y\}$

$A \times B = \{(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)\}$

$B \times A = \{(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3)\}$

Na osnovu ovog primera može se zaključiti da Dekartov proizvod nije komutativan, tj. $A \times B \neq B \times A$, ~~$A \neq B$~~ .

Dekartov kvadrat skupa A je $A^2 = A \times A = \{(a_1, a_2) \mid a_1, a_2 \in A\}$.

Primer: $A = \{1, 2, 3\}$, $A^2 = \{1, 2, 3\}^2$

$A^2 = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$.

$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$
 $B \times A = \{(1, a), (1, b), (2, a), (2, b)\}$ } $\Rightarrow A \times B \neq B \times A$, $A \neq B$

$$A = \{a, b\}$$

$$A = \{a, b\}$$

$$A^2 = A \times A = \{(a, a), (a, b), (b, a), (b, b)\}$$

Binarna relacija je bilo koji podskup od $A \times B$, tj. $\rho \subseteq A \times B$.

Ako uređen par (x, y) pripada relaciji ρ kaže se da su x i y u relaciji ρ i piše se $(x, y) \in \rho$ ili $x\rho y$.

Binarna relacija skupa A , je bilo koji podskup od A^2 , tj. $\rho \subseteq A^2$.

Kako je $\emptyset \subseteq A^2$ i $A^2 \subseteq A^2$ to su \emptyset i A^2 sigurno relacije skupa A , i one se nazivaju prazna i puna relacija.

$$A = \{a, b\}$$

$$B = \{1, 2\}$$

$$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$$

$$\rho_1 = \{(a, 2)\}$$

$$\rho_2 = \{(b, 2), (a, 1)\}$$

$$\rho_3 = \emptyset$$

$$\rho_4 = A \times B$$

\vdots

$$(x, y) \in \rho$$

$$x \rho y$$

$$(1, 2) \in \leq$$

$$1 \leq 2$$

$$(5 > 3)$$

$$(5, 3) \in >$$

$$S = \{5, 7, 9\}, \quad S = \{5, 7, 9\}$$

$$S^2 = S \times S = \{(5, 5), (5, 7), (5, 9), (7, 5), (7, 7), (7, 9), (9, 5), (9, 7), (9, 9)\}$$

$$P_1 = \{(5, 5)\}$$

$$P_2 = \{(5, 7), (5, 9)\}$$

$$P_3 = \emptyset$$

$$P_4 = S^2$$

$$P_5 = \{(5, 9), (7, 9), (9, 9)\}$$

Relacije koje imaju konačno mnogo elemenata se mogu zadati na više načina. Neka je $A = \{1, 2, 3\}$ i $\rho \subseteq A^2$ tada se ρ može zadati na sledeće načine:

- ▶ nabrojanjem elemenata: $\rho = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2)\}$
- ▶ pomoću drugih relacija: $\rho = \{(x, y) \in A^2 \mid x^2 + y \leq 6\}$

▶ tablično:

ρ	1	2	3
1	⊤	⊤	⊤
2	⊤	⊤	⊥
3	⊥	⊥	⊥

- ▶ grafički.

Relacije koje imaju beskonačno mnogo elemenata mogu se zadati pomoću drugih relacija ili se mogu opisati rečima govornog jezika.

Primer *: $A = \{1, 2, 3\}$

$$\rho_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}$$

$$\rho_2 = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$$

$$\rho_3 = \{(1, 1), (1, 3), (3, 1), (3, 2), (3, 3)\}$$

$$\rho_4 = \{(1, 2), (1, 3), (2, 3)\}$$

$$\rho_5 = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3)\}$$

Inverzna relacija relacije ρ je $\rho^{-1} = \{(y, x) \mid (x, y) \in \rho\}$

Primer: Inverzne relacije relacija iz Primera * su:

$$\rho_1^{-1} = \{(1, 1), (2, 1), (1, 2), (2, 2), (3, 3)\}$$

$$\rho_2^{-1} = \{(2, 1), (3, 1), (2, 2), (3, 2), (2, 3), (3, 3)\}$$

$$\rho_3^{-1} = \{(1, 1), (3, 1), (1, 3), (2, 3), (3, 3)\}$$

$$\rho_4^{-1} = \{(2, 1), (3, 1), (3, 2)\}$$

$$\rho_5^{-1} = \{(1, 1), (2, 1), (3, 1), (2, 2), (3, 2)\}$$

$$f = \{(x, y)\}$$

$$f^{-1} = \{(y, x)\}$$

$$a \leq b$$

$$b \geq a$$



Osnovne osobine binarne relacije ρ skupa $A \neq \emptyset$:

- ▶ **refleksivnost (R):** $(\forall x \in A) x\rho x$ $(\forall x \in A) (x, x) \in A$
- ▶ **simetričnost (S):** $(\forall x, y \in A) (x\rho y \Rightarrow y\rho x)$ $(\forall x, y \in A) (x, y) \in A \Rightarrow (y, x) \in A$
- ▶ **antisimetričnost (A):** $(\forall x, y \in A) (x\rho y \wedge y\rho x \Rightarrow x = y)$ ili $(\forall x, y \in A) (x, y) \in \rho \wedge (y, x) \in \rho \Rightarrow x = y$
- ▶ **tranzitivnost (T):** $(\forall x, y, z \in A) ((x\rho y \wedge y\rho z) \Rightarrow x\rho z)$ $(\forall x, y, z \in A) (x, y) \in \rho \wedge (y, z) \in \rho \Rightarrow (x, z) \in \rho$

RST
RELACIJA
EQUIVALENCIJE
RAT
RELACIJA PORETKA

$A = \{1, 2, 3\}$

$\rho_1 = \{(1,1), (2,2), (3,3), (1,2), (2,1), (3,1)\}$

$\rho_2 = \{(1,1), (2,2), (3,1), (1,3)\}$

$\rho_3 = \{(1,1), (2,2)\}$

$\rho_4 = \{(1,1), (2,2), (3,3)\}$

R ~~X~~ ~~X~~ ~~X~~
S ~~X~~ ~~X~~ ~~X~~

$(3,1) \in \rho_1 \wedge (1,2) \in \rho_1$
 $(3,2) \notin \rho_1$

~~X~~ S A T
R S A T

Primer: Ispitati osobine relacija iz Primera *.

$$\rho_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3)\}, A = \{1, 2, 3\}$$

$$(2, 1) \in \rho_1 \wedge (1, 2) \in \rho_1$$

$$\Rightarrow (2, 2) \in \rho_1$$

R

S

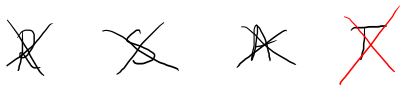
~~A~~

T

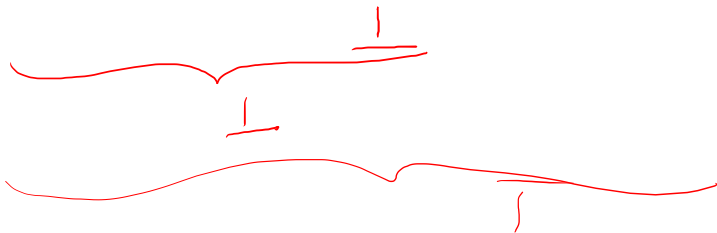
$$\rho_2 = \{(1, 2), (1, \underline{3}), (\underline{2}, 2), \textcircled{(2, 3)}, (\underline{3}, 2), (3, \underline{3})\}, A = \{1, 2, 3\}$$



$$\rho_3 = \{(1, 1), \underbrace{(1, 3)}, \underbrace{(3, 1)}, \underbrace{(3, 2)}, (3, 3)\}, A = \{1, 2, 3\}$$



$$(3, 2) \in \rho_3 \wedge \underline{\underline{(2, ?)}} \in \rho_3 \Rightarrow$$



$$\rho_4 = \{(1, 2), (1, 3), (2, 3)\}, A = \{1, 2, 3\}$$

~~R~~

~~S~~

A

T

$$\rho_5 = \{(1, \underline{1}), (1, 2), (1, 3), (2, 2), (2, \underline{3})\}, A = \{1, 2, 3\}$$

~~R~~

~~S~~

A

T