Bulova algebra- vežbe

November 8, 2021

$+ \rightarrow \cup$

$\cdot \rightarrow \cap$

$' \rightarrow \overline{}$

$0 \rightarrow \emptyset$

$1 \rightarrow \mathcal{U}$

1. Koja od sledećih tvrđenja su tačna u svakoj Bulovoj algebri $(B, +, \cdot, ', 0, 1)$?

- $\cancel{}$ $a + ab = aa'$;
- $a + 1 = 0'$;
- $\cancel{}$ $a \cdot 1 = 0'$;
- $\cancel{}$ $ab = (ab)'$; $(ab)' = a' + b'$
- $a + a'b = a + b$;
- $1 \cdot 0 = 1'$;
- $\cancel{}$ $a + b = (ab)'$; $(ab)' = a' + b'$
- $ab = (a' + b')'$; $(a' + b')' = a \cdot b$
- $\cancel{}$ $a(a + b) = aa'$;
- $\cancel{}$ $a + 1 = a$;

$a + ab = aa'$

$\underbrace{A \cup (A \cap B)}_{A} = A \cap \overline{A} \quad \emptyset$

$A \cup \mathcal{U} = \overline{\emptyset}$

$\mathcal{U} = \mathcal{U}$

$a \cdot 1 = 0'$

$A \cap \mathcal{U} = \overline{\emptyset}$

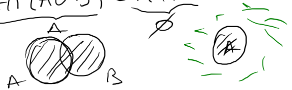
$A = \mathcal{U}$

$a + a'b = a + b$

$A \cup (\overline{A} \cap B) = A \cup B$

$a(a+b) = aa'$

$\underbrace{A \cap (A \cup B)}_{A} = \underbrace{A \cap \overline{A}}_{\emptyset}$



- $1 + c = 1;$
- $1 \cdot 0 = 1;$
- $a + a' = a;$
- $a' + a' = a';$
- $a + bc = (a + b)(a + c);$
- $1 + c = 0;$
- $a \cdot 0 = 0;$
- $a + a' = 0;$
- $a' \cdot a' = a';$
- $a \leq 1;$
- $a \leq 0.$

$a + a' = a$

$A \cup \overline{A} = A$

$U = A$

$a' + a' = a'$

$\overline{A} \cup \overline{A} = \overline{A}$

$\overline{A} = \overline{A}$

$a \cdot (b + c) = a \cdot b + a \cdot c$

$a + (b \cdot c) = (a + b) \cdot (a + c)$

$a \leq 1 \iff a + 1 = 1$

$a \leq 0 \iff a + 0 = 0$

2. Dokazati da su u svakoj Bulovoj algebri $(B, +, \cdot, ', 0, 1)$ sledeći iskazi ekvivalentni:

$a \Rightarrow b \Rightarrow c \Rightarrow d \Rightarrow a$

$X \leq Y$

(a) $xy = x$; (b) $\boxed{x + y = y}$; (c) $x' + y = 1$; (d) $xy' = 0$.

a) $\Rightarrow$ b) : $\boxed{xy = x}$ ← VAŽI

$x + y = y$ ← TREBA

$x + y = xy + y = (x+1)y = 1 \cdot y = y$

b) $\Rightarrow$ c) : $x + y = y$ ← VAŽI

$x' + y = 1$ ← TREBA

$x' + y = x' + (x + y) = (x' + x) + y = 1 + y = 1$

c) $\Rightarrow$ d) $x' + y = 1$ ← VAŽI

$xy' = 0$ ← TREBA

$x' + y = 1 \quad /'$

$(x' + y)' = 1'$

$\boxed{x \cdot y' = 0}$

d) $\Rightarrow$ a) $xy' = 0$ ← VAŽI

$xy = x$ ← TREBA

$xy = xy + 0 = xy + xy'$

$= x(y + y') = x \cdot 1 = x$

$x \cdot (y+z) = xy + xz$

3. Dokazati da su u svakoj Bulovoj algebri $(B, +, \cdot, ', 0, 1)$ za sve $a, b, c \in B$ važi:

$A \cap \overline{A}$

3.1 $a\,(bc') = (ab)\,(ac)'$;

$$(ab)(ac)' = (ab)(a'+c') = (ab)a' + (ab)c' = b(\underline{aa'}) + a(bc') = b \cdot 0 + a(bc')$$

$$= 0 + a(bc')$$

$$= a(bc')$$

3.2 $ab = 0 \iff ab' = a$;

1) $\underbrace{ab = 0}_{\text{VAŽI}} \Rightarrow \underbrace{ab' = a}_{\text{TREBA}}$

$ab' = ab' + 0 = ab' + ab$

$= a(b' + b) = a \cdot 1 = a$

2) $\underbrace{ab' = a}_{\text{VAŽI}} \Rightarrow \underbrace{ab = 0}_{\text{TREBA}}$

$ab = (ab')b = a(b'b) = a \cdot 0 = 0$

3.3 $(ab) + (a' + b') = 1$;

$$(ab) + (a' + b') = (ab) + (ab)' = 1$$

3.4 $(c \leq a \wedge c \leq b) \Longleftrightarrow c \leq ab.$

$(c+a=a \wedge c+b=b) \Leftrightarrow c+ab=ab$

$(\Rightarrow)$: $\underbrace{c+a=a \wedge c+b=b}_{\text{VAŽI}} \Rightarrow \underbrace{c+ab=ab}_{\text{TREBA}}$

$c+ab=(c+a)(c+b)=ab$

$(\Leftarrow)$: $\underbrace{c+ab=ab}_{\text{VAŽI}} \Rightarrow (\underbrace{c+a=a \wedge c+b=b}_{\text{TREBA}})$

$c+a=c+a\cdot 1=c+a(b+b')=c+ab+ab'=ab+ab'=a(b+b')=a\cdot 1=a$

$c+b=c+b\cdot 1=c+b(a+a')=c+ba+ba'=c+ab+a'b=ab+a'b=$
$=(a+a')b=1\cdot b=b$