Крамерово правило

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \overset{x_i}{\cdots} + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
&\vdots \\
a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n
\end{aligned}
$$

$$
D_{x_i} = \begin{array}{ccccc} & & x_i & & \\ \end{array}
\begin{vmatrix}
a_{11} & a_{12} & \cdots & b_1 & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & b_2 & \cdots & a_{2n} \\
\vdots & \vdots & & \vdots & & \\
a_{n1} & a_{n2} & \cdots & b_n & \cdots & a_{nn}
\end{vmatrix}
$$

$$
D_s = \begin{vmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & & \vdots \\
a_{n1} & a_{n2} & \cdots & a_{nn}
\end{vmatrix}
$$

ВРСТЕ → $(i)(j)$ ↑ КОЛОНА

Крамерово правило: Ако е $\boxed{D_s \neq 0}$ систем е одреден и негово единствено решение е

$$(x_1, x_2, \ldots, x_n) = \left(\frac{D_{x_1}}{D_s}, \frac{D_{x_2}}{D_s}, \ldots, \frac{D_{x_n}}{D_s}\right)$$

$$\left(x_1 = \frac{D_{x_1}}{D_s},\ x_2 = \frac{D_{x_2}}{D_s}, \ldots, x_n = \frac{D_{x_n}}{D_s}\right)$$

Ако е $D_s = 0$ систем е или неможн. или неодреден и Крамерово правило не дава одговор.

1) $$\begin{aligned} x+y+z&=3\\ 2x+y-z&=1\\ -x+3y+2z&=5 \end{aligned}$$

$$D_z=\begin{vmatrix} 1 & 1 & 3\\ 2 & 1 & 1\\ -1 & 3 & 5 \end{vmatrix}$$

$$D_s=\begin{vmatrix} 1 & 1 & 1\\ 2 & 1 & -1\\ -1 & 3 & 2 \end{vmatrix}$$

← начин формирања детерминанти

$$D_x=\begin{vmatrix} 3 & 1 & 1\\ 1 & 1 & -1\\ 5 & 3 & 2 \end{vmatrix}$$

$$D_y=\begin{vmatrix} 1 & 3 & 1\\ 2 & 1 & -1\\ -1 & 5 & 2 \end{vmatrix}$$

2)

$$\begin{aligned} 2x - y &= 8 \\ 3x + 2y &= -2 \end{aligned}$$

$$D_s = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 4 - (-3) = 7 \neq 0 \implies \text{систем је одређен}$$

$$D_x = \begin{vmatrix} 8 & -1 \\ -2 & 2 \end{vmatrix} = 16 - 2 = 14$$

$$D_y = \begin{vmatrix} 2 & 8 \\ 3 & -2 \end{vmatrix} = -4 - 24 = -28$$

$$x = \frac{D_x}{D_s} = \frac{14}{7} = 2$$

$$R_s = \{(2, -4)\}$$

$$y = \frac{D_y}{D_s} = \frac{-28}{7} = -4$$

3. $\begin{array}{l} x+y=3 \\ x+y=4 \end{array}$

$$D_S = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1-1 = 0 \Rightarrow$$ систем је или немогућ или неодређен и Крамерово правило не даје одговор

$$\begin{array}{l} x+y=3 \\ x+y=4 \quad /\cdot(-1) \\ \hline x+y=3 \\ 0=1 \\ \hline \end{array}$$

$\lightning$ немогућ

$R_S = \emptyset$

4. $\begin{array}{l} x+y=5 \\ 2x+2y=10 \end{array}$

$$D_S = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2-2 = 0 \Rightarrow$$

$$\begin{array}{l} x+y=5 \\ 2x+2y=10 \quad /\cdot(-2) \\ \hline x+y=5 \\ 0=0 \end{array}$$

$$\begin{array}{l} x+y=5 \\ x=5-y \\ \hline y=t,\ t\in\mathbb{R} \\ x=5-t \end{array}$$

$R_S = \{(5-t, t) \mid t\in\mathbb{R}\}$

5)

$$\begin{aligned} 2x - 7y + z &= -8 \\ 3x \qquad - z &= 1 \\ 2x + 5y + 5z &= 0 \end{aligned}$$

$$x = \frac{D_x}{D_s} = \frac{0}{144} = 0$$

$$y = \frac{D_y}{D_s} = \frac{144}{144} = 1$$

$$z = \frac{D_z}{D_s} = \frac{-144}{144} = -1$$

$$R_s = \{(0, 1, -1)\}$$

$$D_s = \left|\begin{array}{ccc|cc} 2 & -7 & 1 & 2 & -7 \\ 3 & 0 & -1 & 3 & 0 \\ 2 & 5 & 5 & 2 & 5 \end{array}\right| = 0 + 14 + 15 - 0 - (-10) - (-105)$$

$$= 144 \neq 0 \Rightarrow \text{систем је одређен}$$

$$D_x = \begin{vmatrix} -8 & -7 & 1 \\ 1 & 0 & -1 \\ 0 & 5 & 5 \end{vmatrix} = -8(-1)^{1+1} \begin{vmatrix} 0 & -1 \\ 5 & 5 \end{vmatrix} + 1(-1)^{2+1} \begin{vmatrix} -7 & 1 \\ 5 & 5 \end{vmatrix}$$

$$= -8(0 - (-5)) + (-1)(-35 - 5) = -40 + 40 = 0$$

$$D_y = \left|\begin{array}{ccc|cc} 2 & -8 & 1 & 2 & -8 \\ 3 & 1 & -1 & 3 & 1 \\ 2 & 0 & 5 & 2 & 0 \end{array}\right| = 10 + 16 - 2 + 120 = 144$$

$$D_z = \left|\begin{array}{ccc|cc} 2 & -7 & -8 & 2 & -7 \\ 3 & 0 & 1 & 3 & 0 \\ 2 & 5 & 0 & 2 & 5 \end{array}\right| = -14 - 120 - 10 = -144$$

VAŽNO

$$\begin{cases} x+y=1 \\ 2x+2y=2 \quad / \cdot(-1) \end{cases}$$

$$x+y=1$$
$$0=0$$

$$x+y=1$$
$$x=1-y$$

$y=t,\ t\in\mathbb{R},\ x=1-t$

$R_S=\{(1-t,t)\ |\ t\in\mathbb{R}\}$

НЕОДРЕЂЕН СИСТЕМ

$$D_S=\begin{vmatrix}1&1\\2&2\end{vmatrix}=0$$

$$D_x=\begin{vmatrix}1&1\\2&2\end{vmatrix}=0$$

$$D_y=\begin{vmatrix}1&1\\2&2\end{vmatrix}=0$$

$$D_S=D_x=D_y=0$$

$$\begin{cases} x+y+z=1 \\ 2x+2y+2z=3 \quad / \cdot(-2) \\ x+y+z=5 \quad / \cdot(-1) \end{cases}$$

$1=5$ ↯ $R_S=\emptyset$

НЕМОГУЋ СИСТЕМ

$$D_S=\begin{vmatrix}1&1&1\\2&2&2\\1&1&1\end{vmatrix}=0$$

$$D_x=\begin{vmatrix}1&1&1\\3&2&2\\5&1&1\end{vmatrix}=0$$

$$D_y=\begin{vmatrix}1&1&1\\2&3&2\\1&5&1\end{vmatrix}=0$$

$$D_z=\begin{vmatrix}1&1&1\\2&2&3\\1&1&5\end{vmatrix}=0$$

$$D_S=D_x=D_y=D_z=0$$

$$x+y+z=1$$
$$0=1$$
$$0=5 \ ↯$$

б) $2ax + y = a^2$

$2x + ay = -1$, $a \in \mathbb{R}$

$$D_s = \begin{vmatrix} 2a & 1 \\ 2 & a \end{vmatrix} = 2a^2 - 2$$

I $D_s \neq 0$ за $2a^2 - 2 \neq 0$

$a^2 \neq 1$

$a \neq 1 \wedge a \neq -1$

$\Rightarrow$ систем је одређен

II $D_s = 0$ за $a = 1 \wedge a = -1$

$\Rightarrow$ систем или немогућ или неодређен

МОРА СЕ ПРЕЋИ НА ГАУСОВ ПОСТУПАК !

$\boxed{a = 1}$

$2x + y = 1 \quad / \cdot (-1)$

$2x + y = -1$

$-2x - y = -1$

$2x + y = -1$

$0 = -2$

$R_s = \emptyset$

немогућ

$\boxed{a = -1}$

$-2x + y = 1 \quad +$

$2x - y = -1$

$-2x + y = 1$

$0 = 0$ ✓

$y = 1 + 2x$

$x = t, \; t \in \mathbb{R}$

$y = 1 + 2t$

$R_s = \{(t, 1+2t) \mid t \in \mathbb{R}\}$

1 x неодређен

7. $\begin{array}{l} ax - y = 2 \\ 4x + 2y = b \end{array}$, $a, b \in \mathbb{R}$

$$D_s = \begin{vmatrix} a & -1 \\ 4 & 2 \end{vmatrix} = 2a + 4$$

I $D_s \neq 0$ за $2a + 4 \neq 0$

$a \neq -2$

$\Rightarrow$ систем одређен

II $D_s = 0$ за $a = -2$

$\Rightarrow$ систем је или немогућ или неодређен

Може се прећи на Гаусов поступак

$\boxed{a = -2}$

$$\begin{array}{l} -2x - y = 2 \quad \big) \cdot 2 \\ 4x + 2y = b \end{array}$$

$$\begin{array}{l} -2x - y = 2 \\ 0 = b + 4 \end{array}$$

$\boxed{b \neq -4}$

$0 =$ број који није 0

немогућ

$R_s = \emptyset$

$\boxed{b = -4}$

$$\begin{array}{l} -2x - y = 2 \\ 0 = 0 \ \checkmark \end{array}$$

$y = -2 - 2x$

$x = t, \ t \in \mathbb{R}$

$y = -2 - 2t$

$R_s = \{(t, -2 - 2t) \mid t \in \mathbb{R}\}$

$1\times$ неодређен

Квадратне једначине, неједначине и функције

Квадратна једначина

$$ax^2+bx+c=0, \quad a\neq 0$$

1, $b=c=0 \Rightarrow ax^2=0 \Rightarrow x_{1,2}=0$

2, $b=0,\ c\neq 0 \Rightarrow ax^2+c=0 \Rightarrow x^2=-\frac{c}{a} \Rightarrow x_{1,2}=\pm\sqrt{-\frac{c}{a}}$

3, $b\neq 0,\ c=0 \Rightarrow ax^2+bx=0 \Rightarrow x\cdot(ax+b)=0 \Rightarrow x_1=0,\ x_2=-\frac{b}{a}$

4, $b\neq 0,\ c\neq 0 \Rightarrow ax^2+bx+c=0 \Rightarrow \boxed{x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$

① $3x^2-2x=0$

$x(3x-2)=0$

$x=0 \qquad 3x-2=0$

$\boxed{x=0} \qquad \boxed{x=\frac{2}{3}}$

② $x^2-3=0$

$x^2=3$

$\boxed{x=\pm\sqrt{3}}$

③ $6x^2-x-2=0$

$$x_{1,2}=\frac{1\pm\sqrt{1+48}}{12}=\frac{1\pm 7}{12}= \begin{cases} \frac{2}{3} \\ -\frac{1}{2} \end{cases}$$

x_1, x_2 - решења квадратне једначине

$$\Rightarrow \quad ax^2+bx+c = a(x-x_1)(x-x_2)$$

1

$$\frac{3x^2-7x+2}{2x^2-5x+2} = \frac{3\cancel{(x-2)}(x-\frac{1}{3})}{2\cancel{(x-2)}(x-\frac{1}{2})} = \frac{3x-1}{2x-1}$$

услови: $x \neq 2$, $x \neq \frac{1}{2}$

$$3x^2-7x+2=0$$

$$x_{1,2} = \frac{7 \pm \sqrt{49-24}}{6} = \frac{7 \pm 5}{6} = \left\langle \begin{array}{l} 2 \\ \frac{1}{3} \end{array} \right.$$

$$2x^2-5x+2=0$$

$$x_{1,2} = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm 3}{4} = \left\langle \begin{array}{l} 2 \\ \frac{1}{2} \end{array} \right.$$

$D = b^2 - 4ac$ дискримінант

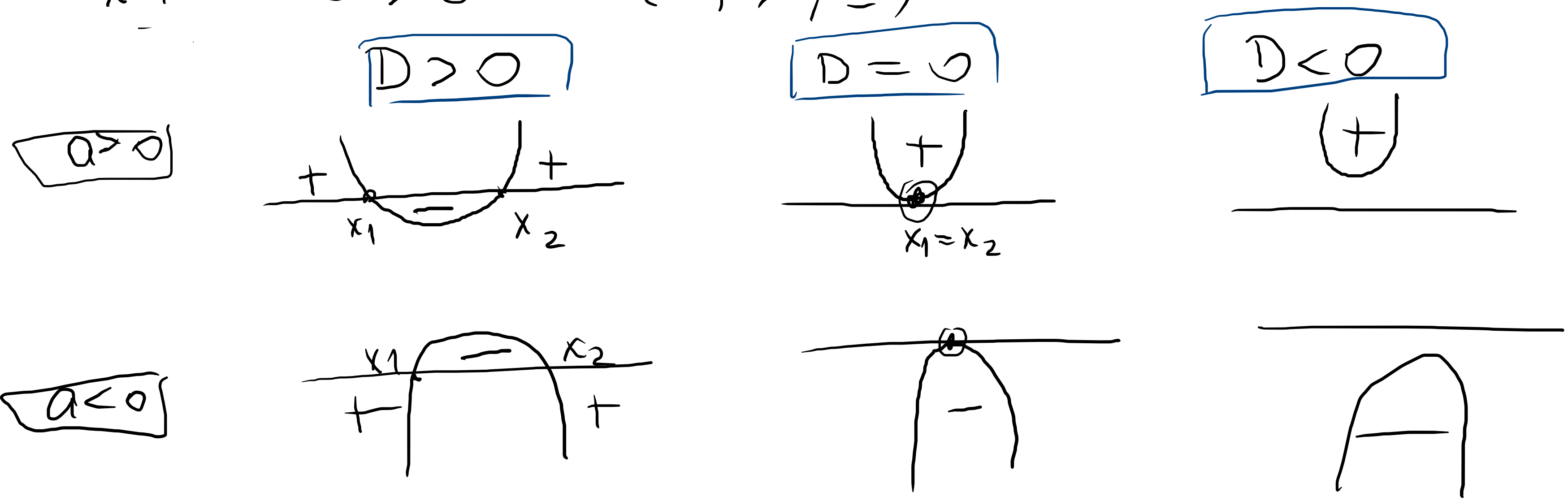
$D > 0 \Rightarrow x_1, x_2 \in \mathbb{R}, \; x_1 \neq x_2$

$D = 0 \Rightarrow x_1, x_2 \in \mathbb{R}, \; x_1 = x_2$

$D < 0 \Rightarrow$ пар кон'юговано комплексних рішань

Квадратні нерівності

$ax^2 + bx + c > 0 \quad (<, \geq, \leq)$



1) $\dfrac{3x^2+6x-9}{2x+1} \geqslant x+1$

$\dfrac{3x^2+6x-9}{2x+1} - x - 1 \geqslant 0$

$\dfrac{3x^2+6x-9-2x^2-x-2x-1}{2x+1} \geqslant 0$

$\dfrac{x^2+3x-10}{2x+1} \geqslant 0$

$\to x^2+3x-10=0$

$x_{1,2} = \dfrac{-3 \pm \sqrt{9+40}}{2} = \dfrac{-3 \pm 7}{2} \Big\langle {-5 \atop 2}$

условие: $2x+1 \neq 0$

$x \neq -\frac{1}{2}$

I $\dfrac{(x+5)(x-2)}{2x+1} \geqslant 0$

	-5		$-\frac{1}{2}$		2	
$x+5$	$-$	•	$+$	$+$		$+$
$x-2$	$-$		$-$	$-$	•	$+$
$2x+1$	$-$		$-$	o	$+$	$+$
	$-$		$+$	$-$		$+$

$x \in [-5, -\frac{1}{2}) \cup [2, +\infty)$

II $\dfrac{x^2+3x-10}{2x+1} \geqslant 0$

	-5		$-\frac{1}{2}$		2	
$x^2+3x-10$	$+$	•	$-$	$-$	•	$+$
$2x+1$	$-$		$-$ o	$+$		$+$
	$-$		$+$	$-$		$+$

2) $\dfrac{1-2x}{x^2-5x+6} < \dfrac{2}{5}$

$\dfrac{1-2x}{x^2-5x+6} - \dfrac{2}{5} < 0$

$\dfrac{5-\cancel{10x}-2x^2+\cancel{10x}-12}{5(x^2-5x+6)} < 0$

$\dfrac{-2x^2-7}{5(x-2)(x-3)} < 0$

$\dfrac{-(2x^2+7)}{5(x-2)(x-3)} < 0$

< 0

услов: $x^2-5x+6 \neq 0$

$\longrightarrow x_{1,2} = \dfrac{5 \pm \sqrt{25-24}}{2} = \dfrac{5\pm 1}{2} = \begin{cases} 3 \\ 2 \end{cases}$

$x \neq 3$ и $x \neq 2$

$(x-2)(x-3) > 0$

$x^2-5x+6 > 0$

$x \in (-\infty, 2) \cup (3, +\infty)$

Квадратне функције

$y=ax^2+bx+c$, $a\neq 0$ ← ПАРАБОЛА

$D>0$ сече x-осу у $(x_1,0)$, $(x_2,0)$ [x_1, x_2 — су решења кв. јед.]

$D=0$ додирује x-осу $(x_1,0)$

$D<0$ не сече (и не додирује) x-осу

y-осу сече у $(0,c)$

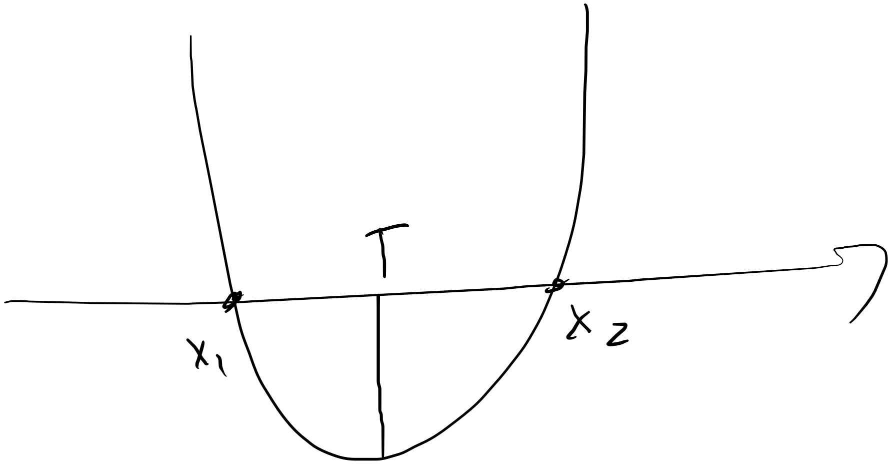
$a>0$ — функција има минимум $\cup$

$a<0$ — функција има максимум $\cap$

Теме параболе $T\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$

$a>0$

$D>0$



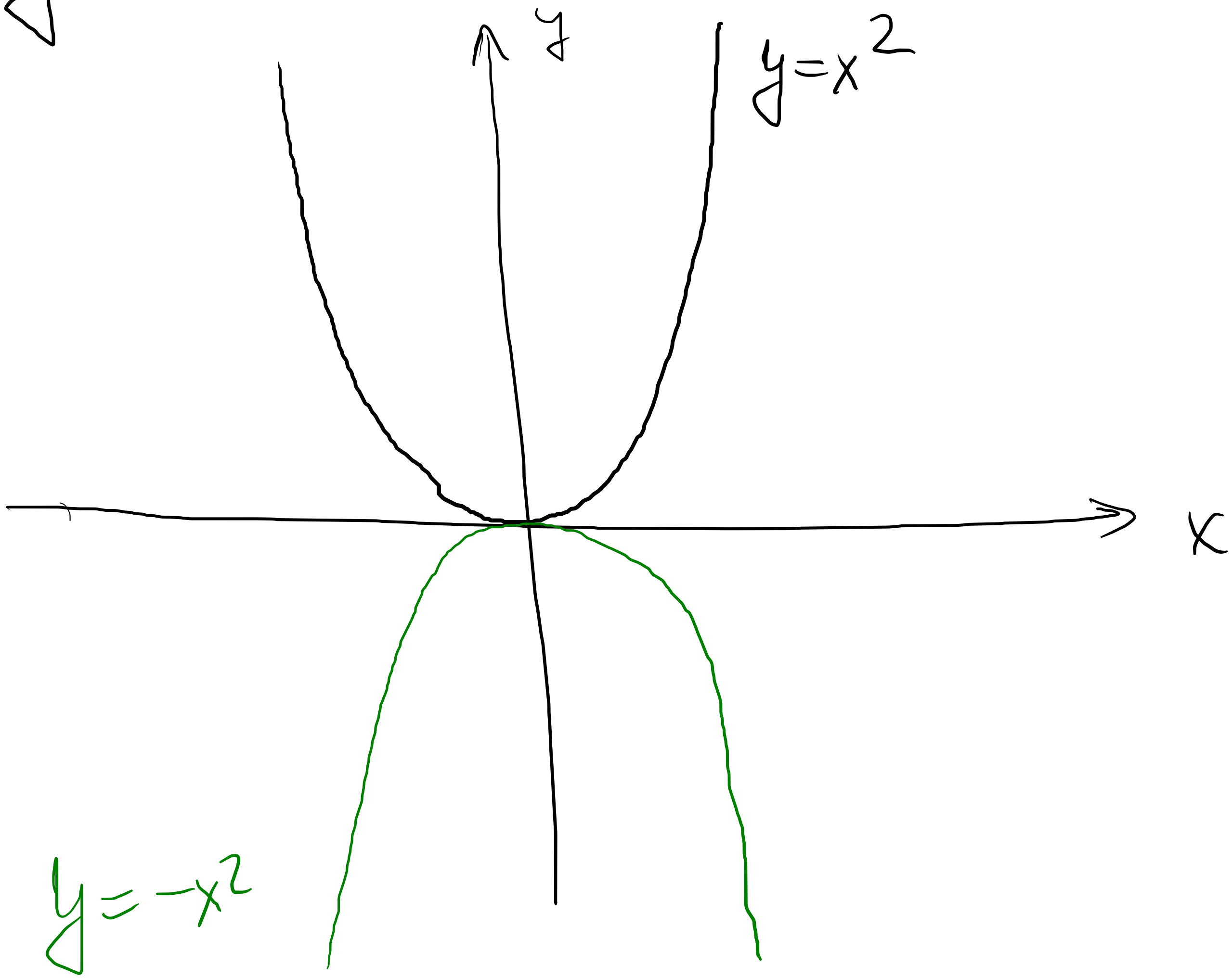
$x < -\frac{b}{2a}$ $\quad y \searrow$

$x > -\frac{b}{2a}$ $\quad y \nearrow$

$x \in (-\infty, x_1) \cup (x_2, +\infty)$ $\quad y>0$

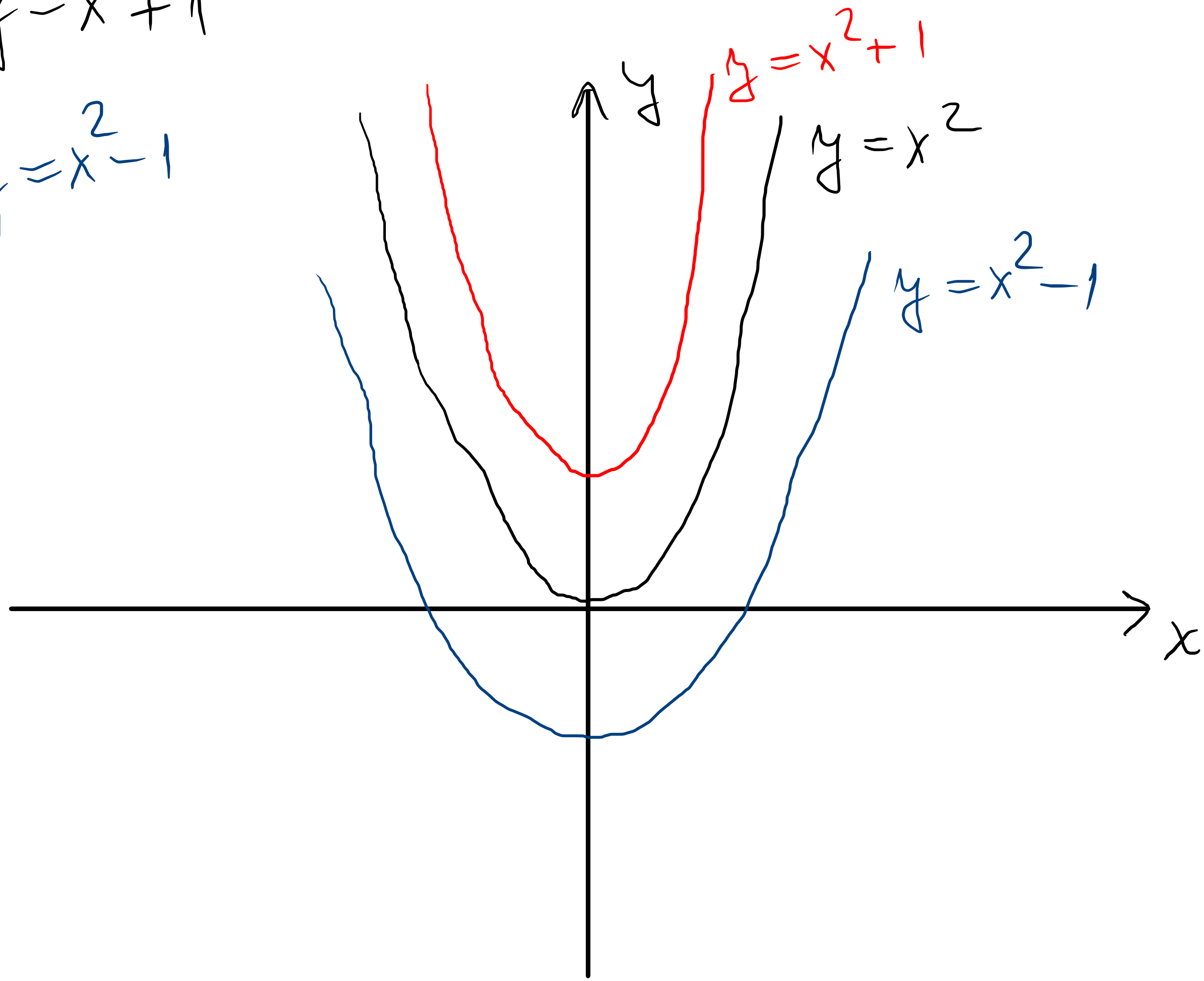
$x \in (x_1, x_2)$ $\quad y<0$

$y = x^2$



$y = x^2 + 1$

$y = x^2 - 1$



$$y = -x^2 + 7x - 12$$

$$x_{1,2} = \frac{-7 \pm \sqrt{49-48}}{-2}$$

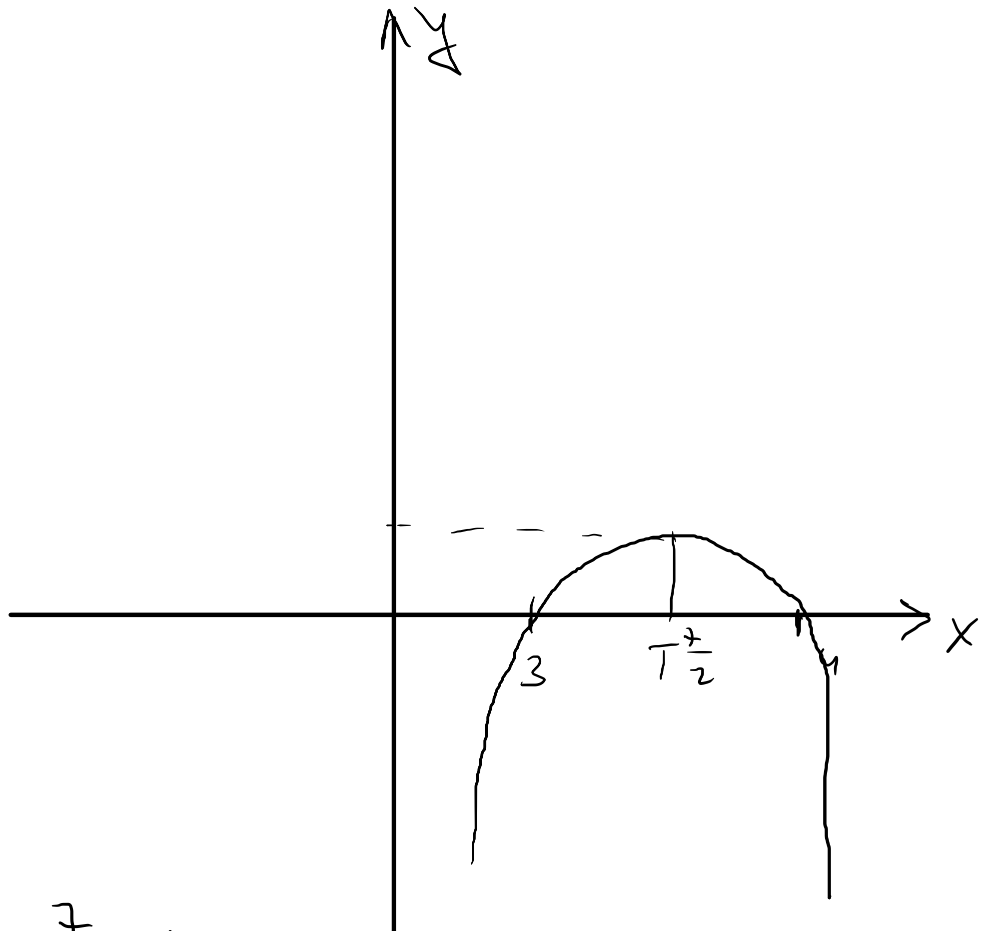
$$= \frac{-7 \pm 1}{-2} = \begin{cases} 4 \\ 3 \end{cases}$$

$$a = -1 < 0$$

$$T\left(-\frac{b}{2a}, \ ?\right)$$

$$-\frac{b}{2a} = -\frac{7}{-2} = \frac{7}{2}$$

$$y\left(\frac{7}{2}\right) = -\left(\frac{7}{2}\right)^2 + 7 \cdot \frac{7}{2} - 12$$



Ирационалне једначине

1) $\sqrt{x+1} = x \quad /^2$

услови: $x+1 \geq 0$

$\boxed{x \geq -1}$

$\boxed{x \geq 0}$

$x+1 = x^2$

$x^2 - x - 1 = 0$

$x_{1,2} = \dfrac{1 \pm \sqrt{1+4}}{2} = \dfrac{1 \pm \sqrt{5}}{2}$

$\boxed{x_1 = \dfrac{1+\sqrt{5}}{2} \geq 0}$ $\quad \cancel{x_2 = \dfrac{1-\sqrt{5}}{2} < 0}$

$2 = \sqrt{4} < \sqrt{5} < \sqrt{9} = 3$

2)

$$\left(\sqrt{x+1}+\sqrt{2x+3}\right)^2=1^2 \quad /^2$$

$$x+1+2\sqrt{(x+1)(2x+3)}+2x+3=1$$

$$3x+3+2\sqrt{(x+1)(2x+3)}=0$$

$$2\sqrt{(x+1)(2x+3)}=-3x-3 \quad /^2$$

$$4(x+1)(2x+3)=9(x+1)^2$$

$$2\cdot 0=0$$

⋮

$$x_{1,2}= \cancel{3},\ -1$$

условия:

$x+1\geq 0$ $\qquad$ $2x+3\geq 0$

$x\geq -1$ $\qquad$ $x\geq -\frac{3}{2}$

$-\frac{3}{2}$ $\quad$ -1

$$\boxed{x\in[-1,+\infty)}$$

услов

$$-3x-3\geq 0$$

$$-3x\geq 3$$

$$\boxed{x\leq -1}$$

услов

$$x=-1$$

Експоненцијалне једначине, неједначине и функција

$$a^x = b, \quad a,b>0, \quad a\neq 1, \quad a,b\in\mathbb{R}$$

$$a^x = a^y \iff x=y$$

$$\boxed{a^x > a^y} \quad (<, \leq, \geq)$$

$a>1: \; x>y$

$0<a<1: \; x<y$

$$2^x > 2^y \overset{2>1}{\Longrightarrow} x>y$$

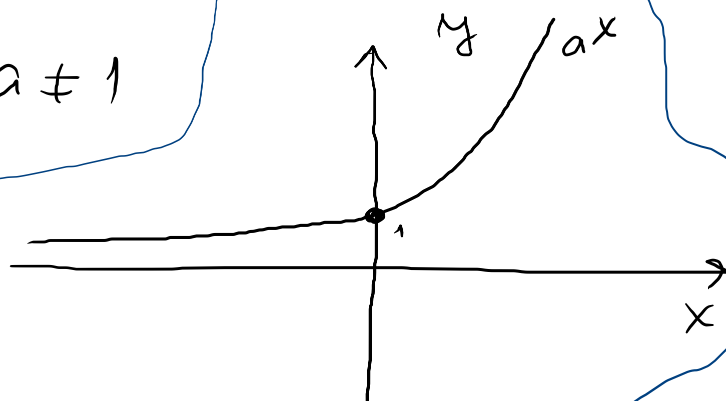
$$\left(\frac{1}{2}\right)^x > \left(\frac{1}{2}\right)^y \overset{\frac{1}{2}<1}{\Longrightarrow} x<y$$

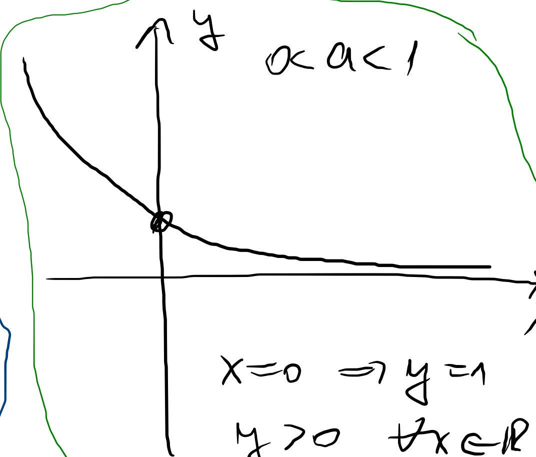
$$y=a^x, \quad a>0, \quad a\neq 1$$

$x=0 \Rightarrow y=1$

$y>0 \quad \forall x\in\mathbb{R}$

$y \nearrow \quad \forall x\in\mathbb{R}$





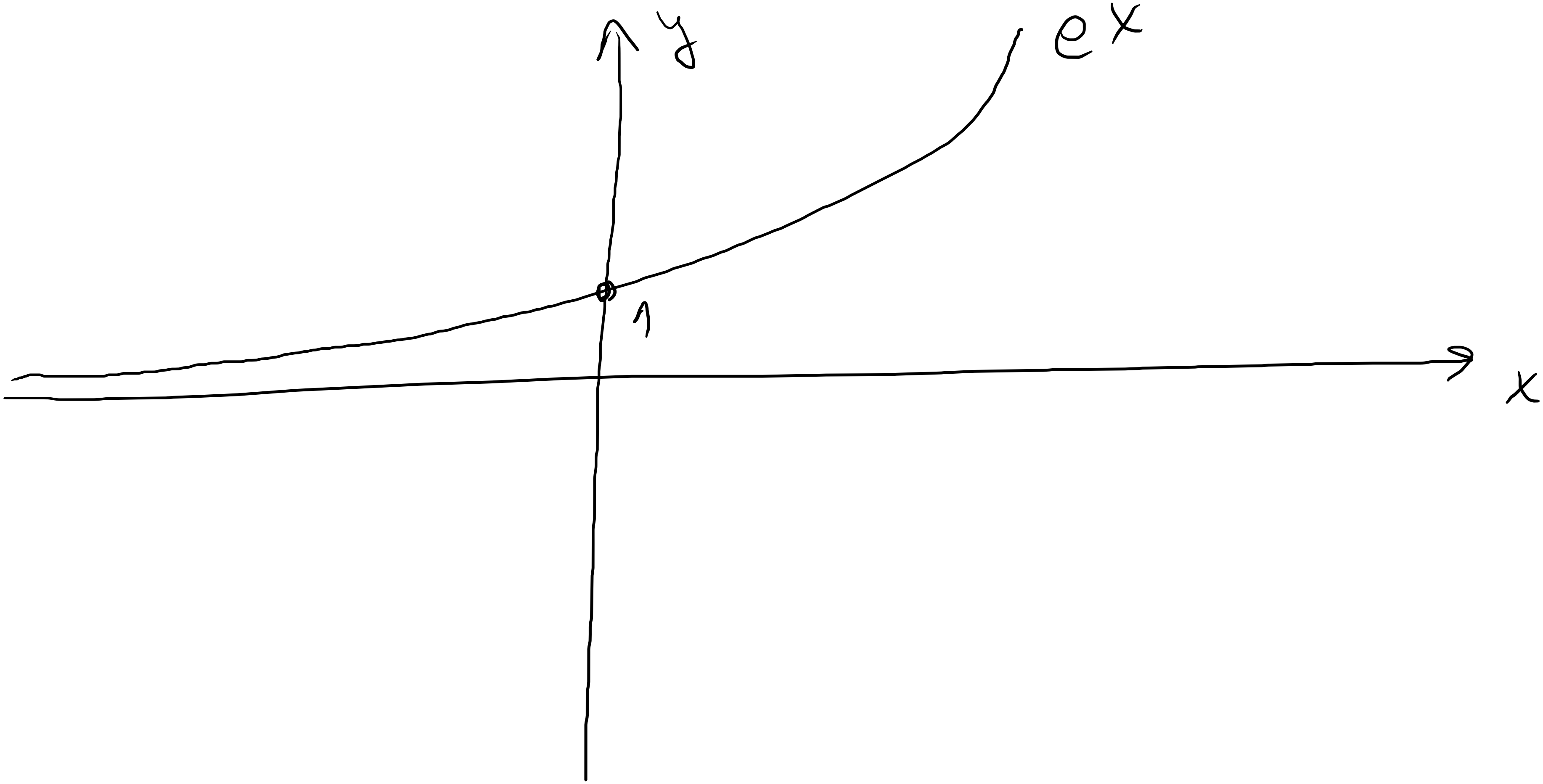
$x=0 \Rightarrow y=1$

$y>0 \quad \forall x\in\mathbb{R}$

$y \searrow \quad \forall x\in\mathbb{R}$

Важно!

$$\boxed{y = e^x}$$



① $(\sqrt{3})^{x^2-x}=27$

$$\left(3^{\frac{1}{2}}\right)^{x^2-x}=3^3$$

$$3^{\frac{x^2-x}{2}}=3^3$$

$$\frac{x^2-x}{2}=3$$

$$x^2-x-6=0$$

$$x_{1,2}=\frac{1\pm\sqrt{1+24}}{2}=\frac{1\pm5}{2}=\begin{cases}3\\-2\end{cases}$$

⑫ $8^x=7^{x-1}+7^x$

$$8^x=7^x\cdot\frac{1}{7}+7^x$$

$$8^x=7^x\,\frac{8}{7}\quad\Big|:7^x\neq0$$

$$\left(\frac{8}{7}\right)^x=\frac{8}{7}$$

$$x=1$$

$$2\cdot2^x\neq4^x$$

$$2\cdot2^x=2^1\cdot2^x=2^{1+x}$$

(3) $4^{x^2+2} - 9 \cdot 2^{x^2+2} + 8 = 0$

$2^{x^2+2} = t$ $\quad$ $2^{2(x^2+2)} - 9 \cdot 2^{x^2+2} + 8 = 0$

$t > 0$ $\quad$ $t^2 - 9t + 8 = 0$

$$t_{1,2} = \frac{9 \pm \sqrt{81-32}}{2} = \frac{9 \pm 7}{2} = \begin{cases} 1 \\ 8 \end{cases}$$

$t_1 = 1 \Rightarrow$ $\quad$ $2^{x^2+2} = 1$

$2^{x^2+2} = 2^0$

$\cancel{x^2+2=0}$

$t_2 = 8$ $\quad$ $2^{x^2+2} = 8$

$2^{x^2+2} = 2^3$

$x^2+2 = 3$

$x^2 = 1$

$x_1 = 1 \qquad x_2 = -1$

4

$$\left(\frac{1}{2}\right)^{2x+3} > \frac{1}{32}$$

$$\left(\frac{1}{2}\right)^{2x+3} > \left(\frac{1}{2}\right)^{5}$$

$\frac{1}{2} < 1$

$$2x+3 < 5$$

$$2x < 2$$

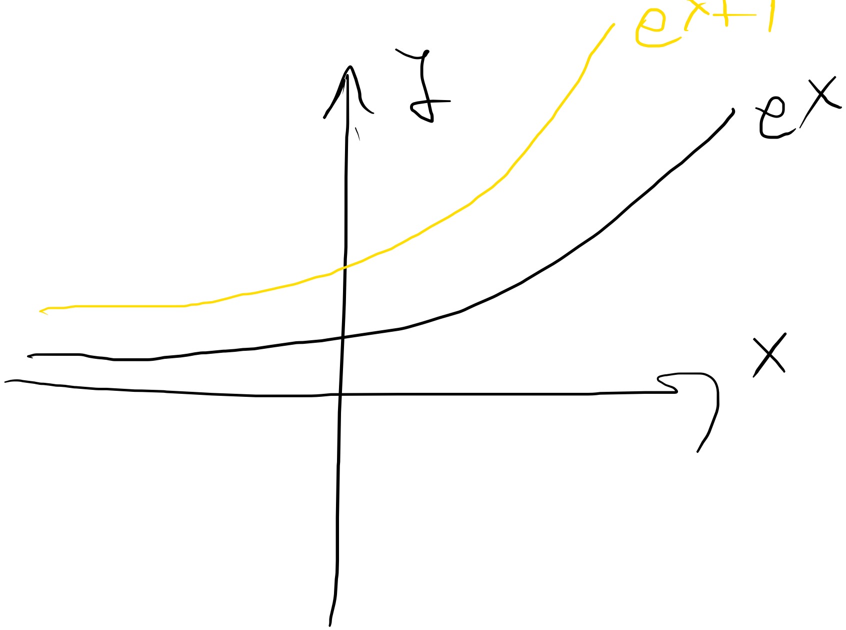
$$x < 1$$

$$2^{-2x-3} > 2^{-5}$$

$$-2x-3 > -5$$

$$-2x > -2$$

$$x < 1$$



Логаритамске једначине, неједначине и функције

$$\boxed{x=\log_a b \iff a^x=b}$$, $a>0,\ a\neq 1,\ b>0$

Особине:

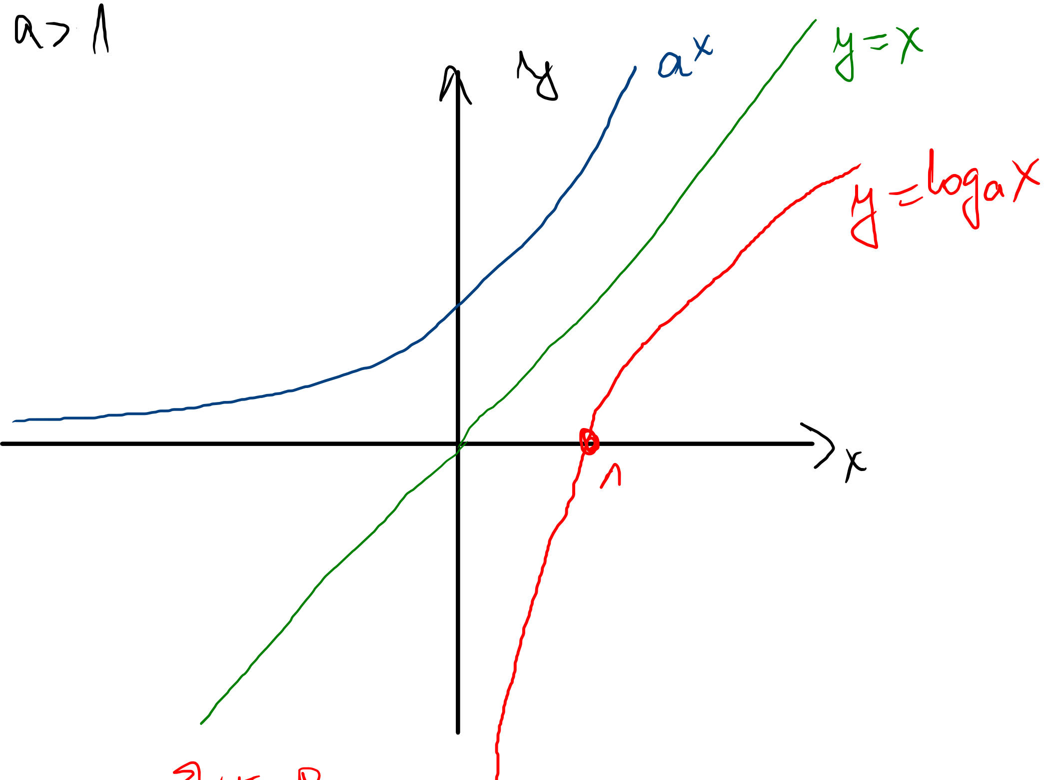
1. $\log_a a^x = x$
2. $a^{\log_a x} = x$
3. $\log_a x = \log_a y \iff x=y$
4. $\log_a (xy) = \log_a x + \log_a y$
5. $\log_a \frac{x}{y} = \log_a x - \log_a y$
6. $\log_a x^n = n\log_a x$, $n\in\mathbb{R}$
7. $\log_a 1 = 0$
8. $\log_a a = 1$
9. $\log_{a^k} x = \frac{1}{k}\log_a x$

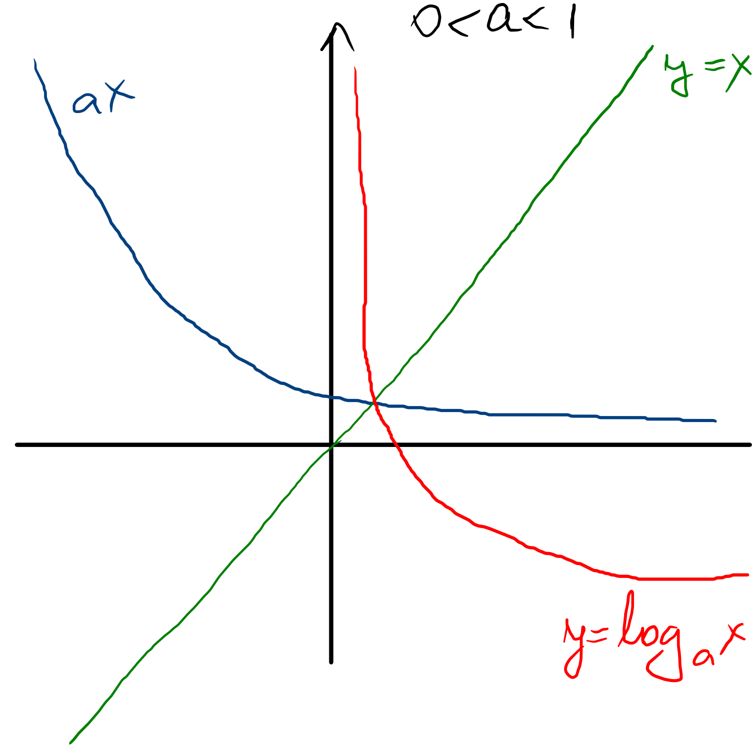
$\log_a x > \log_a y$ $(<, \geq, \leq)$

$\boxed{a>1}$: $x>y$

$\boxed{a<1}$: $x<y$

$y = \log_a x \quad , \quad a > 0 \; , \; a \neq 1 \; , \; x > 0$





$y \nearrow \quad \forall x \in \mathbb{R}$

$x \in (0,1) \quad y < 0$

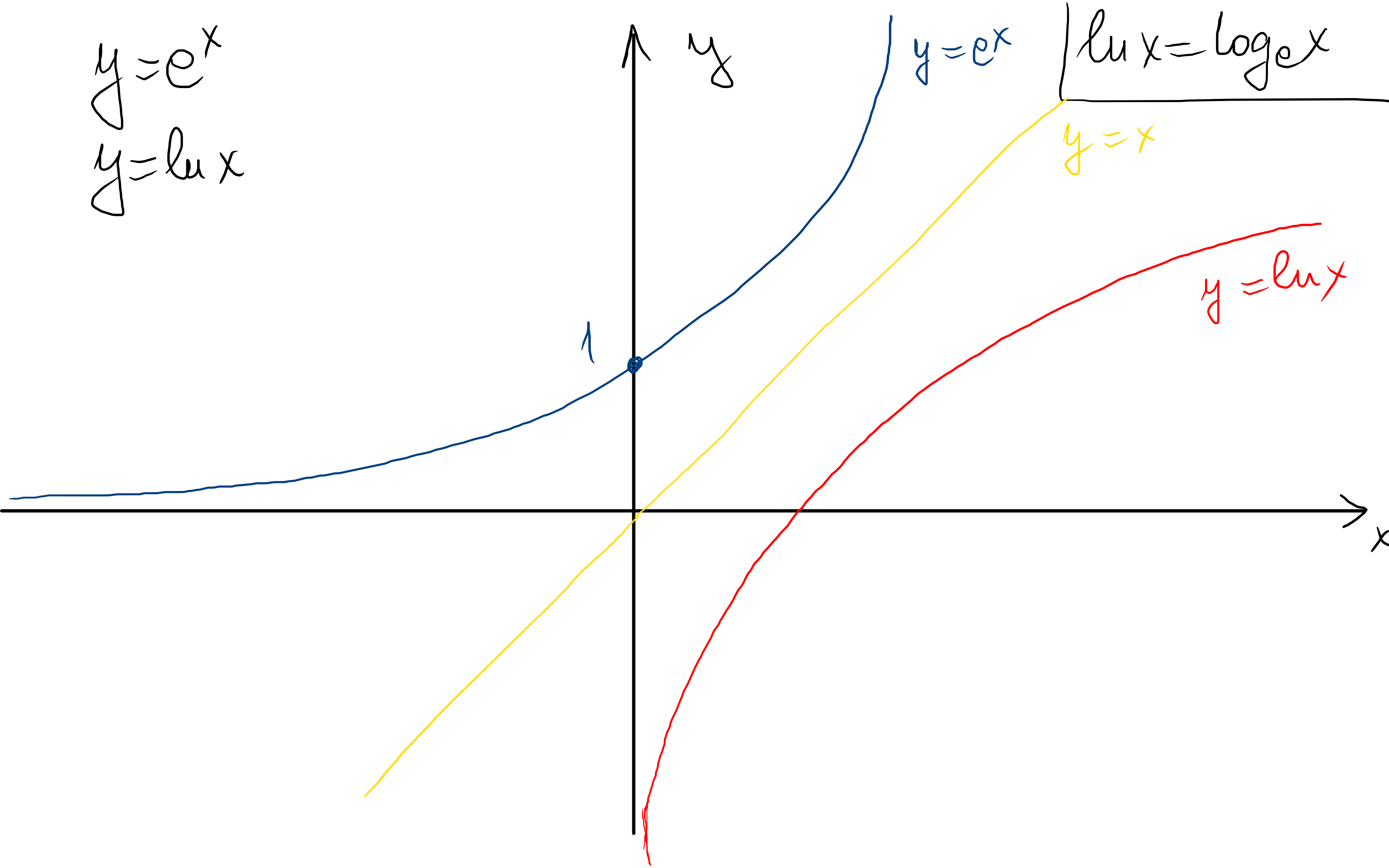
$y = 0 \quad x = 1$

$x \in (1, +\infty) \quad y > 0$

$y = e^x$

$y = \ln x$

$\ln x = \log_e x$



① $\log_5 x + \log_{25} x = \log_{0,2} \sqrt{3}$ умов: $x > 0$

$$\log_5 x + \log_{5^2} x = \log_{5^{-1}} 3^{\frac{1}{2}}$$

$$\underbrace{\log_5 x} + \frac{1}{2}\underbrace{\log_5 x} = -\frac{1}{2}\log_5 3$$

$$\frac{3}{\cancel{2}}\log_5 x = -\frac{1}{\cancel{2}}\log_5 3$$

$$\log_5 x^3 = \log_5 3^{-1}$$

$$x^3 = 3^{-1}$$

$$x^3 = \frac{1}{3}$$

$$x = \frac{1}{\sqrt[3]{3}}$$

2

условие:

$1-x>0$

$\boxed{x<1}$

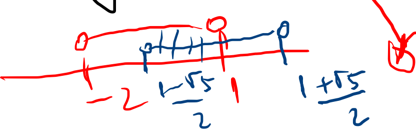
$x+2>0$

$\boxed{x>-2}$

$3>1$

$\boxed{x\in(-2,1)}$

условие



$x\in\left(\frac{1-\sqrt5}{2},1\right)$

$$\log_3(1-x)<\log_{\frac13}(x+2)$$

$$\log_3(1-x)<\log_{3^{-1}}(x+2)$$

$$\log_3(1-x)<-\log_3(x+2)$$

$$\log_3(1-x)<\log_3(x+2)^{-1}$$

$$1-x<(x+2)^{-1}$$

$$1-x<\frac{1}{x+2}$$

$$1-x-\frac{1}{x+2}<0$$

$$\frac{x+2-x^2-2x-1}{x+2}<0$$

$$\frac{-x^2-x+1}{x+2}<0$$

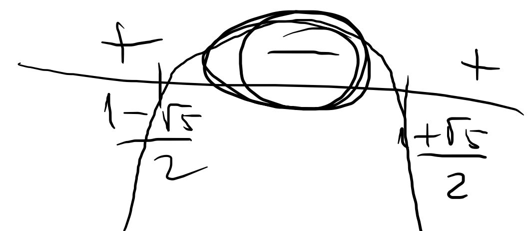
$$x_{1,2}=\frac{1\pm\sqrt{1+4}}{2}=\frac{1\pm\sqrt5}{2}$$

из условия до же

$x>-2 \Rightarrow x+2>0$

$x+2>-2+2=0$

$$-x^2-x+1<0$$



$$\boxed{x\in\left(\frac{1-\sqrt5}{2},\frac{1+\sqrt5}{2}\right)}$$

③ $f(x) = \sqrt{\log_{10} \dfrac{x^2-2x+3}{x+1}}$ Домен?

$\log_a b = x \Leftrightarrow a^x = b$

$\log_{10} \dfrac{x^2-2x+3}{x+1} \geq 0$ и $\dfrac{x^2-2x+3}{x+1} > 0$ и $x+1 \neq 0$

$x \neq -1$

$\dfrac{x^2-2x+3}{x+1} \geq 10^0$

$\dfrac{x^2-2x+3}{x+1} \geq 1$

$\dfrac{x^2-2x+3}{x+1} - 1 \geq 0$

$\dfrac{x^2-2x+3-x-1}{x+1} \geq 0$

$\dfrac{x^2-3x+2}{x+1} \geq 0$

$x_{1,2} = \dfrac{3 \pm \sqrt{9-8}}{2} = \dfrac{3 \pm 1}{2} = \begin{cases} 2 \\ 1 \end{cases}$

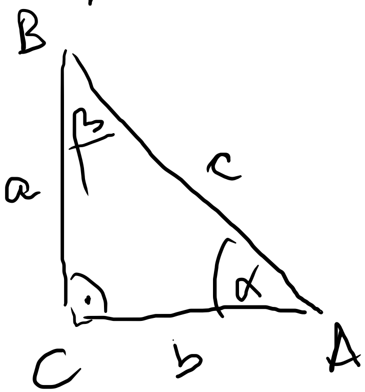
$\dfrac{(x-1)(x-2)}{x+1} \geq 0$

	-1		1		2	
$x-1$	$-$		$-$	0	$+$	$+$
$x-2$	$-$		$-$		$- \;0$	$+$
$x+1$	$-$	0	$+$		$+$	$+$
	$-$		$+$		$-$	$+$

$x \in (-1, 1] \cup [2, +\infty)$

ТРИГОНОМЕТРИЈА

1, Тригонометрија правоуглог троугла



$$c^2=a^2+b^2$$

$$\alpha+\beta=\frac{\pi}{2} \Rightarrow \beta=\frac{\pi}{2}-\alpha$$

$$\boxed{\sin\alpha}=\frac{\text{наспрамна катета}}{\text{хипотенуза}}=\frac{a}{c}=\cos\beta=\cos\left(\frac{\pi}{2}-\alpha\right)$$

$$\boxed{\cos\alpha}=\frac{\text{налегла катета}}{\text{хипотенуза}}=\frac{b}{c}=\sin\beta=\sin\left(\frac{\pi}{2}-\alpha\right)$$

$$\boxed{\operatorname{tg}\alpha}=\frac{\text{наспрамна катета}}{\text{налегла катета}}=\frac{a}{b}=\operatorname{ctg}\beta=\operatorname{ctg}\left(\frac{\pi}{2}-\alpha\right)$$

$$\boxed{\operatorname{ctg}\alpha}=\frac{\text{налегла катета}}{\text{наспрамна катета}}=\frac{b}{a}=\operatorname{tg}\beta=\operatorname{tg}\left(\frac{\pi}{2}-\alpha\right)$$

2) Основне релације између тригонометријских функција.

1) $\sin^2\alpha + \cos^2\alpha = 1$

2) $\operatorname{tg}\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

3) $\operatorname{ctg}\alpha = \dfrac{\cos\alpha}{\sin\alpha}$

4) $\operatorname{tg}\alpha \cdot \operatorname{ctg}\alpha = 1$

5) $-1 \le \sin\alpha \le 1$

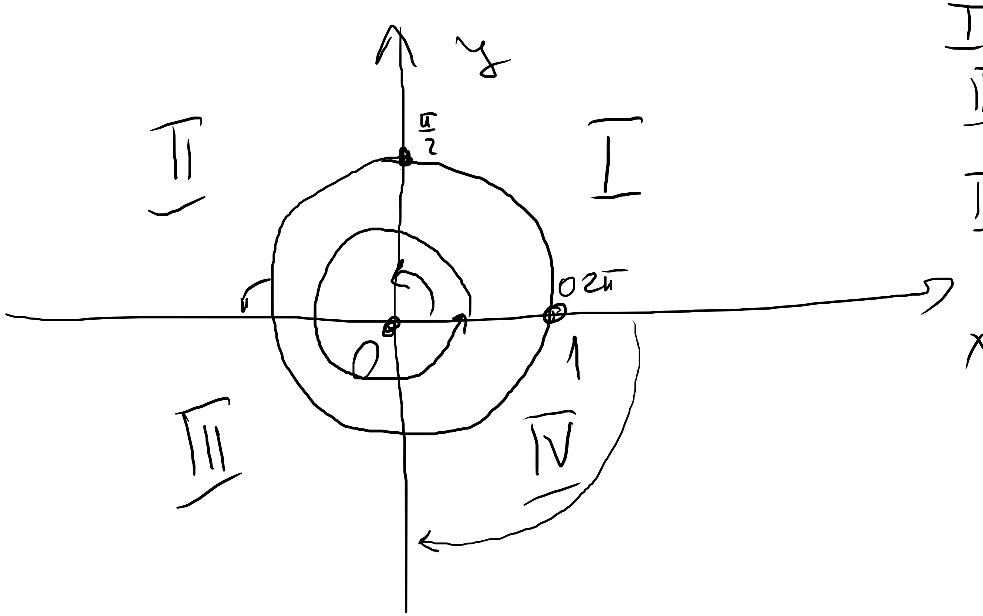
6) $-1 \le \cos\alpha \le 1$

3. Тригонометријски круг

$360^\circ = 2\pi$

$180^\circ = \pi$

$\frac{\pi}{6} = 30^\circ$

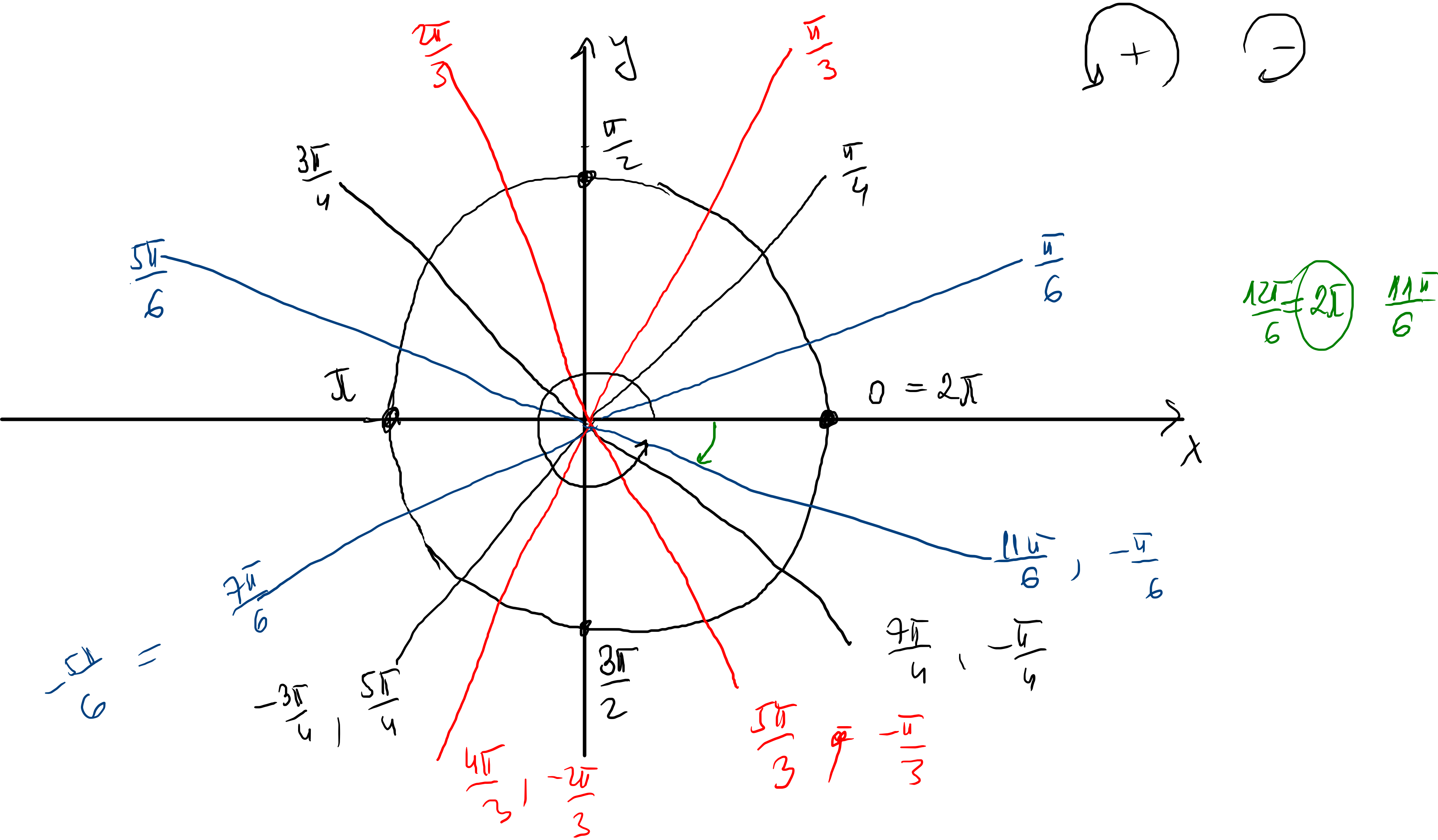


I квадрант $0 < \alpha < \frac{\pi}{2}$

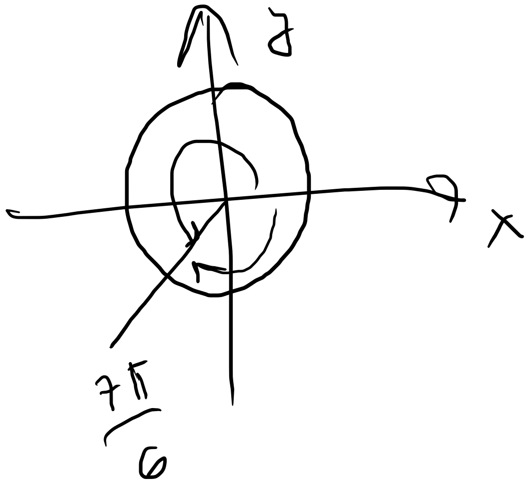
II квадрант $\frac{\pi}{2} < \alpha < \pi$

III квадрант $\pi < \alpha < \frac{3\pi}{2}$

IV квадрант $\frac{3\pi}{2} < \alpha < 2\pi$

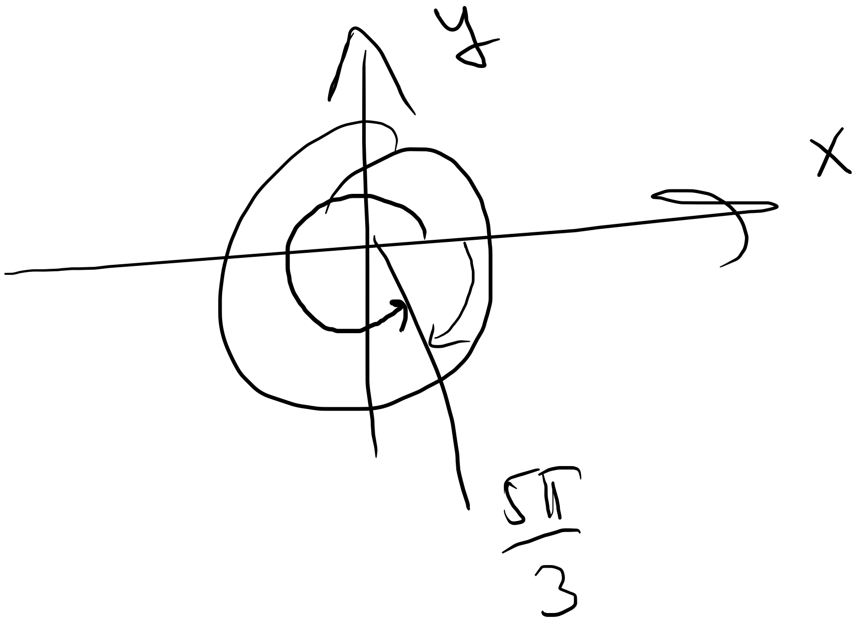


y
x
+
−
π/2
π/3
2π/3
π/4
3π/4
π/6
5π/6
π
0 = 2π
7π/6
11π/6 , −π/6
7π/4 , −π/4
3π/2
−3π/4 , 5π/4
4π/3 , −2π/3
5π/3 = −π/3
−5π/6 =
12π/6 = 2π
11π/6



$$2\pi = \frac{12\pi}{6}$$

$$\frac{7\pi}{6} \qquad -\frac{5\pi}{6}$$

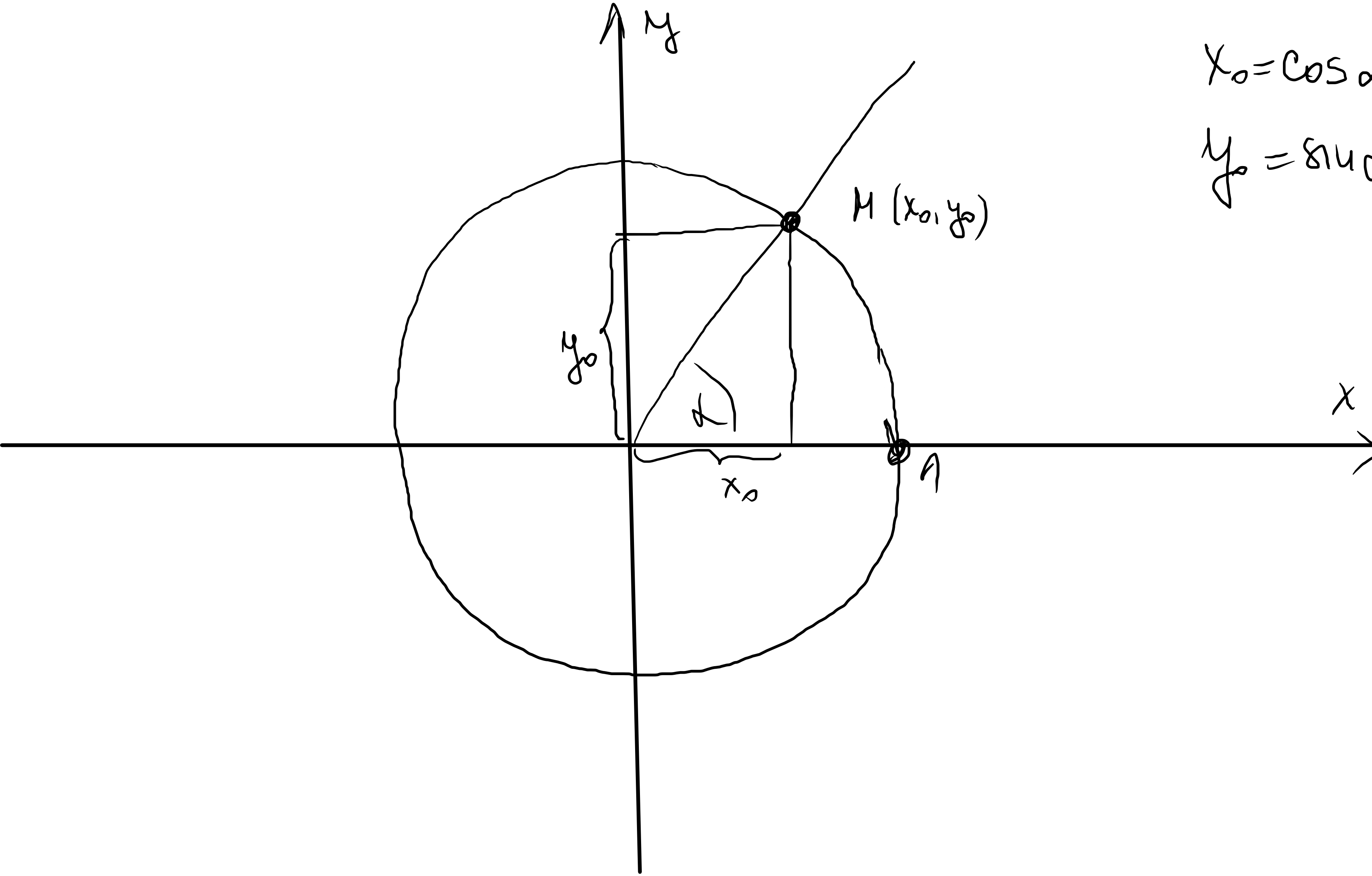


$$2\pi = \frac{6\pi}{3}$$

$$\frac{5\pi}{3} \qquad -\frac{\pi}{3}$$

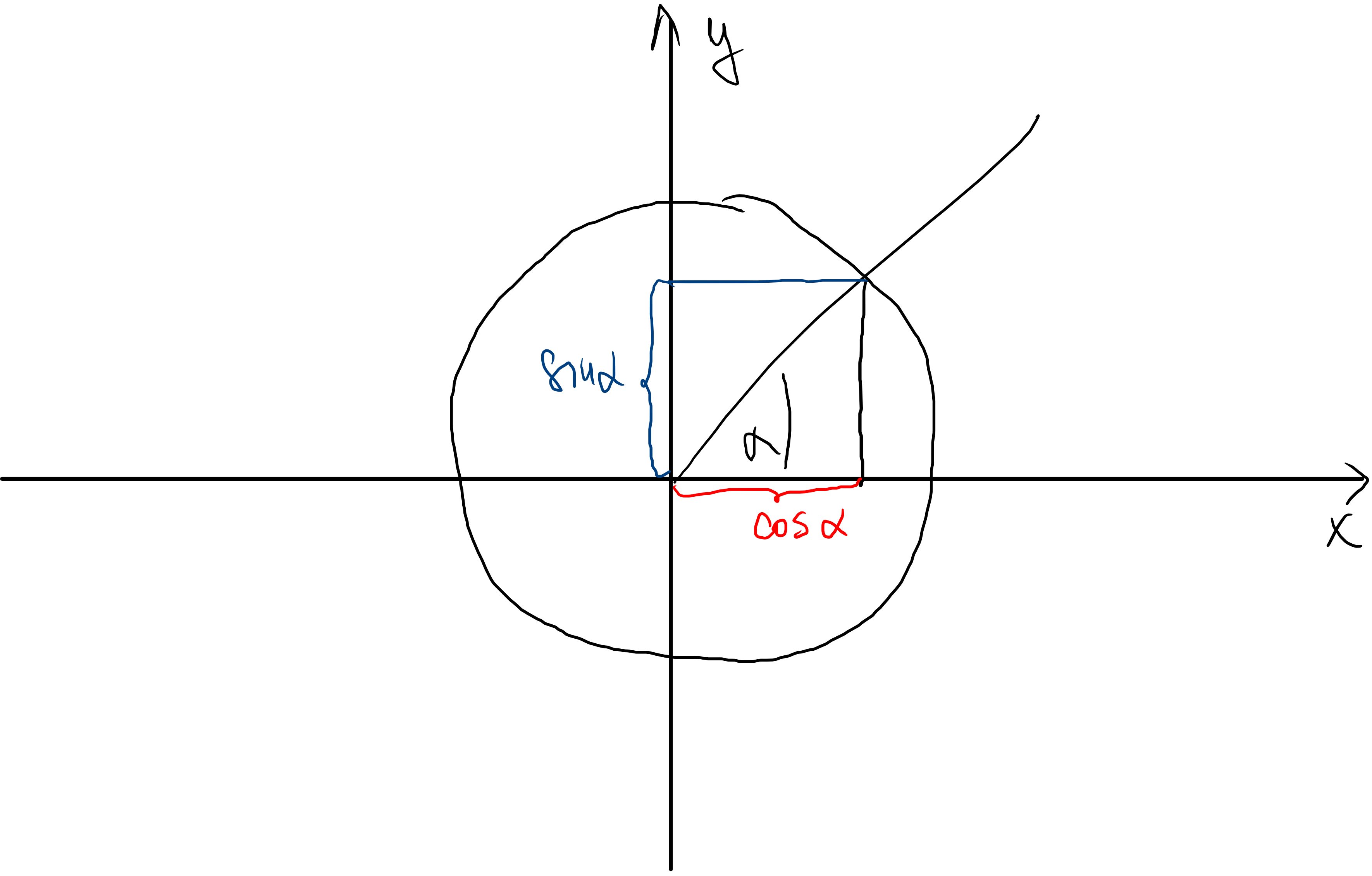
$$\frac{11\pi}{17} \qquad -\frac{23\pi}{17}$$

$$2\pi = \frac{34\pi}{17}$$

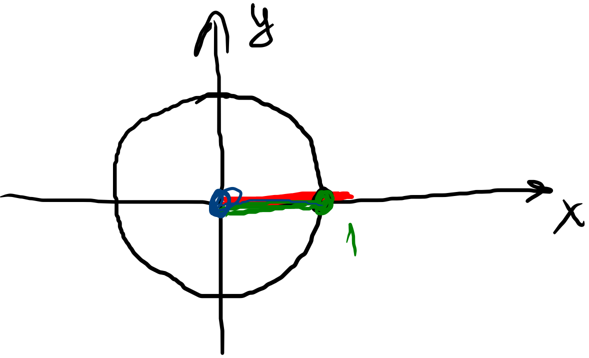


$x_0 = \cos\alpha$

$y_0 = \sin\alpha$

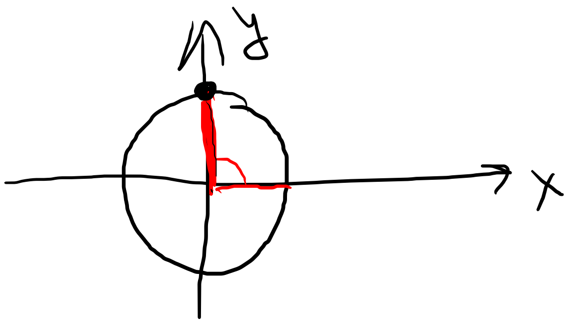


y
sin α
α
cos α
x



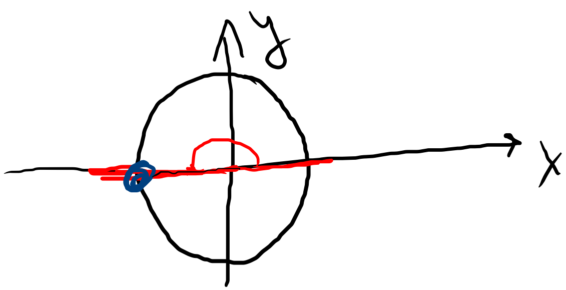
$\sin 0 = 0$

$\cos 0 = 1$



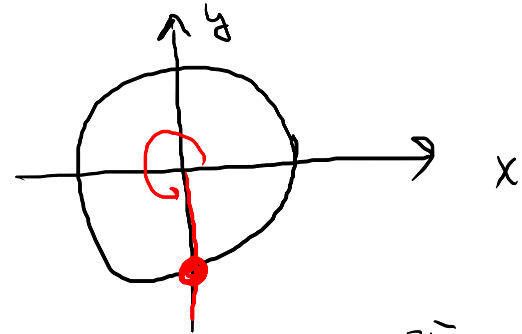
$\sin \frac{\pi}{2} = 1$

$\cos \frac{\pi}{2} = 0$



$\sin \pi = 0$

$\cos \pi = -1$



$\sin \frac{3\pi}{2} = -1$

$\cos \frac{3\pi}{2} = 0$

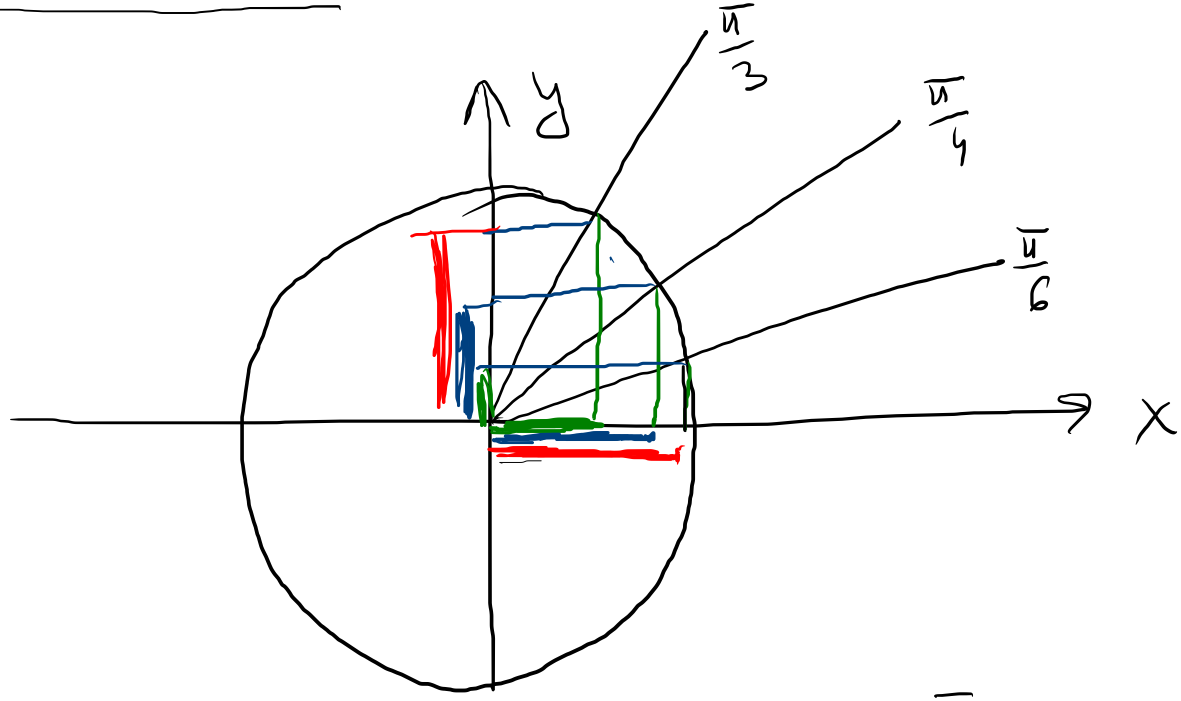
$$\frac{1}{2}, \quad \frac{\sqrt{2}}{2}, \quad \frac{\sqrt{3}}{2}$$

$$\frac{\pi}{6}, \quad \frac{\pi}{4}, \quad \frac{\pi}{3}$$

$$\sin\frac{\pi}{6} = \frac{1}{2}$$

$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

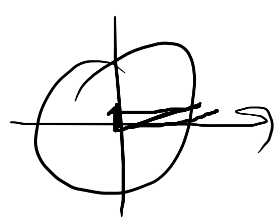


$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

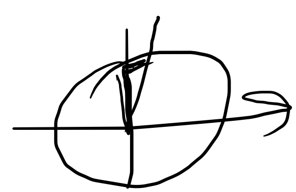
$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos\frac{\pi}{3} = \frac{1}{2}$$

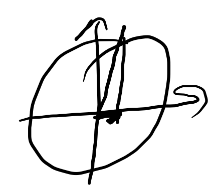
$$\sin\frac{\pi}{6}$$



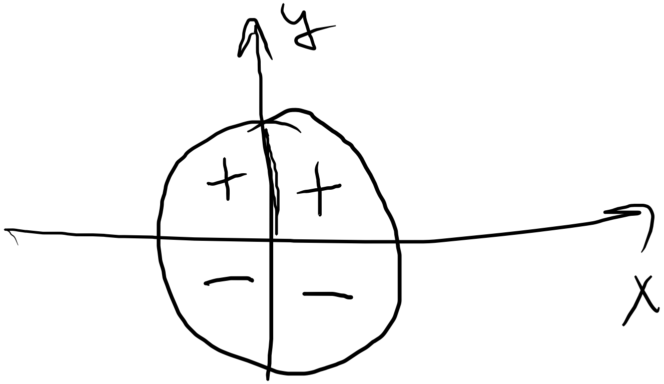
$$\sin\frac{\pi}{3}$$



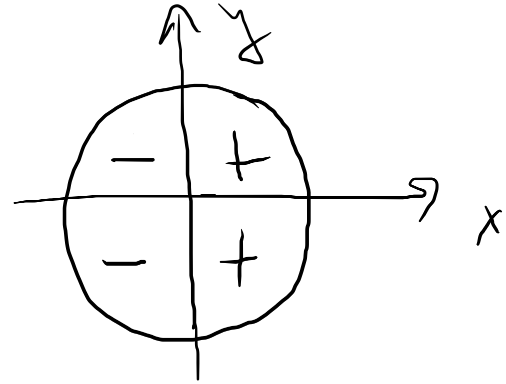
$$\cos\frac{\pi}{3}$$



sin α



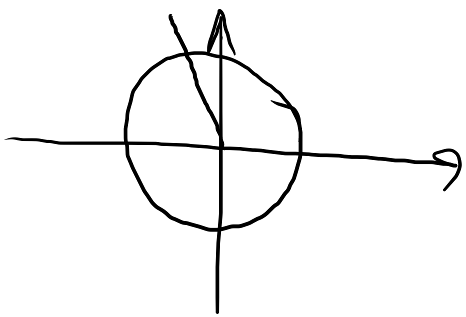
cos α

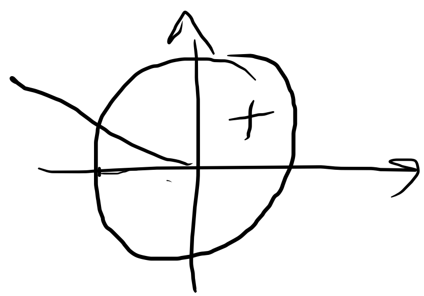


$$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\pi = \frac{6\pi}{6}$$

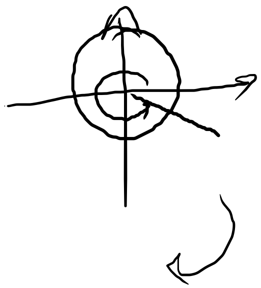




$$\sin\frac{11\pi}{6} = -\sin\frac{\pi}{6} = -\frac{1}{2}$$

$$\sin(-\alpha) = -\sin\alpha$$

$$\cos(-\alpha) = \cos\alpha$$



$$\sin\frac{11\pi}{6} = \sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6}$$

$$2\pi = \frac{12\pi}{6}$$

$$\pi = \frac{6\pi}{6}$$

$$\cos\frac{7\pi}{6} = \cos\left(-\frac{5\pi}{6}\right) = \cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$2\pi = \frac{6\pi}{3}$$

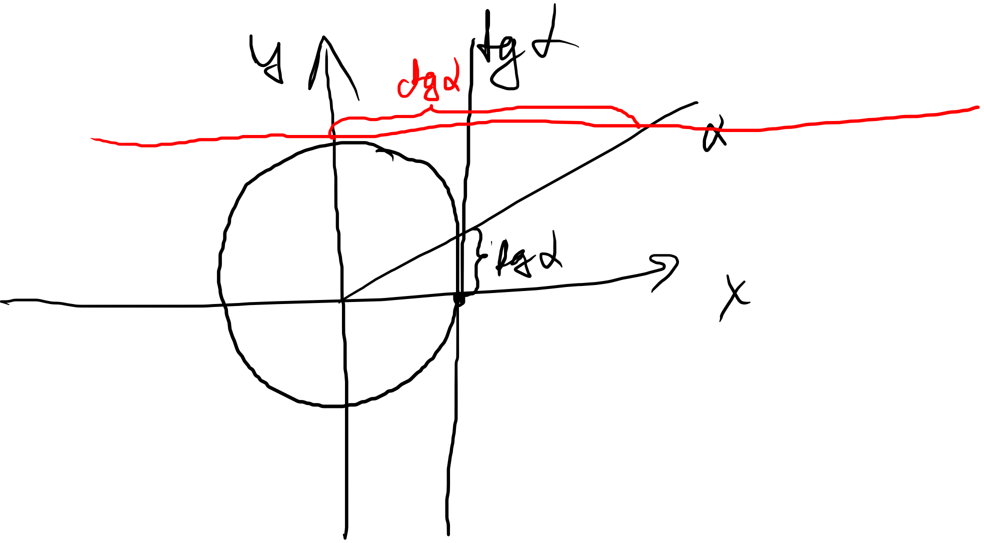
$$\sin\frac{5\pi}{3} = \sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\operatorname{tg}\frac{\pi}{6}=\frac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}}=\frac{\frac{1}{\cancel{2}}}{\frac{\sqrt{3}}{\cancel{2}}}=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

$$\operatorname{tg}\frac{\pi}{3}=\frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$$

$$\operatorname{tg}\frac{\pi}{4}=\frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}}=\frac{\frac{\cancel{\sqrt{2}}}{\cancel{2}}}{\frac{\cancel{\sqrt{2}}}{\cancel{2}}}=1$$

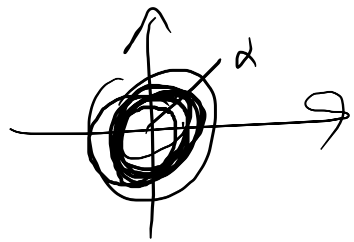
$$\operatorname{tg}\frac{5\pi}{6}=\frac{\sin\frac{5\pi}{6}}{\cos\frac{5\pi}{6}}=\frac{\sin\frac{\pi}{6}}{-\cos\frac{\pi}{6}}=\frac{\frac{1}{\cancel{2}}}{-\frac{\sqrt{3}}{\cancel{2}}}=-\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$$



$$\frac{\sqrt{3}}{3},\ 1,\ \sqrt{3}$$

$$\uparrow$$

$$\frac{\pi}{6},\ \frac{\pi}{4},\ \frac{\pi}{3}$$



$$\sin\left(\frac{\pi}{6}+16\pi\right)=\sin\frac{\pi}{6}$$

$$\sin(\alpha+2k\pi)=\sin\alpha$$

$$\cos(\alpha+2k\pi)=\cos\alpha$$

$$\operatorname{tg}(\alpha+k\pi)=\operatorname{tg}\alpha$$

$$\operatorname{ctg}(\alpha+k\pi)=\operatorname{ctg}\alpha$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

①

$$\sin\frac{\pi}{4}\cos\frac{\pi}{4}+\cos^2\frac{\pi}{6}=\frac{\sqrt2}{2}\cdot\frac{\sqrt2}{2}+\left(\frac{\sqrt3}{2}\right)^2=\frac{1}{2}+\frac{3}{4}=\frac{5}{4}$$

$$\cos^2\frac{\pi}{6} = \left(\cos\frac{\pi}{6}\right)^2$$

②

$$\frac{3\,tg^2\frac{\pi}{6}+ctg^2\frac{\pi}{6}}{\sin^2\frac{\pi}{3}-\cos^2\frac{\pi}{4}}=\frac{3\left(\frac{\sqrt3}{3}\right)^2+\left(\sqrt3\right)^2}{\left(\frac{\sqrt3}{2}\right)^2-\left(\frac{\sqrt2}{2}\right)^2}=\frac{\cancel{3}\,\frac{3}{\cancel{9}}+3}{\frac{3}{4}-\frac{2}{4}}=\frac{4}{\frac{1}{4}}=16$$

$$tg\frac{\pi}{6}=\frac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}}=\frac{\frac{1}{\cancel{2}}}{\frac{\sqrt3}{\cancel{2}}}=\frac{1}{\sqrt3}=\frac{\sqrt3}{3}$$

③ $$\left(1-\sin\frac{\pi}{8}\right)\left(1+\sin\frac{\pi}{8}\right) = 1-\sin^2\frac{\pi}{8} = 1-\sin^2\frac{\frac{\pi}{4}}{2} = \qquad \alpha$$

$$= 1 - \frac{1-\cos\frac{\pi}{4}}{2} = \frac{1}{2}+\frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{1}{2}+\frac{\sqrt{2}}{4}$$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

④ $$\operatorname{tg}\frac{544\pi}{3} = \operatorname{tg}\left(181+\frac{1}{3}\right)\pi = \operatorname{tg}\left(\underbrace{\cancel{181}}_{k}\pi+\frac{\pi}{3}\right) = \operatorname{tg}\frac{\pi}{3} = \sqrt{3}$$

$544 : 3 = 181$

24

4

1

$$\frac{544}{3} = 181+\frac{1}{3}$$

(5)

$$\cos\frac{29\pi}{4} = \cos\left(7+\frac{1}{4}\right)\pi = \cos\left(7\pi + \frac{\pi}{4}\right) = \cos\left(\cancel{6\pi} + \underbrace{\pi + \frac{\pi}{4}}\right)$$

Чет

$29 : 4 = 7$
1

$$\frac{29}{4} = 7 + \frac{1}{4}$$

$$= \cos\frac{5\pi}{4} = \cos\left(-\frac{3\pi}{4}\right)$$

$$= \cos\frac{3\pi}{4} = -\cos\frac{\pi}{4}$$

$$2\pi = \frac{8\pi}{4}$$

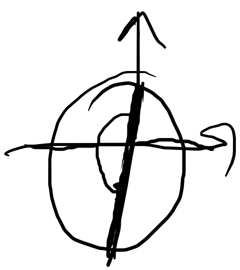
5 2

$$= -\frac{\sqrt{2}}{2}$$

$\frac{1}{2}$, $\frac{\sqrt{2}}{2}$, $\frac{\sqrt{3}}{2}$

(6)

$$\cos 2x = 0$$



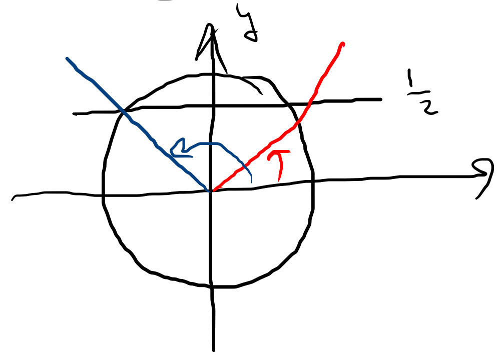
$\frac{\pi}{2}$, $\frac{3\pi}{2}$

$$2x = \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}$$

$$x = \frac{\pi}{4} + \frac{k\pi}{2}$$

(7)

$$\sin 2x = \frac{1}{2}$$



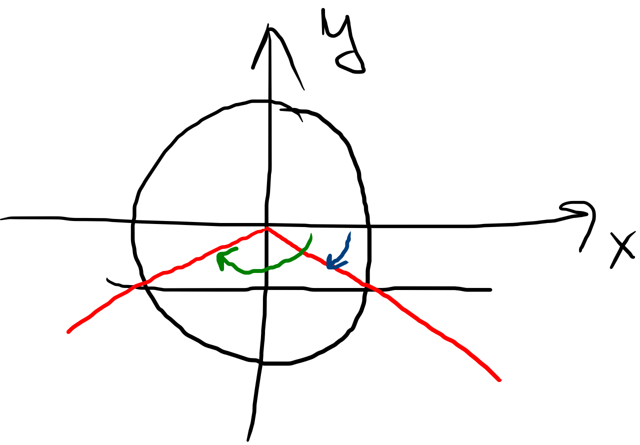
$$2x = \frac{\pi}{6} + 2k\pi$$

$$2x = \frac{5\pi}{6} + 2k\pi$$

$$x = \frac{\pi}{12} + k\pi$$

$$x = \frac{5\pi}{12} + k\pi$$

8) $\sin\alpha = -\frac{1}{2}$

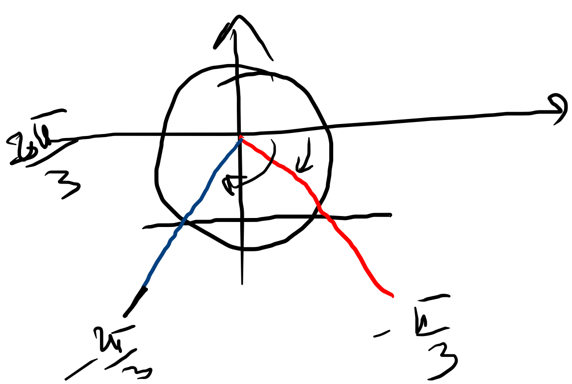


$\alpha = -\frac{\pi}{6} + 2k\pi$

$\alpha = -\frac{5\pi}{6} + 2k\pi$

9) $2\sin x + \sqrt{3} = 0$

$\sin x = -\frac{\sqrt{3}}{2}$

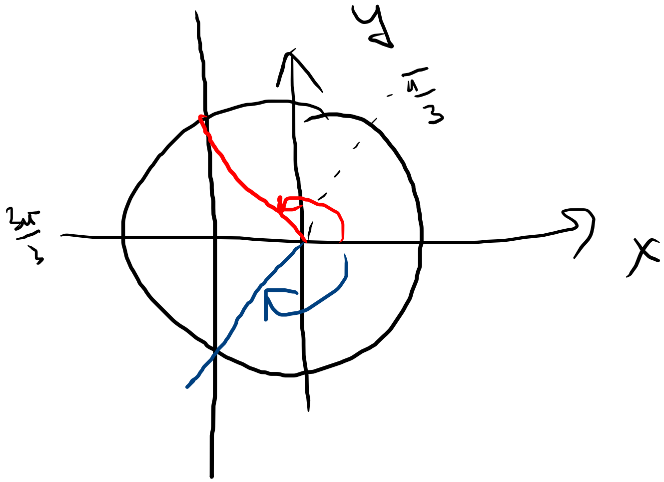


$x = -\frac{\pi}{3} + 2k\pi$

$x = -\frac{2\pi}{3} + 2k\pi$

10

$$\cos x = -\frac{1}{2}$$



$$x = \frac{2\pi}{3} + 2k\pi$$

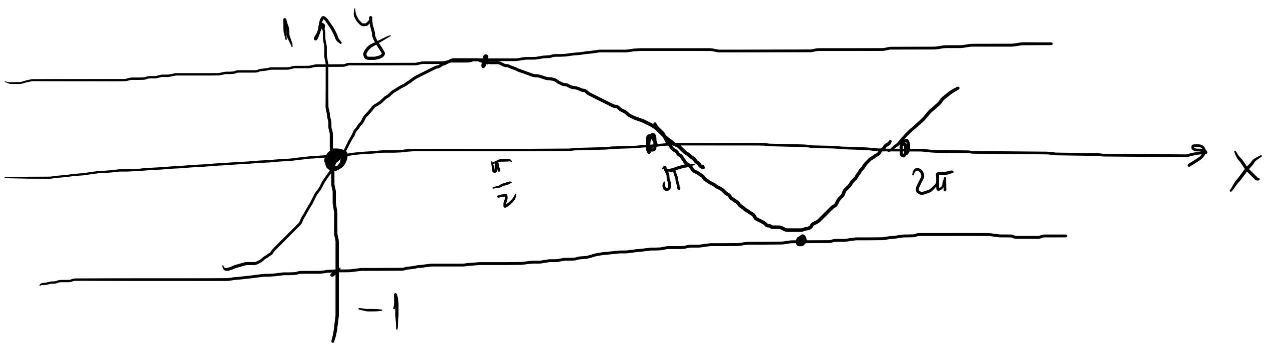
$$x = -\frac{2\pi}{3} + 2k\pi$$

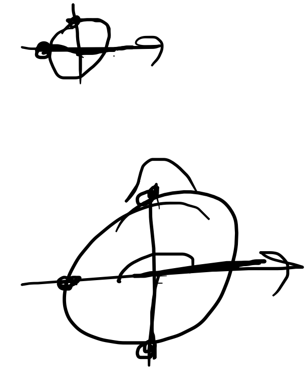
$$\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{1}{2}$$

$$\cos\frac{7\pi}{6} = \cos\left(-\frac{5\pi}{6}\right) = \cos\frac{5\pi}{6} =$$

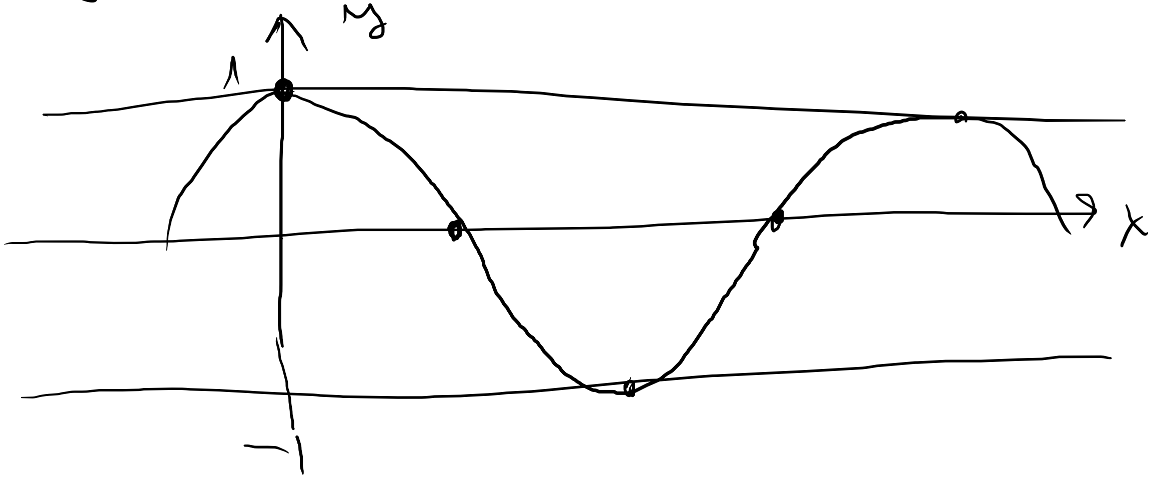
$$= -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$y = \sin x$, $-1 \le \sin x \le 1$, $x \in \mathbb{R}$, период 2π





$y = \cos x$ $-1 \le \cos x \le 1$, $x \in \mathbb{R}$, период 2π



$\sin 0 = 0$

$\cos 0 = 1$

$\sin \frac{\pi}{2} = 1$

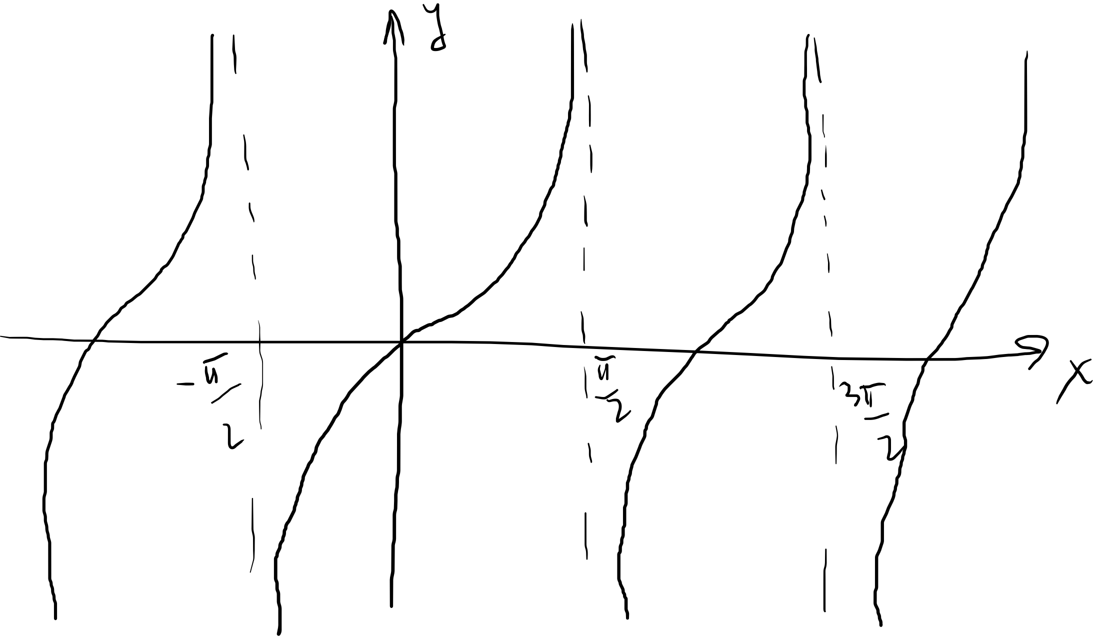
$\cos \frac{\pi}{2} = 0$

$\sin \pi = 0$

$\cos \pi = -1$

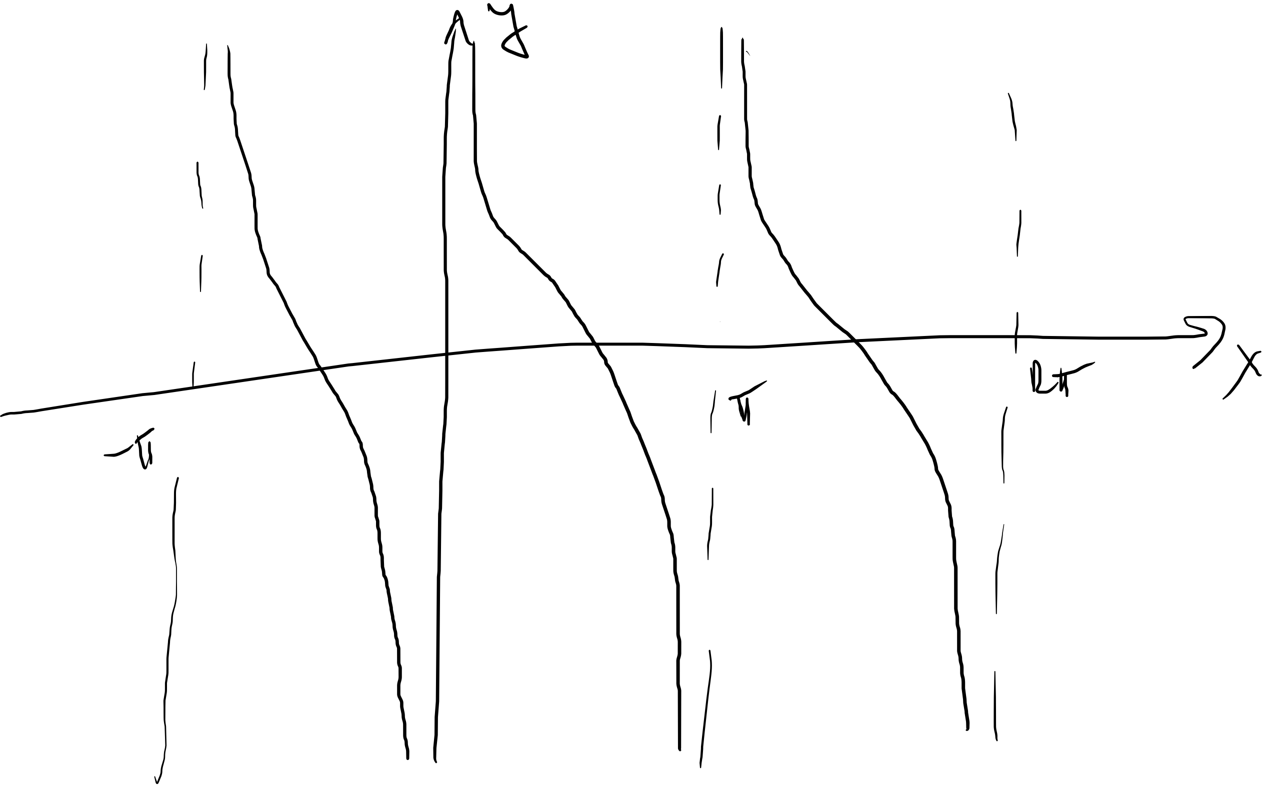
$\sin \frac{3\pi}{2} = -1$

$\cos \frac{3\pi}{2} = 0$



$y = \operatorname{tg} x = \frac{\sin x}{\cos x}$

$x \in \mathbb{R} \setminus \{ \frac{\pi}{2} + k\pi \,,\, k \in \mathbb{Z} \}$

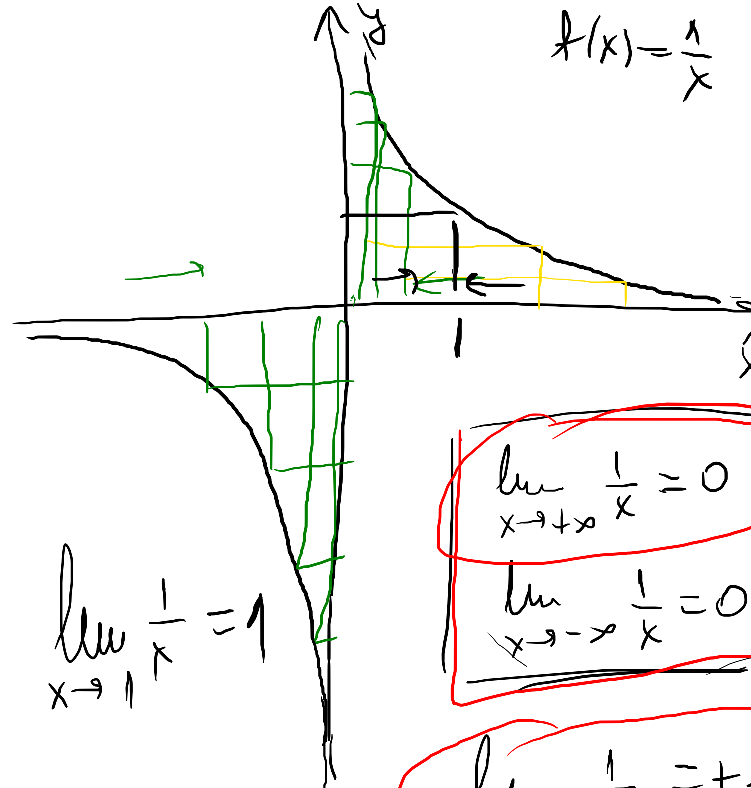


$y = \operatorname{ctg} x = \frac{\cos x}{\sin x}$

$x \in \mathbb{R} \setminus \{k\pi \mid k \in \mathbb{Z}\}$

Граничне вредности функција

$$\lim_{x\to x_0} f(x)$$



$f(x)=\frac{1}{x}$

Т. Нека су $\lim\limits_{x\to x_0} f(x)=a$, $\lim\limits_{x\to x_0} g(x)=b$,

$x_0\in\mathbb{R}\cup\{+\infty,-\infty\}$, $a,b\in(-\infty,\infty)$

$$\lim_{x\to 1}\frac{1}{x}=1$$

$$\lim_{x\to+\infty}\frac{1}{x}=0$$

$$\lim_{x\to-\infty}\frac{1}{x}=0$$

1) $$\lim_{x\to x_0} cf(x)=c\lim_{x\to x_0} f(x)=c\cdot a$$

$$\lim_{x\to 2}\left(3\cdot\frac{1}{x}\right)=3\lim_{x\to 2}\frac{1}{x}=3\cdot\frac{1}{2}$$

$$\lim_{x\to 0^+}\frac{1}{x}=+\infty$$

$$\lim_{x\to 0^-}\frac{1}{x}=-\infty$$

2) $$\lim_{x\to x_0}(f(x)\pm g(x))=\lim_{x\to x_0} f(x)\pm\lim_{x\to x_0} g(x)=a\pm b$$

$$\lim_{x\to 5}\left(\frac{1}{x}+x\right)=\lim_{x\to 5}\frac{1}{x}+\lim_{x\to 5}x=\frac{1}{5}+5$$

$\to \lim\limits_{x\to 0}\frac{1}{x}$ не постоји

3. $\lim\limits_{x \to x_0} f(x) \cdot g(x) = \lim\limits_{x \to x_0} f(x) \cdot \lim\limits_{x \to x_0} g(x) = a \cdot b$

4. $\lim\limits_{x \to x_0} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to x_0} f(x)}{\lim\limits_{x \to x_0} g(x)} = \dfrac{a}{b}$, $\quad b \neq 0$, $\quad g(x) \neq 0$

неодређени изрази

„$\dfrac{0}{0}$", „$\dfrac{\infty}{\infty}$", „$0 \cdot \infty$", „$\infty - \infty$"

„1^{∞}", „0^{0}", „∞^{0}"

одређене (неки)

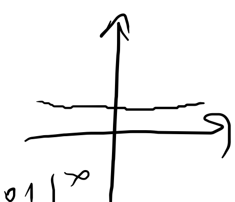
„$\dfrac{2}{\infty} = 0$", „$\dfrac{1}{0^+} = +\infty$"

„$2 \cdot \infty = \infty$" „$\infty + \infty = \infty$"

$\underbrace{1 \cdot 1 \cdot 1 \cdot \ldots \cdot 1}_{\infty} = 1$

„1^{∞}"

$\hookrightarrow (1{,}0\ldots01)^{\infty}$



1

a) $\lim\limits_{x \to 3} (2x-1) = 5$

b) $\lim\limits_{x \to 0} \dfrac{x^2-x+1}{x^2+x-3} = \dfrac{1}{-3}$

2

a) $\lim\limits_{x \to 0} \dfrac{x^2+x^3-2x^4}{2x^2+x^5} \overset{\frac{0}{0}}{=} \lim\limits_{x \to 0} \dfrac{\cancel{x^2}(1+x-2x^2)}{\cancel{x^2}(2+x^3)} = \dfrac{1}{2}$

b) $\lim\limits_{x \to 0} \dfrac{(1+x)(1+3x)-1}{2x} \overset{\frac{0}{0}}{=} \lim\limits_{x \to 0} \dfrac{\cancel{1}+3x+x+3x^2 \cancel{-1}}{2x} = \lim\limits_{x \to 0} \dfrac{4x+3x^2}{2x} \overset{\frac{0}{0}}{=}$

$= \lim\limits_{x \to 0} \dfrac{\cancel{x}(4+3x)}{2\cancel{x}} = \dfrac{4}{2} = 2$

c) $\lim\limits_{x \to 1} \dfrac{x^2-3x+2}{x^2-4x+3} \overset{\frac{0}{0}}{=} \lim\limits_{x \to 1} \dfrac{(x-2)\cancel{(x-1)}}{(x-3)\cancel{(x-1)}} = \dfrac{-1}{-2} = \dfrac{1}{2}$

$x^2-3x+2=0$

$x_{1,2} = \dfrac{3 \pm \sqrt{9-8}}{2} = \dfrac{3 \pm 1}{2} = \begin{cases} 2 \\ 1 \end{cases}$

$x^2-4x+3=0$

$x_{1,2} = \dfrac{4 \pm \sqrt{16-12}}{2} = \dfrac{4 \pm 2}{2} = \begin{cases} 3 \\ 1 \end{cases}$

d)

$$\lim_{x\to 1}\frac{x^2-1}{2x^2-x-1} \overset{\frac{0}{0}}{=} \lim_{x\to 1}\frac{\cancel{(x-1)}(x+1)}{2\cancel{(x-1)}(x+\frac{1}{2})} = \frac{\cancel{2}}{2(1+\frac{1}{2})} = \frac{2}{3}$$

$$2x^2-x-1=0$$

$$x_{1,2}=\frac{1\pm\sqrt{1+8}}{4}=\frac{1\pm 3}{4}=\begin{cases}1\\ -\frac{1}{2}\end{cases}$$

$$\lim_{x\to\infty}\frac{1}{x}=0$$

③ a)

$$\lim_{x\to\infty}\frac{x^2+3x+1}{x^4+3x^2+3} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty}\frac{\cancel{x^4}\left(\frac{1}{x^2}+\frac{3}{x^3}+\frac{1}{x^4}\right)}{\cancel{x^4}\left(1+\frac{3}{x^2}+\frac{3}{x^4}\right)} = \frac{0}{1+0}=0$$

$$\left(= \frac{\lim\limits_{x\to\infty}\frac{1}{x^2}+3\lim\limits_{x\to\infty}\frac{1}{x^3}+\lim\limits_{x\to\infty}\frac{1}{x^4}}{1+3\lim\limits_{x\to\infty}\frac{1}{x^2}+3\lim\limits_{x\to\infty}\frac{1}{x^3}}\right)$$

b) $\lim\limits_{x\to\infty} \dfrac{2x^{4}+3x+1}{x^2+5x} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim\limits_{x\to\infty} \dfrac{\cancel{x^4}\left(2+\frac{3}{x^3}+\frac{1}{x^4}\right)}{\cancel{x^4}\left(\frac{1}{x^2}+\frac{5}{x^3}\right)} = \text{"}\dfrac{2}{0^+}\text{"} = \infty$

c) $\lim\limits_{x\to\infty} \dfrac{7x^3+2x+1}{5x^3+x^2+6} = \lim\limits_{x\to\infty} \dfrac{\cancel{x^3}\left(7+\frac{2}{x^2}+\frac{1}{x^3}\right)}{\cancel{x^3}\left(5+\frac{1}{x}+\frac{6}{x^3}\right)} = \boxed{\dfrac{7}{5}}$

d) $\lim\limits_{x\to\infty} \dfrac{17x^{15}+3x^6+2}{23x^4+5x^{15}} = \dfrac{17}{5}$

f) $\lim\limits_{x\to\infty} \dfrac{2x^3+5x^2}{6x^{7}+4x^5} = 0$

e) $\lim\limits_{x\to\infty} \dfrac{13x^5+4x^{6}+19x}{5x^3+2x^4} = \infty$