①

$y = x^2 + x + 1$

$x = 0$

$X = 1$

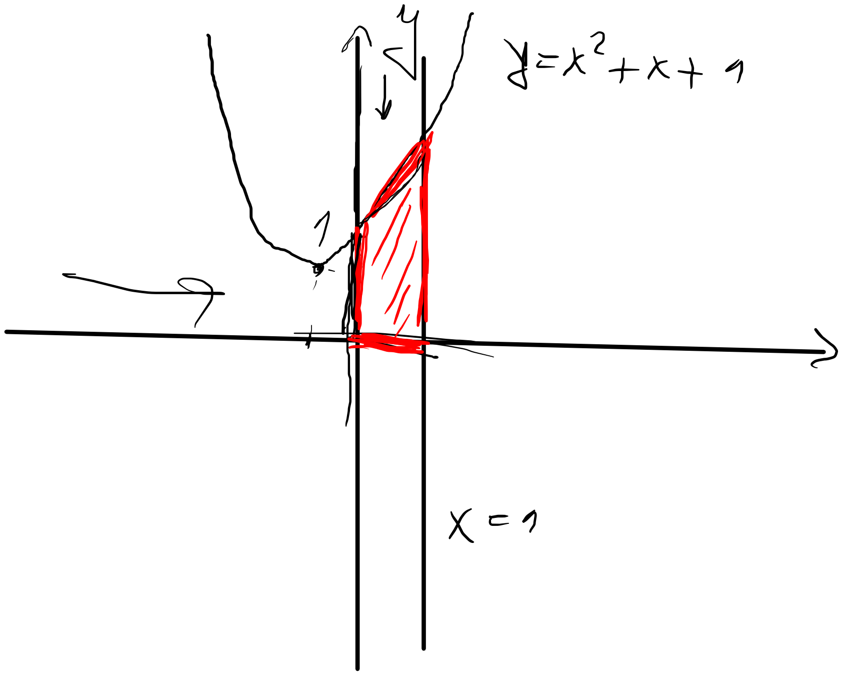
$y = 0$

$x_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} \notin \mathbb{R}$

$y' = 2x + 1$

$y' = 0 \qquad x = -\frac{1}{2}$

$y = \frac{1}{4} - \frac{1}{2} + 1 = \frac{1-2+4}{4} = \frac{3}{4}$



$$P = \int_0^1 (x^2 + x + 1)\,dx$$

$$= \left(\frac{x^3}{3} + \frac{x^2}{2} + x\right)\Big|_0^1$$

$$= \frac{1}{3} + \frac{1}{2} + 1$$

② a) $\int \frac{1}{\sqrt{x^2+2x+7}}\,dx = \int \frac{1}{\sqrt{(x+1)^2+6}}\,dx = \int \frac{1}{\sqrt{t^2+6}}\,dt = \ln|t+\sqrt{t^2+6}|+C$

$= \ln|x+1+\sqrt{(x+1)^2+6}|+C$

$x^2+2x+7 = x^2+2x+1+6 = (x+1)^2+6$

$x+1=t$
$dx=dt$

1 верно!? это верно!?

b) $\int \frac{\boxed{x+3}}{\sqrt{x^2+2x+7}}\,dx = \boxed{A}\cdot\sqrt{x^2+2x+7} + \lambda \int \frac{1}{\sqrt{x^2+2x+7}}\,dx \;/'$

const

$\frac{x+3}{\sqrt{x^2+2x+7}} = A\,\frac{1}{2\sqrt{x^2+2x+7}}\cdot\left(\overset{2(x+1)}{2x+2}\right) + \lambda\,\frac{1}{\sqrt{x^2+2x+7}} \;/\sqrt{x^2+2x+7}$

$x+3 = A(x+1)+\lambda$

$x+3 = Ax+A+\lambda$

$\boxed{A=1}$ $\quad A+\lambda=3 \quad$ $\boxed{\lambda=2}$

$\int \frac{x+3}{\sqrt{x^2+2x+4}}\,dx = \sqrt{x^2+2x+4} + 2\underbrace{\int \frac{1}{\sqrt{x^2+2x+7}}\,dx}_{a)}$

2 степень   1 степень

c) $$\int \frac{x^2+3}{\sqrt{x^2+2x+7}}\,dx = (Ax+B)\cdot\sqrt{x^2+2x+7} + \lambda\int\frac{1}{\sqrt{x^2+2x+7}}\Bigg)'$$

$$\frac{x^2+3}{\sqrt{x^2+2x+7}} = A\sqrt{x^2+2x+7} + (Ax+B)\frac{1}{2\sqrt{x^2+2x+7}}\cdot\overbrace{(2x+2)}^{\cancel{2}(x+1)} + \lambda\frac{1}{\sqrt{x^2+2x+7}}\Bigg/\sqrt{x^2+2x+7}$$

$$x^2+3 = A(x^2+2x+7) + (Ax+B)(x+1) + \lambda$$

$$x^2+3 = Ax^2+2Ax+7A+Ax^2+Ax+Bx+B+\lambda$$

$$x^2+3 = 2Ax^2+(3A+B)x + B+\lambda+7A$$

$2A=1$ $3A+B=0$ $7A+B+\lambda=3$

$\boxed{A=\frac{1}{2}}$ $\boxed{B=-\frac{3}{2}}$ $\lambda=3-\frac{7}{2}+\frac{3}{2}=1$

$$\int\frac{x^2+3}{\sqrt{x^2+2x+7}}\,dx = \left(\frac{1}{2}x-\frac{3}{2}\right)\sqrt{x^2+2x+7} + \int\frac{1}{\sqrt{x^2+2x+7}}$$

a)

③ $$\int \frac{\operatorname{arctg} e^{\frac{x}{2}}}{e^{\frac{x}{2}} \cdot (1+e^{x})}\, dx = \int \frac{\operatorname{arctg} t}{t(1+t^2)} \cdot \frac{2}{t}\, dt =$$

smena :

$$e^{\frac{x}{2}} = t$$

$$e^{\frac{x}{2}}\, \frac{1}{2}\, dx = dt$$

$$t \cdot \frac{1}{2}\, dx = dt$$

$$dx = \frac{2}{t}\, dt$$

$$= 2 \int \frac{\overset{u}{\operatorname{arctg} t}}{\boxed{t^2}(1+t^2)}\, \overset{du}{dt}$$

$$\operatorname{arctg} t = u$$

$$\frac{1}{1+t^2}\, dt = du$$

$$u = \operatorname{arctg} t \qquad dv = \frac{1}{t^2(1+t^2)}\, dt$$

$$du = \frac{1}{1+t^2}\, dt$$

$$\int \frac{1}{t^2(1+t^2)}\, dt = \int \left(\frac{A}{t} + \frac{B}{t^2} + \frac{Ct+D}{1+t^2} \right) dx$$

④

$$\left.\begin{array}{l} y = -x^2 + 6x - 5 \\ y = 3x - 9 \end{array}\right\} \begin{array}{l} -x^2 + 6x - 5 = 3x - 9 \\ -x^2 + 3x + 4 = 0 \end{array}$$

$$x_{1,2} = \frac{-3 \pm \sqrt{9+16}}{-2} = \frac{-3 \pm 5}{-2} \left< \begin{array}{l} -1 \\ 4 \end{array} \right.$$

$$P = \int_{-1}^{4} (-x^2 + 6x - 5 - 3x + 9)\,dx$$

$$x_{1,2} = \frac{-6 \pm \sqrt{36-20}}{-2} = \frac{-6 \pm 4}{-2} \left< \begin{array}{l} 1 \\ 5 \end{array} \right.$$

$y' = -2x + 6$

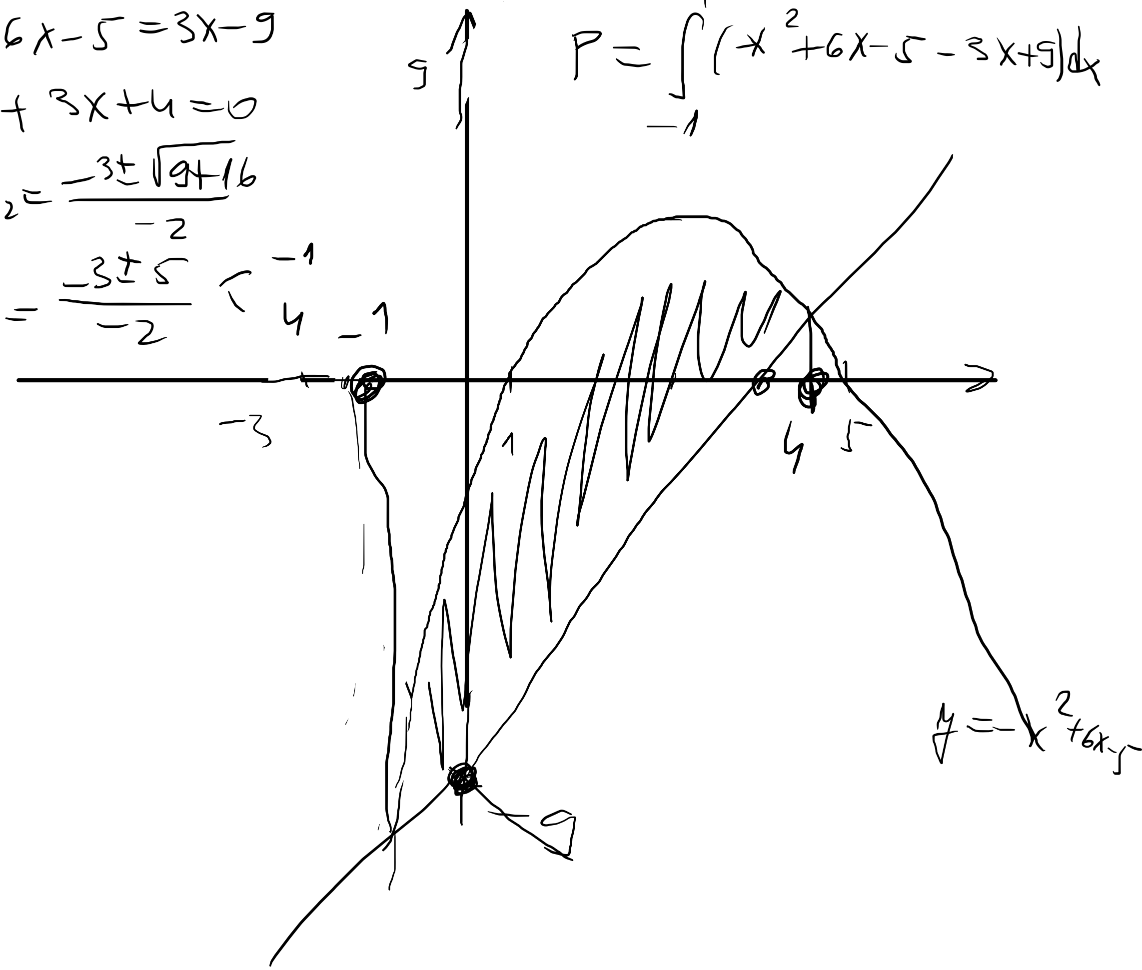
$y' = 0 \quad x = 3$

$y = -9 + 18 - 5 = 4$

$y = 3x + 9$

$x = 0 \quad y = 9$

$y = 0 \quad x = -3$



⑤ $$\int \frac{\sin x}{(2+\cos x)(2-\sin^2 x)}\,dx = \int \underbrace{\frac{1}{(2+\cos x)(2-\sin^2 x)}}_{f(\cos x)} \cdot \sin x\,dx$$

$$= \int \frac{1}{(2+\cos x)(2-(1-\cos^2 x))} \sin x\,dx = \int \frac{1}{(2+\cos x)(1+\cos^2 x)} \sin x\,dx$$

Smeno:
$\cos x = t$
$-\sin x\,dx = dt$
$\sin x\,dx = -dt$

$$= -\int \frac{1}{(2+t)(1+t^2)}\,dt = -\frac{1}{5}\int \frac{1}{2+t}\,dt - \int \frac{-\frac{1}{5}t + \frac{2}{5}}{1+t^2}\,dt = -\frac{1}{5}\int \frac{1}{2+t}\,dt + \frac{1}{5}\int \frac{t-2}{t^2+1}\,dt =$$

$$\frac{1}{(2+t)(1+t^2)} = \frac{A}{2+t} + \frac{Bt+D}{1+t^2} \quad \Big|\,(2+t)(1+t^2)$$

$$1 = A + At^2 + 2Bt + Bt^2 + 2D + Dt$$

$A+B=0 \rightarrow B = -A$
$2B+D=0 \qquad -2A+D=0$
$A+2D=1 \qquad A+2D=1$

$5A = 1$

$\boxed{A=\frac{1}{5}}$ $\boxed{B=-\frac{1}{5}}$ $\boxed{D=\frac{2}{5}}$

$$= -\frac{1}{5}\int \frac{1}{2+t}\,dt + \boxed{\frac{1}{5}\int \frac{t}{t^2+1}\,dt} - \frac{2}{5}\int \frac{1}{t^2+1}\,dt = \ldots\ldots$$

Smeno: $t^2+1 = u$
$2t\,dt = du$
$t\,dt = \frac{1}{2}du$

$$= -\frac{1}{5}\int \frac{1}{2+t} + \frac{1}{5}\int \frac{\frac{1}{2}du}{u} - \frac{2}{5}\int \frac{1}{t^2+1}\,dt$$

$$= -\frac{1}{5}\ln|t+2) + \frac{1}{10}\ln|u| - \frac{2}{5}\operatorname{arctg} t + C$$

$$= -\frac{1}{5}\ln|t+2| + \frac{1}{10}\ln|t^2+1| - \frac{2}{5}\operatorname{arctg} t + C$$

$$= -\frac{1}{5}\ln|\cos x+2| + \frac{1}{10}\ln|\cos^2 x+1) - \frac{2}{5}\operatorname{arctg} \cos x + C$$

⑥ $\int \frac{2+\cos^2 x}{\underline{\cos x}\,\underline{(2-\sin x)}}\,dx = \int \boxed{\frac{2+\cos^2 x}{\cos^2 x(2-\sin x)}}^{\sin x}\,\underline{\cos x\,dx}$

$= \int \frac{2+1-\sin^2 x}{(1-\sin^2 x)(2-\sin x)}\cos x\,dx = \int \frac{3-t^2}{(1-t^2)(2-t)}\,dt = \int \frac{3-t^2}{(1-t)(1+t)(2-t)}\,dt = \dots$

$\sin x = t$

$\cos x\,dx = dt$

$\int \frac{2+\cos^2 x}{\cos x(2-\sin x)}\,dx = \int \frac{2\cos x}{\cos^2 x(2-\sin x)}\,dx + \int \frac{\cos^{\cancel{2}} x}{\cancel{\cos x}(2-\sin x)}\,dx$

$\boxed{\sin^2 \alpha} + \underline{\cos^2 \alpha} = 1$

⑦ $$\int \frac{\cos^3 x}{\sin^3 x} dx = \int \underset{\text{od cosx}}{\boxed{\frac{\cos^3 x}{\underbrace{\sin^4 x}_{(\sin^2 x)^2}}}} \cdot \sin x\, dx$$

$$= \int \frac{\cos^3 x}{(1-\cos^2 x)^2} \sin x\, dx = -\int \frac{t^3}{(1-t^2)^2} dt = -\int \frac{t^3}{(1-t)^2(1+t)^2} dt$$

$$\frac{A}{1-t} + \frac{B}{(1-t)^2} + \frac{D}{1+t} + \frac{E}{(1+t)^2} \cdots$$

$\cos x = t$

$-\sin x\, dx = dt$

$\sin x\, dx = -dt$

⑧ $$\int \frac{\overset{2x-5}{3x+2}}{x^2-5x+8}\,dx = 3\int \frac{x+\frac{2}{3}}{x^2-5x+8}\,dx = \frac{3}{2}\int \frac{2\left(x+\frac{2}{3}\right)}{x^2-5x+8}\,dx =$$

$$x_{1,2} = \frac{5 \pm \sqrt{25-32}}{2} \notin \mathbb{R}$$

$$x^2-5x+8 = x^2 - 2x\cdot\boxed{\frac{5}{2}} + \frac{25}{4} - \frac{25}{4} + 8 = \left(x-\frac{5}{2}\right)^2 + \frac{7}{4}$$

Smena: $x^2-5x+8 = t$

$$(2x-5)\,dx = dt$$

$$= \frac{3}{2}\int \frac{2x+\frac{4}{3}}{x^2-5x+8}\,dx = \frac{3}{2}\int \frac{2x-5+5+\frac{4}{3}}{x^2-5x+8}\,dx = \frac{3}{2}\int \frac{2x-5}{x^2-5x+8}\,dx + \frac{3}{2}\cdot\frac{19}{3}\int \frac{dx}{x^2-5x+8}$$

$$= \frac{3}{2}\int \frac{dt}{t} + \frac{19}{2}\int \frac{dx}{\left(x-\frac{5}{2}\right)^2+\frac{7}{4}} = \frac{3}{2}\ln|t| + \frac{19}{2}\,\frac{1}{\frac{\sqrt{7}}{2}}\,\mathrm{arctg}\,\frac{x-\frac{5}{2}}{\frac{\sqrt{7}}{2}} + C$$

⑨ $\frac{x^3+5x+2}{x^2+3} = x + \frac{2x+2}{x^2+3}$

$(x^3+5x+2):(x^2+3)=\boxed{x}$

$-(x^3+3x)$

$\boxed{2x+2}$

$$\int\frac{dx}{x^2+a^2} = \frac{1}{a}\operatorname{arctg}\frac{x}{a}+C$$

$$\int\frac{dx}{x^2+3} = \int\frac{dx}{x^2+(\sqrt{3})^2} = \frac{1}{\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}}+C$$

$$\int\frac{x^3+5x+2}{x^2+3}dx = \int x\,dx + \int\frac{2x+2}{x^2+3}dx =$$

$$= \int x\,dx + \underline{\int\frac{2x}{x^2+3}dx} + \int\frac{2}{x^2+3}dx = \int x\,dx + \int\frac{dt}{t} + 2\int\frac{1}{x^2+3}dt$$

smen $x^2+3=t$

$2x\,dx=dt$

$$= \frac{x^2}{2} + \ln|x^2+3| + 2\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}}+C$$

⑩ $z = \underline{x^3 y^5} + \sin(x^2+6y^3) + \ln(x^2 \underline{y^3})$

$$\frac{\partial z}{\partial x} = y^5 \cdot 3x^2 + \cos(x^2+6y^3)\cdot(2x+0) + \frac{1}{x^2y^3}\cdot y^3 \cdot 2x$$

$$\frac{\partial z}{\partial y} = x^3 \cdot 5y^4 + \cos(x^2+6y^3)\cdot(0+\underline{6\cdot 3}y^2) + \frac{1}{\cancel{x^2}y^{\cancel{3}}}\cdot \cancel{x^2}\cdot 3y^{\cancel{2}}$$

$$\boxed{\frac{\partial z}{\partial y} = 5x^3y^4 + 18y^2\cos(x^2+6y^3) + \frac{3}{y}}$$

$$\frac{\partial^2 z}{\partial y \partial x} = 5y^4 \cdot 3x^2 + 18y^2\left(-\sin(x^2+6y^3)\cdot(2x+0)\right) + 0$$

11)

$$y' - \frac{2}{x+1} \cdot y - \boxed{(x+1)^3} = 0$$

$$y' - \frac{2}{x+1} y = (x+1)^3$$

Smena $y = uv \Rightarrow y' = u'v + uv'$

$$u'v + \underbrace{uv' - \frac{2}{x+1} uv}_{} = (x+1)^3$$

$$u'v + u\underbrace{\left(v' - \frac{2}{x+1} v\right)}_{=0} = (x+1)^3$$

$$v' - \frac{2}{x+1} v = 0$$

$$v' = \frac{2}{x+1} v$$

$$\frac{dv}{dx} = \frac{2}{x+1} v$$

$$dv = \frac{2}{x+1} \boxed{v}\, dx$$

$$\int \frac{1}{v} dv = \int \frac{2}{x+1} dx$$

$$\ln v = 2 \ln(x+1)$$

$$\boxed{v = (x+1)^2}$$

$$u' \cdot \cancel{(x+1)^2} = (x+1)^{\cancel{3}}$$

$$u' = x+1$$

$$\frac{du}{dx} = x+1$$

$$\int du = \int (x+1)\, dx$$

$$\boxed{u = \frac{x^2}{2} + x + C}$$

$$y = uv$$

$$y = \left(\frac{x^2}{2} + x + c\right)(x+1)^2$$

$f(x) = \sin x$ $F(x) = -\cos x$

$F(x) = \sin x$ $f(x) = \cos x$

$F(x)$ primitivna za $f(x)$

$$\boxed{F'(x) = f(x)}$$

① $z = 4x^2 + 3y^3 + 16x - 9y + 5$

$\frac{\partial z}{\partial x} = 8x + 16$ $\qquad \frac{\partial^2 z}{\partial x^2} = 8$ $\qquad \frac{\partial^2 z}{\partial x \partial y} = 0$ $\qquad \frac{\partial^2 z}{\partial y^2} = 18y$

$\frac{\partial z}{\partial y} = 9y^2 - 9$

$8x + 16 = 0$

$9y^2 - 9 = 0$

$x = -2$

$y^2 = 1$

$y = 1 \quad y = -1$

$$\boxed{M_1(-2,1),\ M_2(-2,-1)}$$

stacionarne

I $M_1(-2,1)$

$r = \frac{\partial^2 z}{\partial x^2}(M_1) = 8$

$s = \frac{\partial^2 z}{\partial y^2}(M_1) = 18$

$t = \frac{\partial^2 z}{\partial x \partial y}(M_1) = 0$

$rs - t^2 = 8 \cdot 18 - 0 > 0$

$\Rightarrow$ ima e.v.

$r = 8 > 0 \Rightarrow$ lokalni minimum

$($ili $s = 18 > 0)$

$z_{min}(M_1) = 4 \cdot (-2)^2 + 3 \cdot 1^3 + 16 \cdot (-2) - 9 \cdot 1 + 5$

$M_2(-2,-1)$

$r = \frac{\partial^2 z}{\partial x^2}(M_2) = 8$

$s = \frac{\partial^2 z}{\partial y^2}(M_2) = -18$

$t = \frac{\partial^2 z}{\partial x \partial y}(M_2) = 0$

$rs - t^2 = 8 \cdot (-18) - 0 < 0$

$\Rightarrow$ nema e.v.

$\underline{II}$ $$d^2z = \frac{\partial^2 z}{\partial x^2}dx^2 + 2\frac{\partial^2 z}{\partial x \partial y}dx\,dy + \frac{\partial^2 z}{\partial y^2}dy^2$$

$$= 8dx^2 + 18y\,dy^2$$

$$d^2z(\overbrace{-2,1}^{M_1}) = 8dx^2 + 18dy^2 > 0 \quad (dx,dy) \neq (0,0)$$

$\Rightarrow$ lokalni minimum $z_{min}(-2,1) = \ldots$

$$d^2z(\overbrace{-2,-1}^{M_2}) = \underline{8dx^2} - 18dy^2 \leftarrow$$ menja znak

$\Rightarrow$ nema e.v.

② a) $\int_0^1 (x^2-3)e^{2x}dx = \frac{1}{2}(x^2-3)e^{2x}\Big|_0^1 - \int_0^1 \cancel{2}x \cdot \frac{1}{\cancel{2}}e^{2x}dx$

$= \frac{1}{2}(1-3)e^2 - \frac{1}{2}(0-3)e^0 - \int_0^1 xe^{2x}dx$

$u = x^2-3 \quad dv = e^{2x}dx$

$du = 2x\,dx \quad v = \frac{1}{2}e^{2x}$

$\int e^{2x}dx = \frac{1}{2}\int e^t dt = \frac{1}{2}e^t + C = \frac{1}{2}e^{2x} + C$

smena $2x = t$

$2dx = dt$

$dx = \frac{1}{2}dt$

$\int e^{\alpha x}dx = \frac{1}{\alpha}e^{\alpha x} + C$

$\int \sin \alpha x\, dx = -\frac{1}{\alpha}\cos \alpha x + C$

$\int \cos \alpha x\, dx = \frac{1}{\alpha}\sin \alpha x + C$

$u = x \quad dv = e^{2x}dx$

$du = dx \quad v = \frac{1}{2}e^{2x}$

$= -e^2 + \frac{3}{2} - \left(\frac{1}{2}xe^{2x}\Big|_0^1 - \frac{1}{2}\int_0^1 e^{2x}dx\right)$

$= -e^2 + \frac{3}{2} - \left(\frac{1}{2}e^2 - \frac{1}{2}\,\frac{1}{2}e^{2x}\Big|_0^1\right) =$

$= -e^2 + \frac{3}{2} - \frac{1}{2}e^2 + \frac{1}{4}(e^2 - e^0)$

$= -e^2 + \frac{3}{2} - \frac{1}{2}e^2 + \frac{1}{4}e^2 - \frac{1}{4}$

b) $\overset{I}{\int} \frac{x+\overset{x+1}{2}}{\sqrt{\underbrace{x^2+2x-3}_{t}}}dx = \int \frac{x+1+1}{\sqrt{x^2+2x-3}}dx = \int \frac{x+1}{\sqrt{x^2+2x-3}}dx + \int \frac{1}{\sqrt{x^2+2x-3}}dx$

змена: $x^2+2x-3=t$

$(2x+2)\,dx = dt$

$2(x+1)dx = dt$

$(x+1)dx = \frac{1}{2}dt$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t}} + \int \frac{1}{\sqrt{(x+1)^2-4}}dx$$

$x^2+2x-3 = x^2+2x\cdot 1+1-1-3$

$= (x+1)^2-4$

змена: $x+1 = u$

$dx = du$

$$= \frac{1}{2}\int t^{-\frac{1}{2}}dt + \int \frac{1}{\sqrt{u^2-4}}du$$

$$= \frac{1}{\cancel{2}}\frac{t^{\frac{1}{2}}}{\frac{1}{\cancel{2}}} + \ln\left|u+\sqrt{u^2-4}\right| + C$$

$$= \sqrt{x^2+2x-3} + \ln\left|x+1+\sqrt{(x+1)^2-4}\right| + C$$

$\overline{\underline{I}}$ $$\int \frac{x+2}{\sqrt{x^2+2x-3}}\,dx = A\sqrt{x^2+2x-3} + \lambda \int \frac{1}{\sqrt{x^2+2x-3}}\,dx \quad \Big/ \Big)$$

$$\frac{x+2}{\sqrt{x^2+2x-3}} = A\,\frac{1}{\cancel{2}\sqrt{x^2+2x-3}} \cdot \overset{\cancel{2}(x+1)}{(2x+2)} + \lambda\,\frac{1}{\sqrt{x^2+2x-3}} \Big/ \cdot \sqrt{x^2+2x-3}$$

$$x+2 = A(x+1)+\lambda$$

$$x+2 = Ax + A + \lambda$$

$$\boxed{A=1} \qquad A+\lambda = 2$$

$$\boxed{\lambda = 1}$$

$$\int \frac{x+2}{\sqrt{x^2+2x-3}}\,dx = \sqrt{x^2+2x-3} + \int \frac{1}{\sqrt{x^2+2x-3}}\,dx$$

$$= \sqrt{x^2+2x-3} + \int \frac{1}{\sqrt{(x+1)^2-4}}\,dx$$

$$= \sqrt{x^2+2x-3} + \ln\left|x+1+\sqrt{(x+1)^2-4}\right| + C$$

c) $$\int \frac{\cos x - 9}{(\cos x + 1)(10 - \sin^2 x)} \boxed{\sin x \, dx} = \int \frac{\cos x - 9}{(\cos x + 1)(10 - (1 - \cos^2 x))} \sin x \, dx$$

$$= \int \frac{\cos x - 9}{(\cos x + 1)(9 + \cos^2 x)} \sin x \, dx = -\int \frac{t - 9}{(t+1)(9+t^2)} dt =$$

Smena : $\cos x = t$

$-\sin x \, dx = dt$

$\sin x \, dx = -dt$

$$= -\int \frac{-1}{t+1} dt - \int \frac{t}{9+t^2} dt$$

$9 + t^2 = u$

$2t \, dt = du$

$t \, dt = \frac{1}{2} du$

$$= \int \frac{1}{t+1} dt - \frac{1}{2} \int \frac{du}{u} = \ln|t+1| - \frac{1}{2} \ln|u| + C$$

$$= \ln|t+1| - \frac{1}{2} \ln|9+t^2| + C$$

$$= \ln|\cos x + 1| - \frac{1}{2} \ln|9 + \cos^2 x| + C$$

$$\frac{t-9}{(t+1)(9+t^2)} = \frac{A}{t+1} + \frac{Bt+D}{9+t^2} \Big/ (t+1)(9+t^2)$$

$$t - 9 = 9A + At^2 + Bt^2 + Bt + Dt + D$$

$A + B = 0$

$B + D = 1$

$9A + D = -9$ $\leftarrow -1$

$9A - B = -10$

$A + B = 0$ $+$

$10A = -10$

$\boxed{A = -1}$

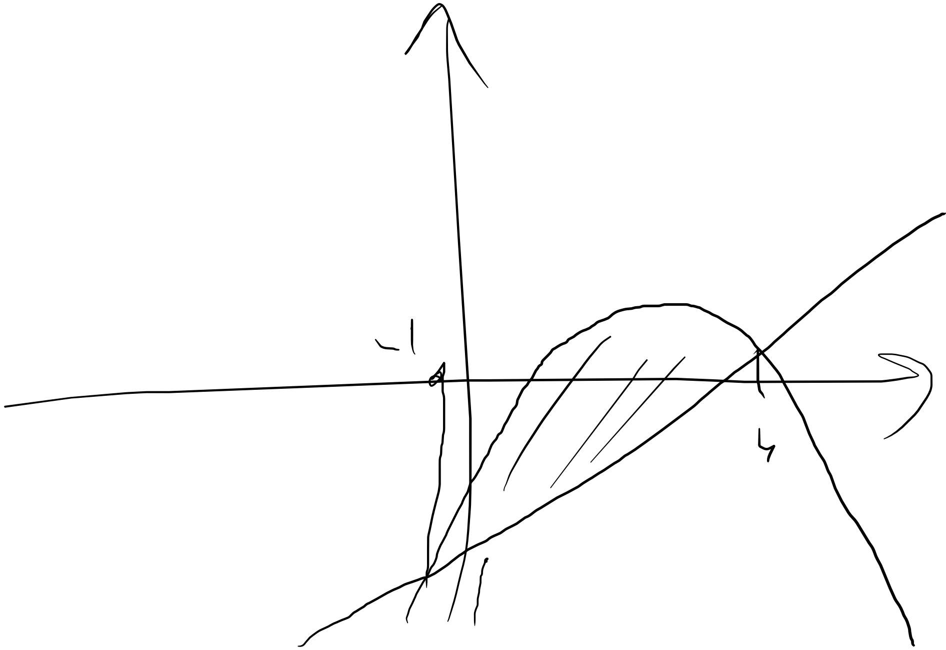
$\boxed{B = 1}$

$D = 0$

③ $y = -x^2 + 6x - 5$

$y = 3x - 9$

URADIU RANIJE



④ a) $xy' - (y + \sqrt{xy}) = 0 \quad / : x$

$$y' - \frac{y}{x} - \frac{\sqrt{xy}}{x} = 0$$

$$y' - \frac{y}{x} - \sqrt{\frac{\cancel{x}y}{x^{\cancel{2}}}} = 0$$

$$y' - \frac{y}{x} - \sqrt{\frac{y}{x}} = 0$$

$$y' = \frac{y}{x} + \sqrt{\frac{y}{x}}$$

$$t'x + \cancel{t} = \cancel{t} + \sqrt{t}$$

$$t'x = \sqrt{t}$$

HOMOGENA

$y' = f\left(\frac{y}{x}\right)$

$\frac{y}{x} = t$

$y = tx \quad y' = t'x + t$

$$\frac{dt}{dx} x = \sqrt{t}$$

$$dt\, x = \sqrt{t}\, dx$$

$$\int \frac{1}{\sqrt{t}}\, dt = \int \frac{1}{x}\, dx$$

$$\frac{t^{\frac{1}{2}}}{\frac{1}{2}} = \ln|x| + C$$

$$2\sqrt{\frac{y}{x}} = \ln|x| + C$$

b) $y' - 2\frac{1}{x}y = -\frac{3}{x^2}$, $y(1) = 5$

smena: $y = uv \Rightarrow y' = u'v + uv'$

$$u'v + \underbrace{uv' - 2\frac{1}{x}uv}\ = -\frac{3}{x^2}$$

$$u'v + u\underbrace{\left(v' - 2\frac{1}{x}v\right)}_{=0} = -\frac{3}{x^2}$$

$$v' - 2\frac{1}{x}v = 0$$

$$v' = 2\frac{1}{x}v$$

$$\frac{dv}{dx} = 2\frac{1}{x}v$$

$$dv = 2\frac{1}{x}\,v\,dx$$

$$\int\frac{1}{v}dv = 2\int\frac{1}{x}dx$$

$$\ln v = 2\ln x$$

$$\boxed{v = x^2}$$

$$u' \cdot x^2 = -\frac{3}{x^2}$$

$$u' = -\frac{3}{x^4}$$

$$\frac{du}{dx} = -\frac{3}{x^4}$$

$$\int du = -\int\frac{3}{x^4}dx$$

$$u = \cancel{-3}\,\frac{x^{-3}}{\cancel{-3}} + C$$

$$\boxed{u = \frac{1}{x^3} + C}$$

$$y = uv$$

$$y = \left(\frac{1}{x^3} + C\right)x^2$$

$$\boxed{y = \frac{1}{x} + Cx^2}$$

opšte rešenje

$y(1) = 5$

$\Rightarrow x = 1,\ y = 5$

$$5 = \frac{1}{1} + C \cdot 1$$

$$\boxed{C = 4}$$

$$\boxed{y = \frac{1}{x} + 4x^2}$$

partikularno rešenje