$$z = x^3 y + 2xy$$

$$\frac{\partial z}{\partial x} = \boxed{y \cdot 3x^2 + 2y}$$

$$\frac{\partial z}{\partial y} = x^3 + 2x$$

$$\frac{\partial^2 z}{\partial x^2} = 3y \cdot 2x$$

$$\frac{\partial^2 z}{\partial y^2} = 0$$

$$\frac{\partial^2 z}{\partial x \partial y} = 3x^2 + 2$$

$$d^2 z = \frac{\partial^2 z}{\partial x^2} dx^2 + 2 \frac{\partial^2 z}{\partial x \partial y} dx dy + \frac{\partial^2 z}{\partial y^2} dy^2$$

$$= 6xy\, dx^2 + (3x^2 + 2)\, dx dy$$

$$\int f(x)\,dx = F(x)\Big|_a^b = F(b)-F(a)$$

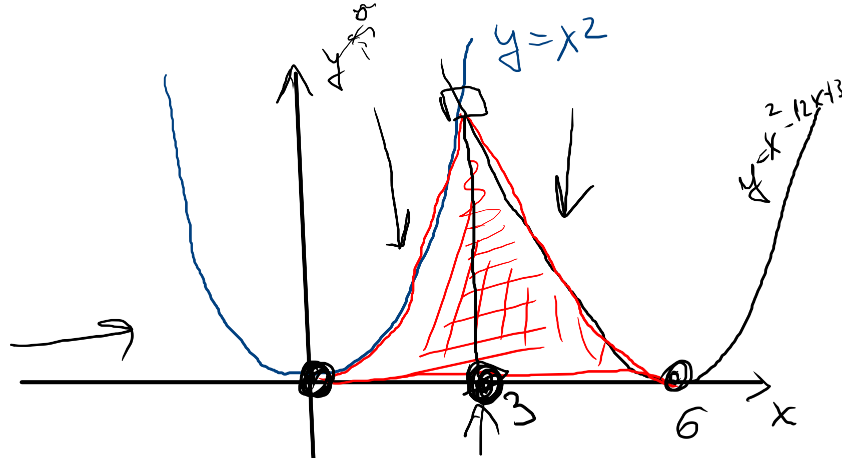
1) $y = x^2 - 12x + 36$

$y = x^2$

x-oso

$$y = x^2 - 12x + 36 = (x-6)^2$$

$$x_{1,2} = \frac{12 \pm \sqrt{144-144}}{2} = 6$$



presek $y = x^2$

$y = x^2 - 12x + 36$

$$\cancel{x^2} = \cancel{x^2} - 12x + 36$$

$$12x = 36$$

$$x = 3$$

$$P = \int_0^3 x^2\,dx + \int_3^6 (x^2 - 12x + 36)\,dx$$

$$= \frac{x^3}{3}\Big|_0^3 + \left(\frac{x^3}{3} - \cancel{12}\,6\,\frac{x^2}{\cancel{2}} + 36x\right)\Big|_3^6 = 9 + \left(\frac{6^3}{3} - \cancel{6^3} + \cancel{6^3}\right.$$

$$\left. - (9 - 54 + 108)\right)$$

$$\int \sin x\,dx = -\cos x + C$$

2) $z = \ln(y-2x) + xy - x$

$$\frac{\partial z}{\partial x} = \frac{1}{y-2x}\cdot(-2) + y - 1 = \boxed{\frac{-2}{y-2x} + y - 1}$$

$$\frac{\partial z}{\partial y} = \frac{1}{y-2x}\cdot 1 + x = \frac{1}{y-2x} + x$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{0-(-2)\cdot(-2)}{(y-2x)^2} = \frac{-4}{(y-2x)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{0-(-2)\cdot 1}{(y-2x)^2} + 1 = \frac{2}{(y-2x)^2} + 1$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{0-1\cdot 1}{(y-2x)^2} = \frac{-1}{(y-2x)^2}$$

3)

$$z = ye^{x^3} + \sin x \ln y + \operatorname{ctg}(x+y) \qquad (5 \cdot e^{x^3})' = 5(e^{x^3})'$$

$$\frac{\partial z}{\partial x} = y e^{x^3} \cdot 3x^2 + \ln y \cdot \cos x + \frac{-1}{\sin^2(x+y)} \cdot (1+0) = 3yx^2e^{x^3} + \ln y \cos x - \frac{1}{\sin^2(x+y)}$$

$$\sin^{-2}(x+y)$$

$$\frac{\partial z}{\partial y} = e^{x^3} + \sin x \; \frac{1}{y} + \frac{-1}{\sin^2(x+y)} (0+1) = e^{x^3} + \sin x \cdot \frac{1}{y} - \frac{1}{\sin^2(x+y)}$$

$$\frac{\partial^2 z}{\partial x^2} = 3y\left(2x e^{x^3} + x^2 \cdot e^{x^3} \cdot 3x^2\right) + \ln y(-\sin x) - \frac{0 - 1 \cdot 2 \sin(x+y) \cdot \cos(x+y) \cdot 1}{\sin^4(x+y)}$$

$$\frac{\partial^2 z}{\partial x \partial y} = 3x^2e^{x^3} + \cos x \cdot \frac{1}{y} - \left(-2 \sin^{-3}(x+y) \cos(x+y) \cdot 1\right)$$

$$\frac{\partial^2 z}{\partial y} = \sin x \; \frac{-1}{y^2} - \frac{0 - 1 \cdot 2 \sin(x+y) \cos(x+y) \cdot 1}{\sin^4(x+y)}$$

* $d^2 z(M) > 0$ lokalni min

$d^2 z(M) < 0$ lokalni max

$d^2 z(M)$ – neodr. znak ⟹ nema ekstremuma

$\boxed{\underline{\underline{dx\,dy}}}$

*

$d^2 z(M) = 4\underline{\underline{dx\,dy}}$

$d^2 z(M) = \underline{dx^2} - \underline{dy^2}$

$rs - t^2 > 0$ – ima e.v.

$r > 0$ lokalni min

$r < 0$ lokalni max

$rs - t^2 < 0$ – nema e.v.

$rs - t^2 = 0$ – nema odgovora, tj. nemamo pojma

4) $d^2z = -dx^2 + 2dxdy - \frac{1}{4}dy^2$ $\qquad A^2 - 2AB + B^2$

$= -\left(dx^2 - 2dxdy + dy^2 - dy^2 + \frac{1}{4}dy^2\right)$

$= -\left((dx-dy)^2 - dy^2 + \frac{1}{4}dy^2\right)$

$= -\left((dx-dy)^2 - \frac{3}{4}dy^2\right)$ $\leftarrow$ неопр знак

$3dx^2 + 6dxdy + 4dy^2$

$= 3\left(dx^2 + 2dxdy + dy^2 - dy^2 + \frac{4}{3}dy^2\right) = 3\left((dx+dy)^2 \underbrace{- dy^2 + \frac{4}{3}dy^2}\right)$

$= 3\left((dx+dy)^2 + \frac{1}{3}dy^2\right) > 0$

5) $\boxed{z = x^3 + 3xy^2 - 12x}$

$\frac{\partial z}{\partial x} = \boxed{3x^2 + 3y^2 - 12}$

$\frac{\partial z}{\partial y} = 3x \cdot 2y = 6xy$

$3x^2 + 3y^2 - 12 = 0$

$6x \cdot y = 0$

$x = 0 \quad \vee \quad y = 0$

$3y^2 - 12 = 0$

$3y^2 = 12$

$y^2 = 4$

$y = \pm 2$

$3x^2 - 12 = 0$

$x^2 = 4$

$x = \pm 2$

$M_1(0,2),\ M_2(0,-2)$

$M_3(2,0),\ M_4(-2,0)$

$\frac{\partial^2 z}{\partial x^2} = 6x \qquad \frac{\partial^2 z}{\partial x \partial y} = 6y \qquad \frac{\partial^2 z}{\partial y^2} = 6x$

I M_1: $r = \frac{\partial^2 z}{\partial x^2}(M_1) = 0$

$S = \frac{\partial^2 z}{\partial y^2}(M_1) = 0$

$t = \frac{\partial^2 z}{\partial x \partial y}(M_1) = 12$

$rs - t^2 = 0 - 144 = -144 < 0$

нема е.v.

M_2: $r = \frac{\partial^2 z}{\partial x^2}(M_2) = 0$

$S = \frac{\partial^2 z}{\partial y^2}(M_2) = 0$

$t = \frac{\partial^2 z}{\partial x \partial y}(M_2) = -12$

$rs - t^2 = 0 - 144 = -144 < 0$

нема е.v.

M_3:

$$r = \frac{\partial^2 z}{\partial x^2}(M_3) = 12$$

$$s = \frac{\partial^2 z}{\partial y^2}(M_3) = 12$$

$$t = \frac{\partial^2 z}{\partial x \partial y}(M_3) = 0$$

$rs - t^2 = 144 - 0 = 144 > 0 \Rightarrow$ ima e.v.

$r = 12 \Rightarrow$ lokalni minimum

$(s = 12)$ $\quad z_{min}(2,0) = 2^3 + 3 \cdot 2 \cdot 0^2 - 12 \cdot 2$

M_4:

$$r = \frac{\partial^2 z}{\partial x^2}(M_4) = -12$$

$$s = \frac{\partial^2 z}{\partial y^2}(M_4) = -12$$

$$t = \frac{\partial^2 z}{\partial x \partial y}(M_4) = 0$$

$rs - t^2 = 144 - 0 = 144 > 0$ ima e.v.

$r = -12 \rightarrow$ lokalni maximum

$z_{max}(-2,0) = (-2)^3 + 3 \cdot (-2) \cdot 0^2 - 12(-2)$

II $\quad d^2z = \frac{\partial^2 z}{\partial x^2}dx^2 + 2\frac{\partial^2 z}{\partial x \partial y}dx\,dy + \frac{\partial^2 z}{\partial y^2}dy^2$

$= 6x\,dx^2 + 12y\,dx\,dy + 6x\,dy^2$

M_1: $\quad d^2z(M_1) = \cancel{6\cdot 0\,dx^2} + 12\cdot 2\,dx\,dy + \cancel{6\cdot 0\,dy^2} = 24\,dx\,dy \leftarrow$ неод знак

$\Rightarrow$ нема е.в.

M_2: $\quad d^2z(M_2) = \cancel{6\cdot 0\,dx^2} + 12\cdot(-2)\,dx\,dy + \cancel{6\cdot 0\,dy^2} = -24\,\underline{dx\,dy} \leftarrow$ неод знак

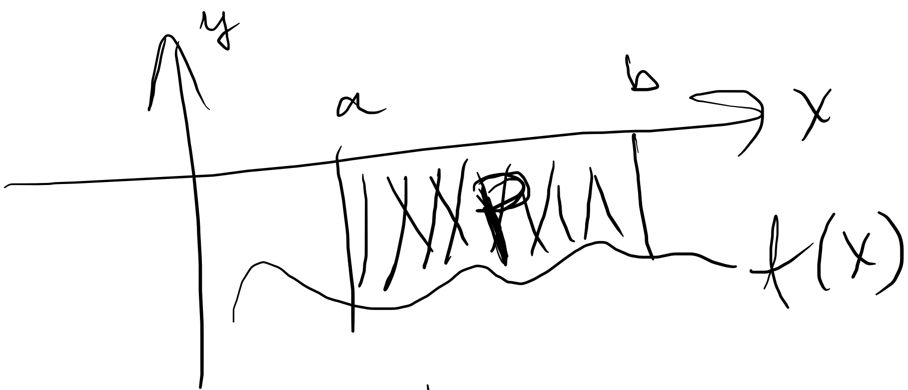
$\Rightarrow$ нема е.в.

M_3: $\quad d^2z(M_3) = 12dx^2 + \cancel{12\cdot 0\,dx\,dy} + 12dy^2 = 12(dx^2+dy^2) > 0 \Rightarrow$ локални минимум

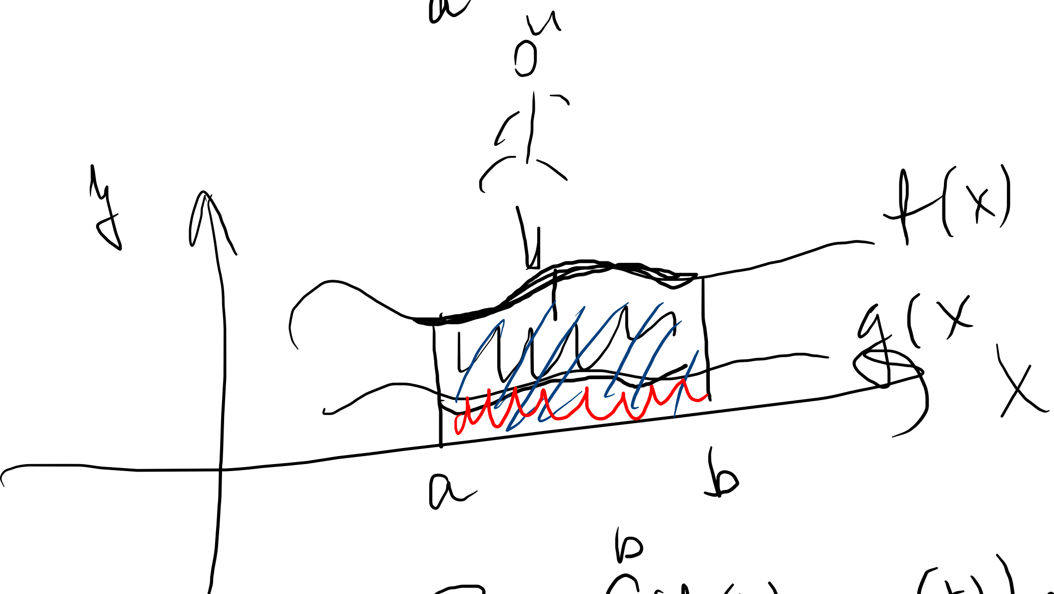
$(dx,dy)\neq(0,0)$ $\qquad z_{min}(2,0) = \ldots$

M_4: $\quad d^2z(M_2) = -12dx^2 - 12dy^2 = -12(dx^2+dy^2) < 0 \rightarrow$ локални max

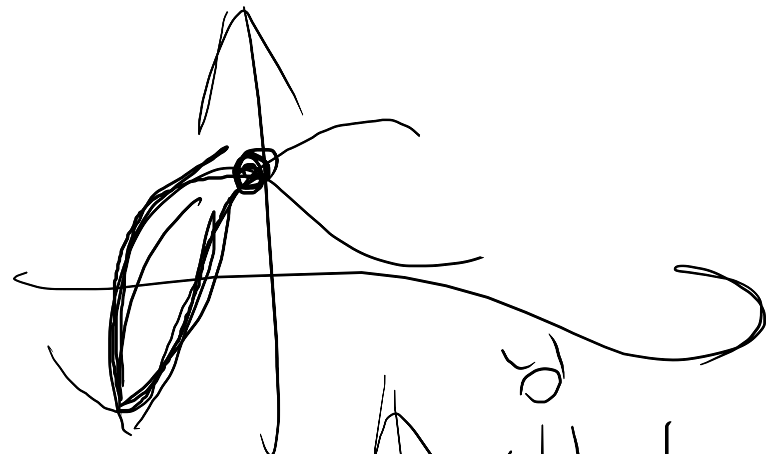
$z_{max}(-2,0) = \ldots$

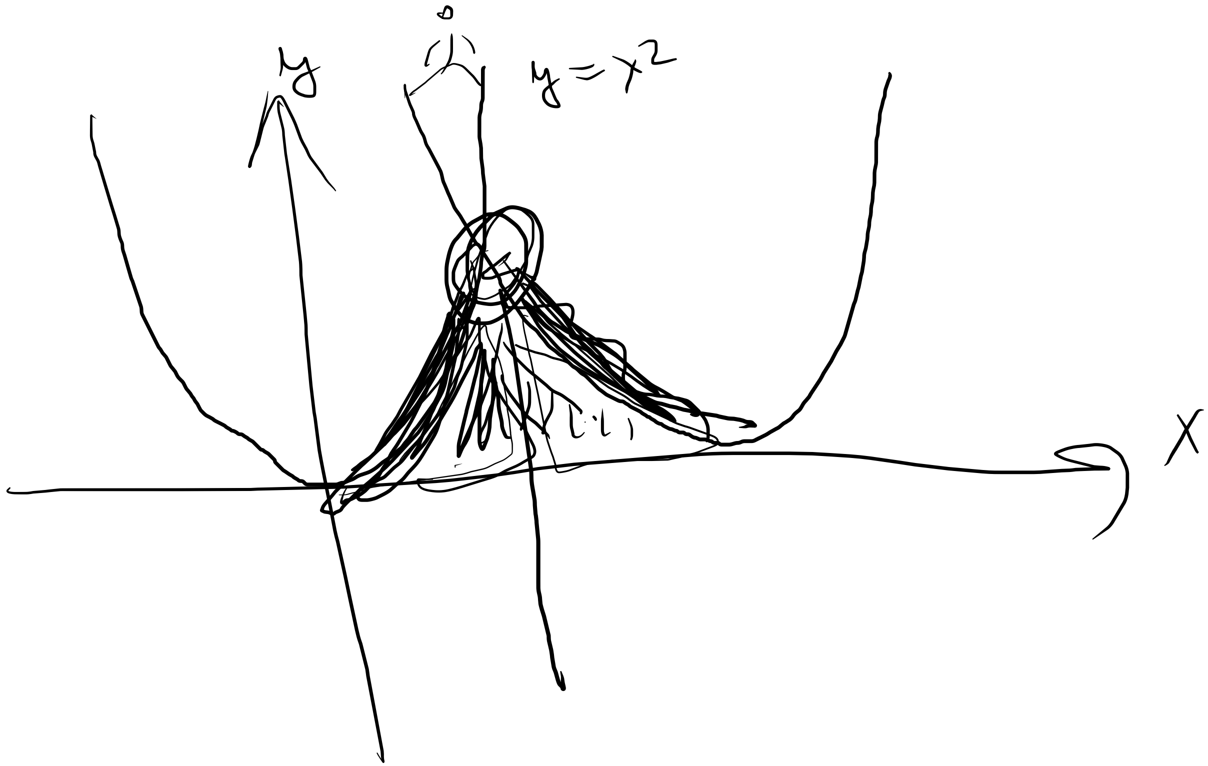


$$P = -\int_a^b f(x)\,dx$$



$$P = \int_a^b \left(f(x) - g(x)\right) dx$$




y
y = x^2
X

6 $\boxed{z = \frac{y}{x}} = y \cdot \frac{1}{x}$

$$\frac{\partial z}{\partial x} = \boxed{y \, \frac{-1}{x^2}} = -y \cdot \frac{1}{x^2}$$

$$\frac{\partial z}{\partial y} = \frac{1}{x}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$

$$\frac{\partial^2 z}{\partial x^2} = -y\left(-2x^{-3}\right)$$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{1}{x^2}$$

$$\frac{\partial^2 z}{\partial y^2} = 0$$

7

$y = 2^x$

$y = 2x - x^2$

$x = 0$

$x = 2$

$P = \int_0^2 (2^x - (2x - x^2)) dx$

$= \int_0^2 2^x dx - 2\int_0^2 x dx + \int_0^2 x^2 dx$

$y = 2x - x^2$

$y = 0 \qquad 2x - x^2 = 0$

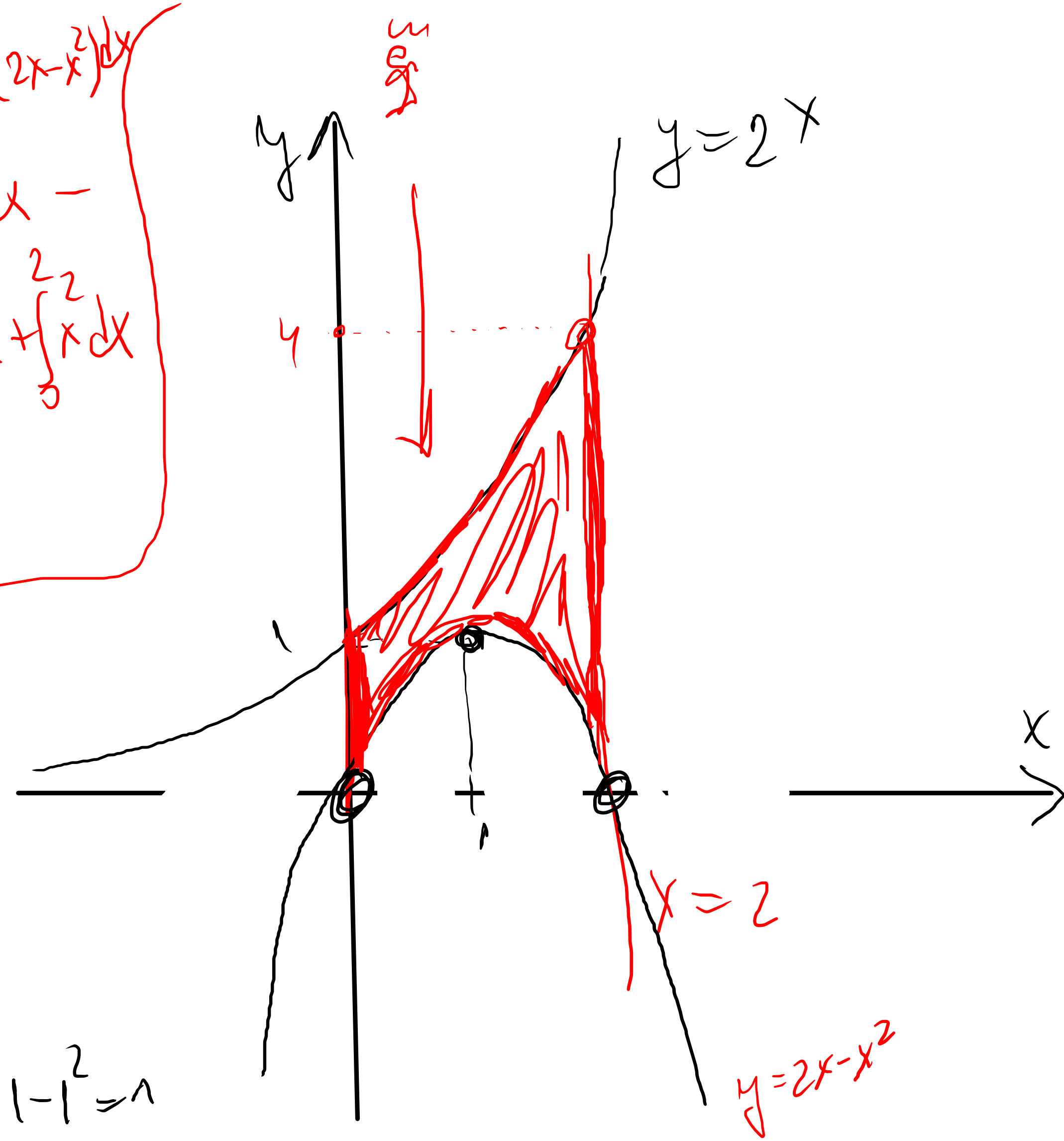
$x(2-x) = 0$

$x = 0 \quad x = 2$

$y' = 2 - 2x$

$y' = 0 \qquad 2 - 2x = 0$

$\boxed{x = 1} \rightarrow y(1) = 2 \cdot 1 - 1^2 = 1$



8) $y' - \frac{2}{x} y = 2x\sqrt{y}$

smena: $y = uv \Rightarrow y' = u'v + uv'$ $\quad x>0$

$u'v + \underbrace{uv' - \frac{2}{x} uv} = 2 \cdot x \sqrt{uv}$

$u'v + u\underbrace{\left(v' - \frac{2}{x} v\right)}_{=0} = 2x\sqrt{uv}$

$v' - \frac{2}{x} v = 0$

$v' = \frac{2}{x} v$

$\frac{dv}{dx} = \frac{2}{x} v$

$dv = \frac{2}{x} v \, dx$

$\int \frac{1}{v} dv = \int \frac{2}{x} dx$

$\ln v = 2 \ln x$

$\ln v = \ln x^2$

$\boxed{v = x^2}$

$u' \cdot \cancel{x^2} = 2\cancel{x}\sqrt{u \cdot x^2}$

$u' \cancel{x} = 2 \cdot \sqrt{u} \cdot \cancel{x}$

$u' = 2 \cdot \sqrt{u}$

$\frac{du}{dx} = 2\sqrt{u}$

$du = 2\sqrt{u}\, dx$

$\int \frac{1}{\sqrt{u}} du = 2\int dx$

$\int u^{-\frac{1}{2}} du = 2\int dx$

$\frac{u^{\frac{1}{2}}}{\frac{1}{2}} = 2x + C$

$\boxed{\begin{array}{l} \ln A = B \Leftrightarrow A = e^B \\ m \ln A = \ln A^m \end{array}}$

$2\sqrt{u} = 2x + C \quad / :2$

$\sqrt{u} = x + \frac{C}{2} \quad /^2$

$\boxed{u = \left(x + \frac{C}{2}\right)^2}$

$y = uv = \left(x + \frac{C}{2}\right)^2 \cdot x^2$

$$\int r(\underbrace{\sin x}_{t})\underbrace{\cos x\,dx}_{dt}$$

$$\int r(\underbrace{\cos x}_{t})\underbrace{\sin x\,dx}_{-dt}$$

$$\sin^2\alpha+\cos^2\alpha=1$$

$$\int \frac{1}{\sin^2 x+\sin x+1}\cdot\cos x\,dx = \int\frac{1}{t^2+t+1}\,dt = \int\frac{1}{(t+\frac{1}{2})^2+\frac{3}{4}}\,dt$$

смена: $\sin x = t$

$\cos x\,dx = dt$

$$t^2+t+1 = t^2+2\cdot t\cdot\frac{1}{2}+\frac{1}{4}-\frac{1}{4}+1$$

$$=(t+\tfrac{1}{2})^2+\frac{3}{4}$$

$$=\frac{1}{\frac{\sqrt{3}}{2}}\operatorname{arctg}\frac{1}{\frac{\sqrt{3}}{2}}\left(t+\frac{1}{2}\right)+C=$$

$$\int \frac{1}{-\cos^2 x + \sin x + 2} \cos x \, dx$$

$$= \int \frac{1}{-(1-\sin^2 x) + \sin x + 2} \cos x \, dx$$

$$= \int \frac{1}{\sin^2 x + \sin x + 1} \cos x \, dx$$

$$A^2 \pm 2AB + B^2 = (A \pm B)^2$$

$$\left.\begin{array}{l}\int p_n(x)\, e^{\alpha x}\, dx \\ \int p_n(x) \sin \alpha x\, dx \\ \int p_n(x) \cos \alpha x\, dx\end{array}\right\} \quad \begin{array}{l} u = p_n(x) \\ \text{ostalo u } \underline{dv}\end{array}$$

$\underline{\ln x}$, arc$\underline{tg\, x}$, $\underline{\text{arc}\sin x}$