

# Lopitalovo pravilo

March 21, 2021

Neka su funkcije  $f$  i  $g$  diferencijabilne u nekoj okolini tačke  $a \in \mathbb{R}$ , sem eventualno u samoj tački  $a$  i neka je  $g'(x) \neq 0$  za svako  $x$  iz te okoline. Ako je  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$ , i ako postoji

$$\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} = A, \quad A \in \mathbb{R}, \text{ tada je:}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} = A.$$

$\frac{0}{0}$ ,  $\frac{\infty}{\infty}$

Lopitalovo pravilo važi i ako je  $A = \pm\infty$  i kada  $x \rightarrow \pm\infty$ .

Takođe važi i u slučaju kada je  $\lim_{x \rightarrow a} f(x) = \pm\infty$  i  $\lim_{x \rightarrow a} g(x) = \pm\infty$ .

**Primer 1.** Izračunati  $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ .

Kako je  $\lim_{x \rightarrow 0} \sin x = 0$  i  $\lim_{x \rightarrow 0} x = 0$  i kako postoji

$$\lim_{x \rightarrow 0} \frac{(\sin x)'}{(x)'} = \lim_{x \rightarrow 0} \frac{\cos x}{1} = 1,$$

to je  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ .

**Primer 2.** Izračunati  $\lim_{x \rightarrow \infty} \frac{\ln x}{x^a}$ ,  $a > 0$ .

Kako je  $\lim_{x \rightarrow \infty} \ln x = \infty$  i  $\lim_{x \rightarrow \infty} x^a = \infty$ , za  $a > 0$  i kako postoji

$$\lim_{x \rightarrow \infty} \frac{(\ln x)'}{(x^a)'} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{ax^{a-1}} = \lim_{x \rightarrow \infty} \frac{1}{ax^a} = 0,$$

to je  $\lim_{x \rightarrow \infty} \frac{\ln x}{x^a} = 0$ .

Obrnuto ne mora da važi, tj. ako ne postoji  $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$  to ne mora da znači da i  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  ne postoji.

Primer 3.

$$\lim_{x \rightarrow \infty} \frac{x + \sin x}{x} = \lim_{x \rightarrow \infty} \left( 1 + \frac{\sin x}{x} \right) = 1$$

jer je funkcija  $\sin x$  ograničena, tj  $\sin x \in [-1, 1]$ , a  $\frac{1}{x} \rightarrow 0$ , kad  $x \rightarrow \infty$ .

Dakle, ova granična vrednost postoji, a ne može se primeniti Lopitalovo pravilo za njeno izračunavanje jer

$$\lim_{x \rightarrow \infty} \frac{x + \sin x}{x} + \lim_{x \rightarrow \infty} \frac{1 + \cos x}{1}$$

ne postoji.

Pored toga što se primenjuje na neodređene izraze oblika „ $\frac{0}{0}$ “ i „ $\frac{\infty}{\infty}$ “, Lopitalovo pravilo se može primeniti i na ostale neodređene izraze („ $0 \cdot \infty$ “, „ $\infty - \infty$ “, „ $1^\infty$ “, „ $0^0$ “, „ $\infty^0$ “) koji se elementarnim aritmetičkim transformacijama svode na prethodna dva slučaja.

- „ $0 \cdot \infty$ “: Ako  $f(x) \rightarrow 0$ ,  $x \rightarrow a$  i  $g(x) \rightarrow \pm\infty$ ,  $x \rightarrow a$  tada je

$$\lim_{x \rightarrow a} f(x) g(x) = \lim_{x \rightarrow a} \frac{f(x)}{\frac{1}{g(x)}} = \frac{0}{0}$$

ili

$$\lim_{x \rightarrow a} f(x) g(x) = \lim_{x \rightarrow a} \frac{g(x)}{\frac{1}{f(x)}} = \frac{\infty}{\infty}.$$

► „ $\infty - \infty$ “! Ako  $f(x) \rightarrow \infty$ ,  $x \rightarrow a$  i  $g(x) \rightarrow \pm\infty$ ,  
 $x \rightarrow a$  tada je

$$\lim_{x \rightarrow a} (f(x) - g(x)) = \lim_{x \rightarrow a} f(x) \left( 1 - \frac{g(x)}{f(x)} \right) = " \infty \cdot 0 ".$$

Može se desiti i da  $1 - \frac{g(x)}{f(x)}$  ne teži u nulu kad  $x$  teži  $a$ . U  
tom slučaju  $f(x) - g(x)$  teži u  $\pm\infty$  kad  $x$  teži  $a$ .

Hurđelja:  
obalno &  
nivo nizke

$$\begin{aligned} & \lim_{x \rightarrow \infty} (\sqrt{x^2 - 1} - \sqrt{x^2 + 4}) \stackrel{\infty}{=} \lim_{x \rightarrow \infty} (\sqrt{x^2 - 1} - \sqrt{x^2 + 4}) \cdot \frac{\sqrt{x^2 - 1} + \sqrt{x^2 + 4}}{\sqrt{x^2 - 1} + \sqrt{x^2 + 4}} \\ &= \lim_{x \rightarrow \infty} \frac{x^2 - 1 - x^2 - 4}{\sqrt{x^2 - 1} + \sqrt{x^2 + 4}} = \frac{-5}{\infty} = 0 \end{aligned}$$

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 - 1} - \sqrt{x^2 + 4}) = \lim_{x \rightarrow \infty} \sqrt{x^2 - 1} \left( 1 - \sqrt{\frac{x^2 + 4}{x^2 - 1}} \right) = \lim_{x \rightarrow \infty} \frac{\left( 1 - \sqrt{\frac{x^2 + 4}{x^2 - 1}} \right)}{\left( \frac{1}{\sqrt{x^2 - 1}} \right)^{-1}} = \frac{0}{0} = \dots$$

nivo nizke  
am voleo

► „ $0^0$ “, „ $\infty^0$ “ i „ $1^\infty$ “: U sva tri slučaja izraz oblika  $\lim_{x \rightarrow a} f(x)^{g(x)}$ ,  $f(x) > 0$  se svodi na oblik „ $0 \cdot \infty$ “ i to na sledeći način

$$\boxed{\lim_{x \rightarrow a} f(x)^{g(x)} = A} \quad \boxed{\ln}$$

$$\ln A^B = B \ln A$$

„PEŠKE“  
JE MOGAO  
SAMO  $1^\infty$

PREGO

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

„ $0^0$ “ i „ $\infty^0$ “  
DO SADA NISAM  
ZNALI DA  
VRADIMO

$$\begin{aligned} \ln A &= \ln \left( \lim_{x \rightarrow a} f(x)^{g(x)} \right) \\ &= \lim_{x \rightarrow a} \ln(f(x)^{g(x)}) \\ &= \lim_{x \rightarrow a} g(x) \ln f(x) \quad \text{(zato što } g(x) \rightarrow 0 \text{ i } \ln f(x) \rightarrow \infty \text{)} \\ &= "0 \cdot \infty". \end{aligned}$$

$$\text{„}0^0\text{“}, \text{ „}\infty^0\text{“}, \text{ „}1^\infty\text{“}$$

Primenom Lopitalovog pravila izračunati granične vrednosti:

1.  $\lim_{x \rightarrow -1} \frac{x^3 + x^2 - x - 1}{x^3 + 2x^2 - x - 2} =$

Unterschl.  
v.z.v.v.

$$2. \lim_{x \rightarrow 4} \frac{x\sqrt{x} - 4\sqrt{x}}{x^2 - 16} =$$

3.  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sin(x - 2)} =$

4.  $\lim_{x \rightarrow \infty} \frac{e^x}{x^3} =$

5.  $\lim_{x \rightarrow 0} \frac{x^3}{x - \sin x} =$

6.  $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\sin 2x} =$



7.  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln \sin x}{(\pi - 2x)^2} = -\frac{1}{4}$



8.  $\lim_{x \rightarrow 0} \frac{\arcsin 2x}{\arcsin 3x} = \frac{2}{3}$

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9.  $\lim_{x \rightarrow 0} \frac{\operatorname{arctg} 5x}{\operatorname{arctg} 2x} = \frac{5}{2}$



10.  $\lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{x^3} = -\frac{1}{3}$

$$f(x) = \frac{\sin x}{\cos x}$$

$$11. \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - x}{x - \sin x} \stackrel{0}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\cos^2 x} - 1}{1 - \cos x} = \lim_{x \rightarrow 0} \left[ \frac{\frac{1 - \cos^2 x}{\cos^2 x}}{1 - \cos x} \right] =$$

$$= \lim_{x \rightarrow 0} \frac{\overbrace{\sin^2 x}^{1 - \cos^2 x}}{\cos^2 x (1 - \cos x)} = \lim_{x \rightarrow 0} \frac{(1 - \cos x)(1 + \cos x)}{\cos^2 x (1 - \cos x)} = \frac{1+1}{1} = 2$$

$$= \lim_{x \rightarrow 0} \frac{\operatorname{tg}^2 x}{1 - \cos x} \stackrel{0}{\geq} \lim_{x \rightarrow 0} \frac{2 \operatorname{tg} x \frac{1}{\cos^2 x}}{8 \sin x} \stackrel{0}{=} \dots$$

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$$12. \lim_{x \rightarrow 0} \frac{2e^x - 2 - 2x - x^2}{2x^3} = \frac{1}{6}$$



13.  $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\ln(e-x) + x - 1} = \underline{\quad}$



14.  $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} = 2$

$$\begin{aligned}
 15. \lim_{x \rightarrow 0} \frac{\cos 2x - 2 \cos x + 1}{x^2} &= \lim_{x \rightarrow 0} \frac{-\sin 2x \cdot 2 + 2 \sin x}{2x} = \\
 &= \lim_{x \rightarrow 0} \frac{2(-\sin 2x + \sin x)}{2x} = \lim_{x \rightarrow 0} \frac{-\cos 2x \cdot 2 + \cos x}{1} \\
 &= \frac{-2+1}{1} = -1
 \end{aligned}$$

$$16. \lim_{x \rightarrow 0} \frac{e^x \cdot \sin x - x}{3x^2 + x^5} \stackrel{0/0}{=} \lim_{x \rightarrow 0} \frac{e^x \cdot \sin x + e^x \cos x - 1}{6x + 5x^4} \stackrel{0/0}{=}$$

$$= \lim_{x \rightarrow 0} \frac{\cancel{e^x \sin x} + e^x \cos x + e^x \cos x - \cancel{e^x \sin x}}{6 + 20x^3} = \frac{1+1}{6} = \frac{1}{3}$$

$$\operatorname{arctg} 0 = ?$$

$\tan \alpha \neq \infty$   
 $\operatorname{tg} \alpha \neq \infty$

$$\operatorname{tg} 0 = 0$$

$$\alpha \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$\operatorname{arctg} 1 = ?$$

$$\operatorname{tg} \alpha = 1$$

$$\alpha = \frac{\pi}{4}$$

$$\operatorname{arctg} \sqrt{3} = ?$$

$$\operatorname{tg} \alpha = \sqrt{3}$$

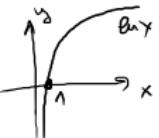
$$\alpha = \frac{\pi}{3}$$

$$17. \lim_{x \rightarrow 0^+} \frac{x \operatorname{arctg} x}{1 - \cos x} \stackrel{0/0}{=} \lim_{x \rightarrow 0^+} \frac{\operatorname{arctg} x + \frac{1}{1+x^2}}{+\sin x} =$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{1+x^2} + \frac{1+x^2 - x \cdot 2x}{(1+x^2)^2}}{\cos x}$$

$$= \frac{1 + \frac{1}{1}}{1} = \frac{2}{1} = 2$$

$$\underbrace{\frac{0}{0}, \frac{\infty}{\infty}}_{0 \cdot \infty, \infty - \infty, 1^\infty, 0^\circ, \infty^\circ}$$



$$18. \lim_{x \rightarrow 0^+} x^2 \ln x = \underset{0 \leftarrow 0^+}{\lim_{x \rightarrow 0^+}} \frac{\ln x}{\frac{1}{x^2}} \stackrel{\infty}{=} \underset{x \rightarrow 0^+}{\lim} \frac{\frac{1}{x}}{-2 \frac{1}{x^3}} =$$

$$A = \left[ \frac{\frac{1}{x}}{-2 \frac{1}{x^3}} \right]_{x \rightarrow 0^+} = \underset{x \rightarrow 0^+}{\lim} \frac{x^2}{-2} = 0$$

$$\underset{x \rightarrow 0^+}{\lim} \frac{x^2}{\frac{1}{\ln x}} = \underset{x \rightarrow 0^+}{\lim} \frac{\frac{2x}{1}}{\frac{-\frac{1}{x}}{\ln^2 x}} = \underset{x \rightarrow 0^+}{\lim} \frac{2x \cdot \frac{\ln^2 x}{1}}{-\frac{1}{x}} =$$

$$= \underset{x \rightarrow 0^+}{\lim} \frac{-2x^2 \ln^2 x}{A} \stackrel{0 \cdot \infty}{=} /$$

TEST, KOMPLIKOVANÍ  
NIE DOBÁR PUT"

$$19. \lim_{x \rightarrow 1} (\ln(x-1) \ln x) \stackrel{x \rightarrow 1}{\stackrel{\infty}{\infty}} = \lim_{x \rightarrow 1} \frac{\ln(x-1)}{\frac{1}{\ln x}} \stackrel{x \rightarrow 1}{\stackrel{\infty}{\infty}} =$$

$$= \lim_{x \rightarrow 1} \left[ \frac{\frac{1}{x-1}}{-\frac{1}{x^2}} \right] \stackrel{x \rightarrow 1}{=} \lim_{x \rightarrow 1} \frac{\ln^2 x}{-\frac{x-1}{x}} \stackrel{x \rightarrow 1}{\stackrel{0}{0}} =$$

$$= \lim_{x \rightarrow 1} \left[ \frac{2 \cdot \ln x \cdot \frac{1}{x}}{-\frac{x-(x-1)}{x^2}} \right] = \lim_{x \rightarrow 1} \frac{2x \ln x}{-1} = \frac{2 \cdot 1 \cdot 0}{-1} = 0$$

$$20. \lim_{x \rightarrow 0} x \cdot \operatorname{ctg} x = \lim_{x \rightarrow 0} \frac{x}{\frac{1}{\operatorname{ctg} x}} = \lim_{x \rightarrow 0} \frac{x}{\frac{1}{\cos^2 x}} = 1$$

$$\operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha = 1$$

$$\operatorname{tg} \alpha = \frac{1}{\operatorname{ctg} \alpha}$$

$$\operatorname{ctg} \alpha = \frac{1}{\operatorname{tg} \alpha}$$

$$\operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\operatorname{tg} \alpha = \frac{1}{\operatorname{ctg} \alpha}$$

$$\operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\operatorname{ctg} \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$21. \lim_{x \rightarrow 0} x^2 e^{\frac{1}{x^2}} = \lim_{x \rightarrow 0} \frac{e^{\frac{1}{x^2}}}{\frac{1}{x^2}} = \lim_{x \rightarrow 0} \frac{e^{\frac{1}{x^2}}}{(\frac{1}{x^2})^1}$$

$$\left. \begin{array}{l} \lim_{x \rightarrow 0^+} \frac{1}{x} = +\infty \\ \lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty \end{array} \right\}$$

$$\lim_{x \rightarrow 0} \frac{1}{x} \text{ -NE POSSII}$$

$$\lim_{x \rightarrow 0} x^2 e^{\frac{1}{x}}$$

$$\begin{aligned} e^{+\infty} &= +\infty \\ e^{-\infty} &= 0 \end{aligned}$$

$$+\infty = \infty$$

22.  $\lim_{x \rightarrow \pi} (\pi - x) \operatorname{tg} \frac{x}{2} = 2$

$$\begin{aligned}
 23. \lim_{x \rightarrow 0^+} \left( \frac{1}{\sin x} - \frac{1}{x} \right) &= \lim_{x \rightarrow 0^+} \frac{x - \sin x}{x \sin x} = \frac{0}{0} \\
 &= \lim_{x \rightarrow 0^+} \frac{1 - \cos x}{\sin x + x \cos x} = \frac{0}{0} \\
 &= \lim_{x \rightarrow 0^+} \frac{+ \sin x}{\cos x + \cos x + x(-\sin x)} \\
 &= \frac{0}{1+1+0} = 0
 \end{aligned}$$

Motore "zeggeren"

oogtak

$\lim_{x \rightarrow 0^+} \left( \operatorname{ctg} x - \frac{1}{x} \right)^{\infty - \infty}$   $\lim_{x \rightarrow 0^+} \left( \frac{\cos x}{\sin x} - \frac{1}{x} \right)$

$\lim_{x \rightarrow 0^+} \frac{x \operatorname{ctg} x - 1}{x}$

$$= \lim_{x \rightarrow 0^+} \frac{x \cos x - \sin x}{x \sin x}$$

$$= \lim_{x \rightarrow 0^+} \frac{\cos x - x \sin x - \cos x}{\sin x + x \cos x}$$

$$= \lim_{x \rightarrow 0^+} \frac{-\sin x - x \cos x}{\cos x + \cos x - x \sin x} = \frac{0}{1+1-0} = 0$$



25.  $\lim_{x \rightarrow 1^+} \left( \frac{1}{\ln x} - \frac{x}{\ln x} \right) = -\gamma$

$$26. \lim_{x \rightarrow 1^+} \left( \frac{x}{x-1} - \frac{1}{\ln x} \right) = \lim_{x \rightarrow 1^+} \frac{x \ln x - x + 1}{(x-1) \ln x} =$$

$$= \lim_{x \rightarrow 1^+} \frac{\cancel{\ln x + x \cdot \frac{1}{x}} - 1}{\ln x + (x-1) \cdot \frac{1}{x}} = \lim_{x \rightarrow 1^+} \frac{\frac{1}{x}}{\frac{1}{x} + \frac{-x+1}{x^2}} =$$

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27.  $\lim_{x \rightarrow 1^+} \left( \frac{2}{x^2 - 1} - \frac{1}{x - 1} \right) = -\frac{1}{2}$

$$28. \lim_{x \rightarrow 0^+} x^{\sin x} \stackrel{0^0}{=} A$$

$$\ln A = B \Leftrightarrow A = e^B$$

$$\boxed{\ln A = \ln \lim_{x \rightarrow 0^+} x^{\sin x}} = \lim_{x \rightarrow 0^+} \ln x^{\sin x} = \lim_{x \rightarrow 0^+} \sin x \cdot \ln x$$

$$\begin{aligned} &= \lim_{x \rightarrow 0^+} \frac{\sin x}{\frac{1}{\ln x}} \stackrel{0^0}{=} \lim_{x \rightarrow 0^+} \frac{-\cos x}{-\frac{1}{x^2}} = \lim_{x \rightarrow 0^+} \frac{\cos x \cdot \ln^2 x}{\frac{1}{x}} \\ &= \lim_{x \rightarrow 0^+} -x \cdot \cos x \cdot \ln^2 x \stackrel{0^0}{=} \end{aligned}$$

sbe oecuho yprofeno  
aw ce uzbog  
zalcomilimkobas uzo  
zhalan qo hau tufa gos  
gordap uszgor

$$\begin{aligned} &\stackrel{0^0}{=} \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{\sin x}} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{-\cos x}{\sin^2 x}} = \lim_{x \rightarrow 0^+} \frac{\sin^2 x}{-x \cos x} \stackrel{0^0}{=} \lim_{x \rightarrow 0^+} \frac{2 \sin x \cdot \cos x}{-\cos x + x \sin x} \\ &= \frac{0}{-1+0} = \boxed{0} \end{aligned}$$

$$\ln A = 0 \rightarrow A = e^0 = 1$$

$$29. \lim_{x \rightarrow 0^+} (\sin x)^x \stackrel{0^0}{=} A$$

~~$\ln A = \ln \lim_{x \rightarrow 0^+} (\sin x)^x = \lim_{x \rightarrow 0^+} \ln(\sin x)$~~

$\ln A = \lim_{x \rightarrow 0^+} \frac{\ln(\sin x)}{\frac{1}{x}} \stackrel{\infty}{=} \lim_{x \rightarrow 0^+} \frac{\frac{1}{\sin x} \cdot \cos x}{-\frac{1}{x^2}} \stackrel{(1)}{=} \lim_{x \rightarrow 0^+} \frac{-x^2 \operatorname{ctg} x}{\cos x} \stackrel{0 \cdot \infty}{=} \lim_{x \rightarrow 0^+} \frac{-2x}{\frac{1}{\cos^2 x}} = 0$

(1)  $= \lim_{x \rightarrow 0^+} \frac{-x^2 \cos x}{\sin x} \stackrel{0/0}{=} \lim_{x \rightarrow 0^+} \frac{-2x \cos x + x^2 \sin x}{\cos x} = \frac{0}{1} = 0$

$$\ln A = 0 \rightarrow A = e^0 = 1$$

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30.  $\lim_{x \rightarrow 0^+} (3x + 1)^{\operatorname{ctg} x} = e^3$

$$31. \lim_{x \rightarrow 0^+} (\operatorname{ctg} x)^{\frac{1}{\ln x}} \stackrel{\stackrel{\infty}{\circ}}{=} A$$

$\ln A = \ln \lim_{x \rightarrow 0^+} (\operatorname{ctg} x)^{\frac{1}{\ln x}}$   $= \lim_{x \rightarrow 0^+} \ln(\operatorname{ctg} x)^{\frac{1}{\ln x}}$   $= \lim_{x \rightarrow 0^+} \frac{1}{\ln x} \cdot \ln(\operatorname{ctg} x)$

$\stackrel{0 \cdot \infty}{=} \lim_{x \rightarrow 0^+} \frac{\ln(\operatorname{ctg} x)}{\ln x} \stackrel{\stackrel{\infty}{\circ}}{=} \lim_{x \rightarrow 0^+} \left[ \frac{1}{\operatorname{ctg} x} \cdot \frac{-1}{\sin^2 x} \right] =$

$= \lim_{x \rightarrow 0^+} \frac{-x}{\frac{\cos x}{\sin x} \cdot \sin^2 x} = \lim_{x \rightarrow 0^+} \frac{-x}{\cos x \cdot \sin x} \stackrel{\stackrel{0}{\circ}}{=} \lim_{x \rightarrow 0^+} \frac{-1}{-\sin^2 x + \cos^2 x}$

$= \frac{-1}{0+1} = -1$

$$\ln A = -1 \Rightarrow A = e^{-1} = \frac{1}{e}$$

(32)  $\lim_{x \rightarrow 0^+} x^{\frac{5}{2+3 \ln x}} = e^{\frac{5}{3}}$

$$33. \lim_{x \rightarrow 0^+} \left( \frac{\sin x}{x} \right)^{\frac{1}{x^2}} = A$$

$\ln A = -\frac{1}{6}$   $\Rightarrow A = e^{-\frac{1}{6}}$

$$\lim_{x \rightarrow 0^+} \ln \left( \frac{\sin x}{x} \right)^{x^2} = \lim_{x \rightarrow 0^+} \ln \left( \frac{1}{x} \right) = \lim_{x \rightarrow 0^+} x^2 \ln \left( \frac{1}{x} \right)$$

$$= \lim_{x \rightarrow 0^+} \frac{\ln(\sin x)}{x^2} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{\sin x} \cdot \cos x - \frac{\sin x}{x^2}}{2x} \quad [$$

$$= \lim_{x \rightarrow 0^+} \frac{x \cos x - \sin x}{2x^2 \sin x} = \lim_{x \rightarrow 0^+} \frac{\frac{0}{0}}{\frac{0}{0}} = \lim_{x \rightarrow 0^+} \frac{\cos x - x \sin x - \cancel{\cos x}}{4x \sin x + 2x^2 \cos x} = \frac{0}{0}$$

$$\begin{aligned} &= \lim_{x \rightarrow 0^+} \frac{-\sin x - x \cos x}{4 \sin x + 4x \cos x + 4x \cos x - 2x^2 \sin x} = \lim_{x \rightarrow 0^+} \frac{-\sin x - x \cos x}{4 \sin x + 8x \cos x - 2x^2 \sin x} \\ &\stackrel{\frac{0}{0}}{=} \lim_{x \rightarrow 0^+} \frac{-\cos x - \cos x + x \sin x}{4 \cos x + 8 \cos x - 8x \sin x - 4x \sin x - 2x^2 \cos x} = \frac{-1 - 1 + 0}{4 + 8 + 0 - 0 + 0} = \frac{-2}{12} \end{aligned}$$

34.  $\lim_{x \rightarrow 0^+} (1 + 2x \sin x)^{\frac{1}{x^2}} = e^2$