Zadaci za samostalan rad:

Zadatak 1. Odrediti prvi izvod sledećih funkcija:

(1) $y = 7x^4$.
R: $y' = 28x^3$.

(2) $y = 2x^3 + 6x - 5$.
R: $y' = 6x^2 + 6$.

(3) $y = \frac{1}{x^4}$.
R: $y' = \frac{-4}{x^5}$.

(4) $y = \frac{1}{x} + \frac{2}{x^3} - \frac{4}{7x^8}$.
R: $y' = -\frac{1}{x^2} - \frac{6}{x^4} + \frac{32}{7x^9}$.

(5) $y = \sqrt{x}$.
R: $y' == \frac{1}{2\sqrt{x}}$.

(6) $y = \sqrt[5]{x^8}$.
R: $y' = \frac{8}{5}\sqrt[5]{x^3}$.

(7) $y = \frac{1}{\sqrt[3]{x}}$.
R: $y' = \frac{-1}{3\sqrt[3]{x^4}}$.

(8) $y = 2\sqrt[4]{x^5} - \frac{1}{3\sqrt[3]{x^7}} + \frac{1}{\sqrt{x}}$.
R: $y' = \frac{-1}{3\sqrt[3]{x^4}}$.

(9) $y = 3e^x + 2\ln x$.
R: $y' = 3e^x + 2\frac{1}{x}$.

(10) $y = e^x \cdot x^2$.
R: $y' = e^x(x^2 + 2x)$.

(11) $y = \frac{3^x}{\ln x}$.
R: $y' = \frac{3^x(\ln 3 \cdot \ln x - \frac{1}{x})}{\ln^2 x}$.

(12) $y = 5\sqrt{x}\ln x$.
R: $y' = \frac{5}{\sqrt{x}}\left(\frac{\ln x}{2} + 1\right)$.

(13) $y = \frac{e^x}{\sqrt{x}} + 3\sqrt[3]{x^7}$.
R: $y' = \frac{e^x\left(\sqrt{x} - \frac{1}{2\sqrt{x}}\right)}{x} + 7\sqrt[3]{x^4}$.

(14) $y = 2\sqrt[3]{x} + \frac{1}{x} - \frac{3}{\sqrt[4]{x^3}}$.
R: $y' = \frac{2}{3\sqrt[3]{x^2}} - \frac{1}{x^2} + \frac{9}{4\sqrt[4]{x^7}}$.

(15) $y = \ln x - \frac{2}{x^3}$.
R: $y' = \frac{1}{x} + \frac{6}{x^4}$.

(16) $y = 3e^x \cdot x^4$.
R: $y' = 3e^x(x^4 + 4x^3)$.

(17) $y = \frac{e^x}{x^4} - \sqrt[5]{x}\ln x$.

R: $y' = \dfrac{e^x(x-4)}{x^5} - \dfrac{1}{5}\dfrac{\ln x}{\sqrt[5]{x^4}} - \dfrac{1}{\sqrt[5]{x^4}}$.

Zadatak 2. Odrediti prvi izvod sledećih složenih funkcija:

(1) $y = \ln^3 x$.
R: $y' = \dfrac{3\ln^2 x}{x}$.

(2) $y = \sqrt{\ln x}$.
R: $y' = \dfrac{1}{2x\sqrt{\ln x}}$.

(3) $y = \sqrt[3]{5x^3 - x}$.
R: $y' = \dfrac{1}{3\sqrt[3]{(5x^3-x)^2}}(15x^2 - 1)$.

(4) $y = (e^x + 6\sqrt{x})^4$.
R: $y' = 4(e^x + 6\sqrt{x})^3\left(e^x + \dfrac{3}{\sqrt{x}}\right)$.

(5) $y = \ln(x^2 + 1)$.
R: $y' = \dfrac{1}{x^2+1}2x$.

(6) $y = \ln\dfrac{1}{x}$.
R: $y' = -\frac{1}{x}$.

(7) $y = \ln(3^x - \sqrt[3]{x})$.
R: $y' = \dfrac{1}{3^x - \sqrt[3]{x}}\left(3^x \ln 3 - \dfrac{1}{3\sqrt[3]{x^2}}\right)$.

(8) $y = \ln\left(\dfrac{\ln x}{\sqrt{x}}\right)$.
R: $y' = \dfrac{2 - \ln x}{2x\ln x}$.

(9) $y = e^{x^3}$.
R: $y' = e^{x^3} \cdot 3x^2$.

(10) $y = e^{e^x}$.
R: $y' = e^{e^x} \cdot e^x$.

(11) $y = e^{\frac{x}{2^x}}$.
R: $y' = e^{\frac{x}{2^x}}\dfrac{1 - x\ln 2}{2^x}$.

(12) $y' = e^{\sqrt[3]{x^4}}$.
R: $y' = \dfrac{4}{3}e^{\sqrt[3]{x^4}}\sqrt[3]{x}$.

(13) $y = \sqrt[3]{\ln x + x}$.
R: $y' = \dfrac{\frac{1}{x} + 1}{3\sqrt{(\ln x + x)^2}}$.

(14) $y = \sqrt[4]{e^{3x}}$.
R: $y' = \dfrac{3e^x}{4\sqrt[4]{e^x}}$.

(15) $y = \ln(x^3 + 2x^2)$.
R: $y' = \dfrac{3x^2 + 4x}{x^3 + 2x^2}$.

(16) $y = e^{\frac{x^2}{e^x}}$.
R: $y' = e^{\frac{x^2}{e^x}} \cdot \dfrac{2x - x^2}{e^x}$.

(17) $y = \ln\sqrt[5]{x^8}$.
R: $y' = \dfrac{8}{5x}$.

(18) $y = 2^{2^x} + 3 \cdot \ln\ln x$.

R: $y' = 2^{2^x} \cdot 2^x \ln^2 2 + \dfrac{3}{x \ln x}$.

(19) $y = \ln(\sqrt{x} - e^{2x} + 3\ln x)$.

R: $y' = \dfrac{1}{\sqrt{x} - e^{2x} + 3\ln x} \cdot \left(\dfrac{1}{2\sqrt{x}} - 2e^{2x} + \dfrac{3}{x}\right)$.

(20) $y = e^{\frac{\ln x}{x^2}}$.

R: $y' = e^{\frac{\ln x}{x^2}} \cdot \dfrac{1 - 2\ln x}{x^3}$.

(21) $y = \ln^8 x + \ln x^8$.

R: $y' = \dfrac{8(\ln^7 x + 1)}{x}$.

Zadatak 3. Za funkciju $f(x) = 3e^x - x^5 + \sqrt{x}$, odrediti $f''(4)$.

R: $f'(x) = 3e^x - 5x^4 + \dfrac{1}{2}x^{-\frac{1}{2}}$,

$f''(x) = = 3e^x - 20x^3 - \dfrac{1}{4}x^{-\frac{3}{2}}$,

$f''(4) = = 3e^4 - 1280 - \dfrac{1}{32}$.

Zadatak 4. Za funkciju $f(x) = e^{2x} - 5\sqrt{x} + 4^x$ odrediti $f''(x)$.

R: $f''(x) = 4e^{2x} - \dfrac{5}{4\sqrt{x^3}} + 4^x \ln^2 4$.

Zadatak 5. Za funkciju $f(x) = \ln^2 x + \ln x^2$ odrediti $f''(x)$

R: $f'(x) = \dfrac{2}{x}(\ln x + 1), \quad f''(x) = -2\dfrac{\ln x}{x^2}$.

Zadatak 6. Za funkciju $f(x) = e^{x^2} + 2x^3 + \sqrt{x}$ odrediti $f''(x)$.

R: $f''(x) = 2e^{x^2}(1 + 4x^2) + 12x - \dfrac{1}{4\sqrt{x^3}}$.