PRVI IZVOD

13. ožujka 2016.

1. Odrediti y' **za:**

1. $y = \frac{1}{\sqrt{x}}$;
2. $y = (x^2-1)(1-x)$;
3. $y = \frac{x^2+1}{3(x^2-1)}$;
4. $y = \frac{\sqrt{x}}{\sqrt{x}+1}$;
5. $y = \frac{1}{x^2+x+1}$;
6. $y = \sin x + \cos x$;
7. $y = \sin^2 x$;
8. $y = e^{-x}$;
9. $y = x^2 e^{-x}$;
10. $y = e^x \cos x$;
11. $y = \frac{e^x}{x-1}$;
12. $y = ln(x^2-2x+1)$;
13. $y = ln \sin x$;
14. $y = x\sin^2(lnx)$;
15. $y = \sqrt{x^2+1} + ln\frac{1-\sqrt{x^2+1}}{x}$.

Rešenje:

1. $y' = (x^{-\frac{1}{2}})' = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2}\frac{1}{\sqrt{x^3}} = -\frac{1}{2x\sqrt{x}}$;
2. $y' = 2x(1-x) + (x^2-1)(-1) = 2x - 2x^2 - x^2 + 1 = -3x^2 + 2x + 1$;
3. $y' = \frac{1}{3}\frac{2x(x^2-1)-2x(x^2+1)}{(x^2-1)^2} = \frac{2x^3-2x-2x^3-2x}{3(x^2-1)^2} = \frac{-4x}{3(x^2-1)^2}$;
4. $y' = (\frac{\sqrt{x}+1-1}{\sqrt{x}+1})' = (1 - \frac{1}{\sqrt{x}+1})' = -\frac{-\frac{1}{2\sqrt{x}}}{(\sqrt{x}+1)^2} = \frac{1}{2\sqrt{x}(\sqrt{x}+1)^2}$;
5. $y' = \frac{-(2x+1)}{(x^2+x+1)^2} = -\frac{2x+1}{(x^2+x+1)^2}$;
6. $y' = \cos x - \sin x$;
7. $y' = 2\sin x\cos x = \sin 2x$;
8. $y' = -e^{-x}$;
9. $y' = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2-x)$;
10. $y' = e^x\cos x - e^x \sin x = e^x(\cos x - \sin x)$;
11. $y' = \frac{e^x(x-1)-e^x}{(x-1)^2} = \frac{e^x(x-2)}{(x-1)^2}$;
12. $y' = \frac{1}{x^2-2x+1}(2x-2) = \frac{2(x-1)}{x^2-2x+1}$;
13. $y' = \frac{1}{\sin x}\cos x = ctgx$;
14. $y' = \sin^2(lnx) + 2x\sin(lnx)\cos(lnx)\frac{1}{x} = \sin^2(lnx) + \sin(2lnx)$;
15. $y' = \frac{2x}{2\sqrt{x^2+1}} + \frac{x}{1-\sqrt{x^2+1}}\frac{-\frac{2x}{2\sqrt{x^2+1}}x-(1-\sqrt{x^2+1})}{x^2} = \frac{x}{\sqrt{x^2+1}} + \frac{x}{1-\sqrt{x^2+1}}\frac{-x^2-\sqrt{x^2+1}+x^2+1}{x^2\sqrt{x^2+1}} =$
$= \frac{x}{\sqrt{x^2+1}} + \frac{1}{x\sqrt{x^2+1}} = \frac{x^2+1}{x\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}}{x}$.

2. Odrediti y'' **za:**

1. $y = x^3 - x + 2$;
2. $y = \sin^2 x$;
3. $y = (x-2)e^{2x}$.

Rešenje:

1. $y' = 3x^2 - 1;$ $y'' = 6x;$
2. $y' = 2\sin x\cos x = \sin 2x;$ $y'' = 2\cos 2x;$
3. $y' = e^{2x} + 2(x-2)e^{2x} = e^{2x}(1 + 2x - 4) = (2x-3)e^{2x};$
$y'' = 2e^{2x} + 2(2x-3)e^{2x} = 2e^{2x}(2x-2) = 4e^{2x}(x-1).$

6. Odrediti jednačinu tangente na krivu $y = 2x^2 - x + 3$ **u tački** $x = 1$.

Rešenje:

Jednačina tangente u tački $x = x_0$ glasi:

$$y - y_0 = y'(x_0)(x - x_0),$$

gde je $y_0 = y(x_0)$.

Kako je nama $x_0 = 1$ imamo:

$y' = 4x - 1 \quad \Rightarrow \quad y'(1) = 3;$ i $y(1) = y_0 = 4;$

pa je jednačina tangente: $y - 4 = 3(x-1),$ tj.

$$y = 3x + 1.$$

L'HOSPITALE-OVO PRAVILO

1.Odrediti granične vrednosti pomoću L'Hospitale-ovog pravila:

1. $\lim_{x\to 0}\frac{e^x-1}{ln(x+1)};$
2. $\lim_{x\to\infty}\frac{\sqrt{x}}{ln^2 x};$
3. $\lim_{x\to 0} x^2 e^{\frac{1}{x^2}};$
4. $\lim_{x\to\infty}(x - xe^{\frac{1}{x-2}})$
5. $\lim_{x\to 0} x^{\sin x};$
6. $\lim_{x\to 1} x^{\frac{1}{1-x}};$
7. $\lim_{x\to 0+}(\frac{1}{x})^x.$

Rešenje:

1. $\lim_{x\to 0}\frac{e^x-1}{ln(x+1)} = \left[\frac{0}{0}\right] = \lim_{x\to 0}\frac{e^x}{\frac{1}{x+1}} = \lim_{x\to 0}(x+1)e^x = 1;$

2. $\lim_{x\to\infty}\frac{\sqrt{x}}{ln^2 x} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty}\frac{\frac{1}{2\sqrt{x}}}{2lnx\cdot\frac{1}{x}} = \lim_{x\to\infty}\frac{x}{4\sqrt{x}lnx} = \lim_{x\to\infty}\frac{\sqrt{x}}{4lnx} =$
$= \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty}\frac{\frac{1}{2\sqrt{x}}}{\frac{4}{x}} = \lim_{x\to\infty}\frac{\sqrt{x}}{8} = \infty;$

3. $\lim_{x\to 0} x^2 e^{\frac{1}{x^2}} = [0\cdot\infty] = \lim_{x\to 0}\frac{e^{\frac{1}{x^2}}}{\frac{1}{x^2}} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to 0}\frac{-\frac{2}{x^3}e^{\frac{1}{x^2}}}{-\frac{2}{x^3}} =$
$= \lim_{x\to 0} e^{\frac{1}{x^2}} = \infty;$

4. $\lim_{x\to\infty}(x - xe^{\frac{1}{x-2}}) = [\infty - \infty] = \lim_{x\to\infty} x(1 - e^{\frac{1}{x-2}}) = [\infty\cdot 0] =$
$= \lim_{x\to\infty}\frac{1-e^{\frac{1}{x-2}}}{\frac{1}{x}} = \left[\frac{0}{0}\right] = \lim_{x\to\infty}\frac{\frac{1}{(x-2)^2}e^{\frac{1}{x-2}}}{-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{-x^2}{(x-2)^2}e^{\frac{1}{x-2}} =$
$= 1;$

5. $\lim_{x\to 0} x^{\sin x} = \left[0^0\right] = A;$
$lnA = ln\lim_{x\to 0} x^{\sin x} = \lim_{x\to 0} lnx^{\sin x} = \lim_{x\to 0}\sin x lnx = [0\cdot\infty] =$
$= \lim_{x\to 0}\frac{lnx}{\frac{1}{\sin x}} = \left[\frac{\infty}{\infty}\right] == \lim_{x\to 0}\frac{\frac{1}{x}}{\frac{-\cos x}{\sin^2 x}} = -\lim_{x\to 0}\frac{\sin^2 x}{x\cos x} =$
$= -\lim_{x\to 0}\frac{\sin x}{x}\frac{\sin x}{\cos x} = -1\cdot 0 = 0;$

$A = e^0 = 1;$

6. $\lim_{x\to 1} x^{\frac{1}{1-x}} = [1^{\pm\infty}] = A;$

   $lnA = ln \lim_{x\to 1} x^{\frac{1}{1-x}} = \lim_{x\to 1} lnx^{\frac{1}{1-x}} = \lim_{x\to 1} \frac{1}{1-x} lnx = \lim_{x\to 1} \frac{lnx}{1-x} =$

   $= \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{\frac{1}{x}}{-1} = -\lim_{x\to 1} \frac{1}{x} = -1;$

   $A = e^{-1};$

7. $\lim_{x\to 0+} (\frac{1}{x})^x = \left[\infty^0\right] = A;$

   $lnA = ln \lim_{x\to 0+} (\frac{1}{x})^x = \lim_{x\to 0+} xln(\frac{1}{x}) = [0 \cdot \infty] = \lim_{x\to 0+} \frac{ln(\frac{1}{x})}{\frac{1}{x}} =$

   $= \left[\frac{\infty}{\infty}\right] == \lim_{x\to 0+} \frac{x(-\frac{1}{x^2})}{-\frac{1}{x^2}} = \lim_{x\to 0+} x = 0;$

   $A = e^0 = 1.$